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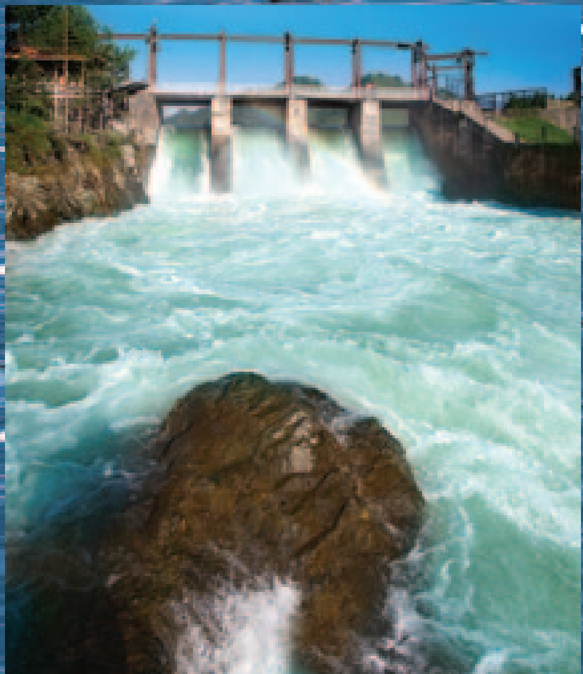
# Water Engineering with the Spreadsheet

## A Workbook for Water Resources Calculations Using Excel



Ashok Pandit, Ph.D., P.E.

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*Water Engineering*
*with the Spreadsheet*

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*To my parents and wife*

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# *Contents*

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# *Preface*



This book was written to show the application of spreadsheets to solve civil engineering problems related to the general area of water resources. The four areas covered are (1) fluid mechanics, (2) hydraulics, (3) hydrology, and (4) stormwater management (or urban hydrology). The purpose of the book is not to replace standard textbooks on fluid mechanics, hydraulics, hydrology, or stormwater management, but to demonstrate how the learning process can be enhanced with the help of spreadsheets. In fact, the book was written with the assumption that readers are at least somewhat knowledgeable in the areas of water resources engineering. For example, it is expected that the reader already has been exposed to concepts such as major and minor losses in pipes and reservoir routing and the continuity, momentum, and energy equations. Therefore, the book does not provide lengthy explanations or derivations of equations but does provide general background information in the four chapters and specific background information within each example description. To develop a deeper understanding of the material covered in this book, you are encouraged to read some of the books listed in the Bibliography.

Most lecturers adopt a two-stage technique when presenting material to students. Background information, important theoretical concepts, and equations are presented in the first stage, usually followed by a second stage in which example problems are solved to illustrate the applications of the material presented in Stage 1. This book is written for readers who have completed Stage 1 and are in Stage 2. Professors can use the material and examples presented in this book to enhance their lecture material and provide more examples to students.

Both students and professionals will find the book useful for these reasons:

- The book provides a large spectrum of examples, from easy to difficult, to explain key fundamentals, analyses, and engineering design.
- All examples are solved with the help of spreadsheets, and readers will have access to these spreadsheets.
- Background information is provided before each section to discuss the theory and equations covered in the examples.
- Background information is also provided within each problem to emphasize some key points regarding the example.
- Where needed, key assumptions are provided within each example.
- All key equations needed to solve the problem are provided.
- A solution procedure is provided within each example. In some problems, readers are led through the *thinking* process needed to come up with an adequate solution procedure (e.g., Examples 1-12 to 1-18).
- Students, and even professors, can benefit from the "what-if" analysis associated with every problem. The "what-if" analysis provides an in-depth

understanding of the problem being examined and can be easily conducted with the help of the spreadsheets.

- The trial-and-error solution procedures adopted in this book best utilize the strength of the spreadsheets and show how even complex solutions can be easily solved using spreadsheets without excessive equation manipulation.
- Trial-and-error solutions are iterative by nature. For the sake of providing clarity, most spreadsheets show solutions where the iterations are done manually until conversion occurs. However, spreadsheet features Goal Seek and Circulation Cells are utilized in some problems to demonstrate how these features can lead to instantaneous conversion.
- Another spreadsheet feature known as "trendlines" is used in several problems. Using trendline equations avoids the tedious task of interpolation while maintaining accuracy. Use of trendline equations goes well with trial-and-error solutions.
- "Assigned" problems are provided in most examples. Readers will have to create new spreadsheets for some of the assigned problems, and this exercise will enhance their understanding of the material.
- If used in the correct way, this book can be an excellent preparation tool for the fundamentals of engineering (FE) and professional engineering (PE) exams.

Finally, all readers, especially students, are strongly encouraged to work the assigned problems that follow most examples. You will require a thorough understanding of the principles and equations to find solutions. You will need to think through the solution process and create new spreadsheets; simply cutting and pasting the spreadsheets provided in this book will not work. Your knowledge will be greatly enhanced if you try to do all the assigned problems.

As a side note, in Excel, the terms Sin$\alpha$, Cos$\alpha$, etc., are calculated by substituting the angle $\alpha$ in radians. An easy way to convert degrees to radians in Excel is by using the radians formula. Alternatively, you can convert degrees into radians by simply multiplying the angle by $\pi/180$.

# 1

# *Fluid Mechanics*

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# Background

This chapter covers two important aspects of fluid mechanics: calculation of forces applied by a fluid on an object (Problems 1-2 through 1-6) and flow through pipe networks (Problems 1-7 through 1-18). A brief background is provided for the material covered in these two areas.

# Force Calculations in Fluid Mechanics

### *Surface and Body Forces*

All civil engineers should be clear about the concepts of "surface" and "body" forces. When two objects are in contact, they *must* apply a force on one another. These forces are called "contact" or "surface" forces. For example, in Fig. 1-1, the fluid is in contact with a rectangular and a triangular door, whereas in Figs. 1-2 and 1-3, it is in contact with the inside of a pipe and the outside of a building. Therefore, the fluid must apply a force on the doors, the pipe, and the building, and these three objects must apply an equal and opposite force on the fluid. The only exception is when engineers are trying to evaluate a limiting situation. In this case, the engineer can make an assumption that the two objects are "just" touching, which implies that they are in contact but not applying a force on one another. As opposed to surface forces, "body" forces are forces that can be applied to an object without contact. Two good examples of a body force are the gravitational force (weight) and the magnetic force.

***Fig. 1-1*** **Fluid inside a confined container (no flow or static condition)**

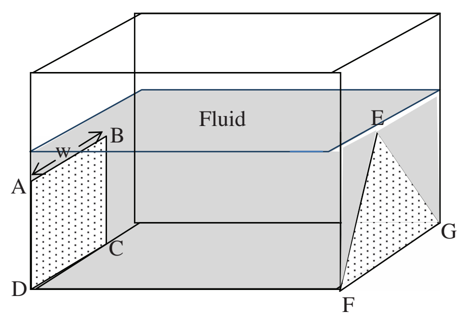



***Fig. 1-2*** **Fluid flow in a pipe (internal flow)**

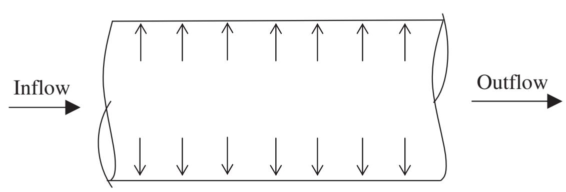


***Fig. 1-3* Fluid flow around a building (external flow)**

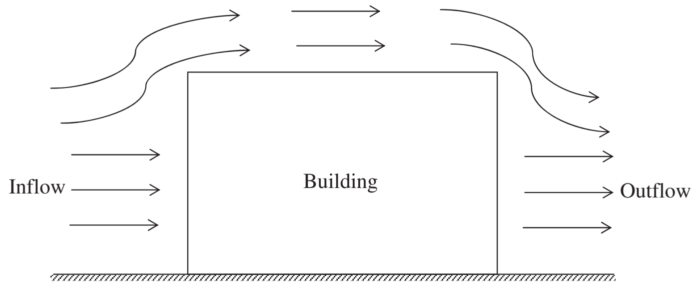
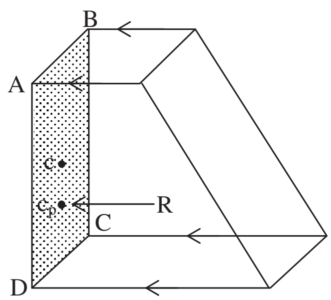



***Fig. 1-4* Pressure distribution (pressure prism) on surface ABCD of Fig. 1-1**




*Distributed and Resultant (Point) Forces in a Static Fluid*

The contact force applied by fluids is a distributed force as shown in Fig. 1-4. The pressure (or force per unit area) distribution on the rectangular door of Fig. 1-1 is shown in Fig. 1-4. A figure showing the pressure distribution on an area is known as a "pressure prism." Thus, Fig. 1-4 depicts the pressure prism on the rectangular door of Fig. 1-1. The distributed force is usually converted to a single resultant (point) force for convenience, and this force is shown as $R$ in Fig. 1-4. Both the distributed force and the resultant force always act *perpendicular* to the plane of the object in *static* fluids. When the fluid is water, the applied resultant force is known as a "hydrostatic force." The magnitude of the resultant concentrated force can be determined by several methods, and two methods are described in Problem 1-2. The point where the resultant concentrated force applies on the surface area is known as the center of pressure, and the method for computing the location of the center of pressure is described in Problem 1-3. As shown in Fig. 1-4, the center of pressure is always *below* the centroid, C, of the submerged object, which in this case is the door, unless the submerged object is *horizontal.*

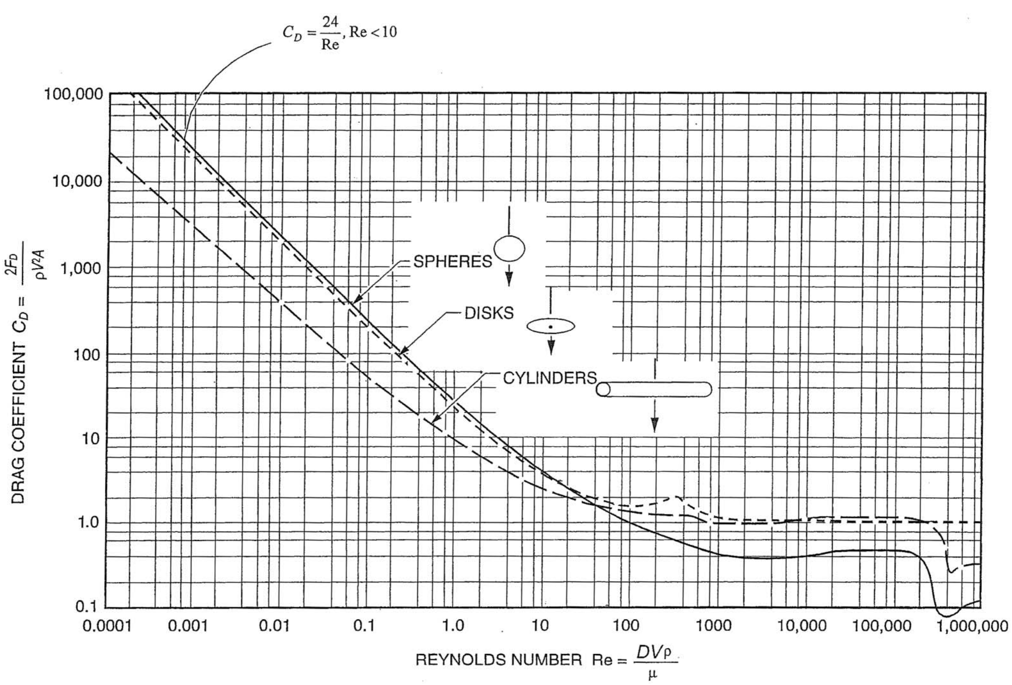
*Internal and External Forces*

The methods and equations used to determine the contact force applied by a static fluid on an object will not work when the fluid is "flowing." Fluid flowing in a restricted space, such as flow inside a conduit, is classified as "internal flow." The force applied by the fluid on the pipe of Fig. 1-2 will be termed an "internal force," as the fluid is confined by a pipe. External flows are defined when there are no spatial constraints on a fluid. The fluid flow around the building in Fig. 1-3 will be classified as "external flow," and the force applied by the fluid on the outer walls of the building will be termed as an "external force." The equations used to calculate contact forces when the fluid is static cannot be used when the fluid is "flowing." Under static conditions, the force applied by a fluid on an object depends *only* on the static pressure distribution. However, when the fluid is flowing, as in internal and external flows, the force applied by the fluid depends on both the static and dynamic pressure distribution.

### *Calculation of Internal and External Forces*

The equation used to calculate internal forces is the "momentum equation." Basically, the momentum equation states that external applied force on an object is equal to the rate of change of momentum. The momentum equation is a vector equation that can be transformed to three scalar equations in the $x$, $y$, and $z$ directions, and the scalar equations typically are used to calculate the internal forces. The external forces can be calculated by the momentum equation and the drag force equation. Use of the drag force equation requires the estimation of the drag coefficient, $C_D$. The value of $C_D$ depends on the Reynolds number ($R_e$) and on the shape of the object on which the force is being exerted. The value of $C_D$ can be read from Fig. 1-5. The drag force equation can be used under the following conditions: (a) the submerged object is stationary and the fluid is moving, such as air flow around a house; (b) the submerged object is moving and the fluid is stationary, such as a moving car when the surrounding air is

***Fig. 1-5* Drag coefficients for spheres, disks, and cylinders (© National Council of Examiners for Engineering and Surveying. Reproduced with permission.)**



stationary; or (c) the submerged object and the fluid are both moving, such as a moving car in a headwind.

### *Problems 1-1 through 1-6*

Problems 1-1 to 1-6 provide examples of how to calculate forces applied by fluids under no flow (static), internal, and external flow conditions.

## General Background for Flow in Pressure Pipes

Pipes flowing full are known as pressure pipes. A brief background of flow through pressure pipes is provided here, as Problems 1-7 through 1-18 are all related to this type of flow. The reader is referred to any standard textbook in fluid mechanics for more detailed information.

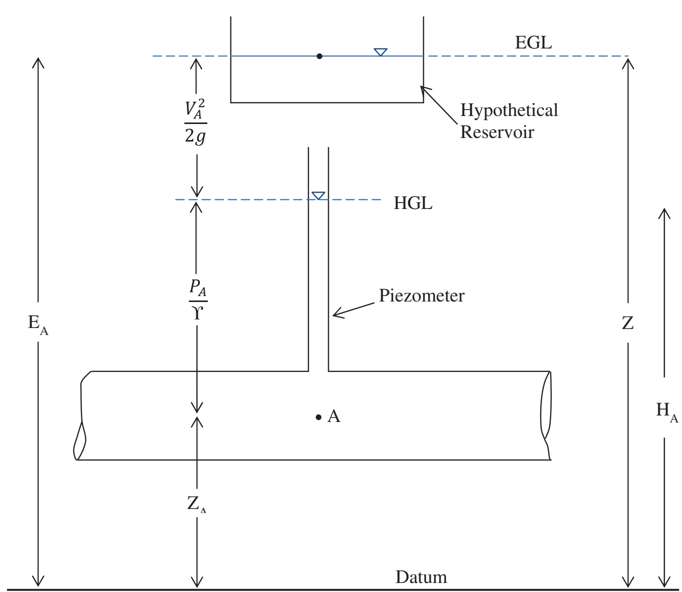
### *Mechanical Energy (ME)*

Fig. 1-6 shows water flowing in a section of a pipe. At any point A in the pipe, the mechanical energy per unit weight of fluid, $E_A$, is given as

$$E_A = p_A/\gamma + V_A^2/(2g) + z_A \tag{1-1}$$

where $p_A$ and $V_A$ are the respective pressure and velocity at A, $z_A$ is the elevation of Point A from an arbitrary datum, $\gamma$ is the specific weight of the fluid, and $g$ is the acceleration due to gravity. The terms $p/\gamma$, $V^2/(2g)$, and $z$ represent the pressure energy, kinetic energy, and potential energy per unit weight of the fluid. Engineers find it convenient to represent mechanical energy as mechanical energy per unit weight of the fluid, because energy per unit weight has

***Fig. 1-6*** **Conversion of any point in a pipe system to a hypothetical reservoir**



convenient units of length. For convenience, the mechanical energy per unit weight from here on will be simply termed as mechanical energy (ME).

As shown in Fig. 1-6, the energy grade line (EGL) represents the mechanical energy at any point in the system from the selected datum. If one installs a piezometer in the pipe, as shown in Fig. 1-6, the water will rise to some level in the piezometer. The level to which water rises indicates the hydraulic head, $H_A$, at Point A and also indicates the location of the hydraulic grade line (HGL). The hydraulic head at any point A is given by

$$H_A = p_A/\gamma + z_A \tag{1-2}$$

Thus, the difference between the EGL and HGL is $V_A^2/(2g)$.

### *Reservoirs*

Under steady state conditions, the velocity inside the reservoir is zero. Therefore, the ME at any location in a reservoir is given as

$$E = p/\gamma + z \tag{1-3}$$

Because the gauge pressure at the surface of a reservoir is zero, $E$ at any point on the surface of a reservoir can be determined as

$$E = z \tag{1-4}$$

Any location in the water distribution system can be converted into a hypothetical "reservoir." For example, Point A in Fig. 1-6 can be converted to a hypothetical reservoir that has an elevation $Z$. A reservoir can represent a fire hydrant, a city, a pump station, or a water treatment plant. Converting certain points of a water distribution system, such as pump stations or water treatment plants, into "reservoirs" sometimes simplifies analyzing the problem.

### *Major and Minor Losses*

A fluid loses mechanical energy as it travels through a pipe. The frictional losses caused by the contact between the fluid and the pipe are known as "major head losses." "Minor head losses," conversely, occur because of several factors, such as pipe expansion, pipe contraction, valves, and fittings. The words "major" and "minor" may imply that major losses are greater than minor losses. However, that is not always true. In a typical sprinkler system, in which there are many bends and valves, the minor losses may exceed the major losses.

Two methods commonly used to calculate major head loss are the Darcy-Weisbach method and the Hazen-Williams method. The advantage of the Darcy-Weisbach method is that it can be used for laminar, transitional, and fully rough turbulent flows, whereas the Hazen-Williams method is used mainly when the flow is expected to be fully turbulent. When using the Darcy-Weisbach method, the friction factor, $f$, can be calculated using several equations (that are discussed in Problem 1-7). Minor losses, $h_{mi}$, are calculated by

$$h_{mi} = K'V^2/(2g) \tag{1-5}$$

where $K'$ is the sum of minor loss coefficients such as the entrance and exit loss coefficients.

### *Important Parameters in Pipe Flow Problems*

There are several parameters in pipe networking problems. Some of the common parameters are flowrate ($Q$), velocity ($V$), mechanical energy (ME), pipe length ($L$), pipe diameter ($d$), friction factor ($f$), the Hazen-Williams constant ($C_H$), minor loss coefficient ($K$), the head provided by a pump ($h_p$ if a pump is being used), major head losses ($h_f$), minor head losses ($h_{mi}$), total head losses ($h_t$), Reynolds number ($\mathrm{R}_e$), and relative roughness ($R_r$). The total head losses are the sum of minor and major losses. In each problem many of the parameters are unknowns. Also, in each problem a number of equations are available to find the values of the unknown parameters. A problem will have a unique solution *only* if the total number of unknowns is equal to the number of available equations.

### *Available Equations*

It becomes easier to find the appropriate solution procedure if one can identify all the unknowns and the available equations. Some of the equations available in pipe networking problems are (a) energy equations, (b) continuity equations, (c) Darcy-Weisbach or Hazen-Williams equations for calculating minor losses, (d) minor loss equations, similar to Eq. (1-4), and (e) friction factor equations, which are discussed in Problem 1-7. The term "friction factor equation" is used in this book to describe an equation that was developed primarily to determine the friction factor. In any problem, the available energy, major head loss, and friction factor equations are equal to the number of pipes; the available continuity equations are equal to the number of junctions in the system; and the available minor loss equations are equal to the number of minor losses being considered. Note that if several energy equations can be written in a problem, the number of *useful* energy equations will be equal to the number of pipes; the remaining equations will be redundant. Also, note that the equations may have to be reorganized to have the unknown variable on the left-hand side of the equation.

### *Problems 1-7 through 1-18*

Problems 1-7 through 1-18 discuss flow through fully flowing or "pressure" pipes. The complexity of these problems increases from Problem 1-7 to 1-18. For example, Problem 1-8 discusses gravity flow between two reservoirs through a single pipe, whereas Problem 1-18 discusses water being *pumped* from a reservoir to two reservoirs through multiple pipes. A summary of the assumptions/specifications, unknowns within each problem, a description of the equations, and the type of solution procedure used to solve Problem 1-8 through 1-11 and Problems 1-12 through 1-14 are provided in Tables A-1 and A-2, respectively. A similar summary for problems 1-15 through 1-18 is provided in Tables A-3 through A-6.

# Problem 1-1: Estimation of Water Density and Dynamic Viscosity Using Trendline Equations

## *Problem Statement*

Determine appropriate trendline equations for density, $\rho_w$, dynamic viscosity, $\mu_w$, and saturated vapor pressure, $e_{sw}$, of water with respect to temperature, $T$. Plot the trendline equations and find the corresponding $R^2$ value. Ensure that the values predicted by the trendline equations are within 3% of the actual values.

## *Background*

A very convenient feature of spreadsheets is the capability of developing polynomials or "trendlines" by using curve-fitting techniques and finding equations for these trendlines. These trendline equations overcome the problem of interpolation. For example, most textbooks provide the values of the density ($\rho_w$), dynamic viscosity ($\mu_w$), and saturated vapor pressure ($e_{sw}$) of water at intervals of 5°C (see "given" values in Spreadsheet 1-1), but one must use interpolation—usually linear interpolation—to find the density and kinematic viscosity values at intermediate values of temperature. The trendline equations overcome this problem of interpolating and are very convenient to use in spreadsheets.

## *Equations*

The trendline equations and equations used to calculate percentage differences are

1. $\rho_{wc} = -0.0036T^2 - 0.0658T + 1{,}000.6$
2. $\%\text{diff.} = (\rho_w - \rho_{wc})/\rho_w * 100$
3. $\mu_{wc} = 3.356 * 10^{-11}T^4 - 9.250 * 10^{-9}T^3 - 9.925 * 10^{-7}T^2 + 1.777 * 10^{-3}$
4. $\%\text{diff.} = (\mu_w - \mu_{wc})/\mu_w * 100$
5. $e_{swc} = 0.1613 * T^3 - 8.5928 * T^2 + 262.33T - 226.7$
6. $\%\text{diff.} = (e_{sw} - e_{swc})/e_{sw} * 100$

where $\rho_w$, $\mu_w$, and $e_{sw}$ are the given values and vapor pressure, and $\rho_{wc}$, $\mu_{wc}$, and $e_{swc}$ are the calculated values of the water density, dynamic viscosity, and vapor pressure, respectively.

## *Solution Procedure*

The graphs between $\rho_w$, $\mu_w$, and, $e_{sw}$ versus $T$ (in °C), along with the trendline equations and $R^2$ values, are respectively shown in Charts 1, 2, and 3 of Spreadsheet 1-1. Water density is plotted using using a second-order polynomial, dynamic viscosity values are plotted using a fourth-order polynomial, and vapor pressure is plotted using a third-order polynomial. These polynomials provide the desired accuracy although other more accurate solutions may be possible.

### *Solution*

The accompanying spreadsheet uses trendline equations that are accurate within 0.1% for water density, within 3% for water dynamic viscosity, and within 1% for vapor pressure for temperature values ranging from 0°C to 100°C (Spreadsheet 1-1). When using trendlines, the accuracy of the results may be highly dependent on the number of decimal points used in generating the equations. For example, Eq. 2 was determined up to 10 decimal figures (Chart 2), although only three decimal figures are reported here. As expected, both the water and air density values decrease with an increase in temperature (Spreadsheet 1-1). The dynamic viscosity of water also decreases with an increase in temperature, as water is a liquid (gasses show an opposite trend). Finally, the vapor pressure increases with an increase in temperature. Note that the vapor pressure has units of absolute pressure.

### *Assigned Problem 1-1-1*

Using the values shown in Spreadsheet 1-1, determine a trendline equation for air density, $\rho_a$, that can predict values within 1% of the given values.

### *Assigned Problem 1-1-2*

Determine the density, dynamic viscosity, and vapor pressure of water at $T = 6°C$, 17°C, and 28°C using the developed trendline equations.

# Problem 1-2: Calculation of Hydrostatic Forces on a Planar Rectangular Surface by Two Methods

## *Problem Statement*

Determine the hydrostatic force and its $x$ and $y$ components, acting on the *submerged* portion, AO, of the inclined rectangular gate, AB, shown in Fig. 1-7, using two methods. The water depth, $y = 6$ m. The gate is 2 m wide (perpendicular to the page) and is inclined at an angle, $\theta = 45$ deg. The unit weight of water is 9.81 kN/m$^3$.

## *Background*

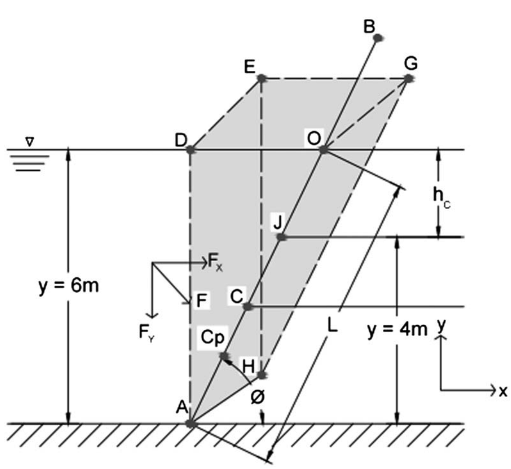
Two methods are described to estimate the magnitude of the resultant hydrostatic force. Both methods can be used to compute the magnitude of the hydrostatic force on an object *of any shape*.

## *Equations for Method 1*

The equations for Method 1 are

1. $L = y/\sin\theta$
2. $h_c = y/2$
3. $p_c = \gamma h_c$
4. $A = lw$
5. $F = p_c A$
6. $F_x = F\sin\theta$
7. $F_y = F\mathrm{Cos}\theta$

***Fig. 1-7*** **Description of Problem 1-2**



where $L = \text{AO}$, $F =$ the hydrostatic force, $p_c$ is the pressure at the centroid of the *submerged* area of the object that is Point C, $A$ is the area of the *submerged* part of the object, $\gamma$ is the unit weight of water, $h_c$ is the vertical distance from the free surface to the centroid of the *submerged* surface, $F_x$ and $F_y$ are the respective horizontal and vertical components of $F$, and $\theta$ is the angle of inclination of the object from the $x$ axis. Note that as shown in Fig. 1-7, the force $F$ is a "point force" that acts at the center of pressure (CP) and not at C. The point of application of $F$ is discussed in Problem 1-3. Also, Eqs. 3 and 5 are applicable to all shapes (not just rectangular objects). A common mistake that students make is thinking that $F = p_{cp}A$ (instead of $p_cA$), because $F$ acts at CP and not at C.

## *Equations for Method 2*

The equations for Method 2 are

8. $p_1 = \gamma h_1$
9. $p_2 = \gamma h_2$
10. $A_v = Lw\text{Sin}\theta$
11. $F_x = p_1 A_v + (p_2 - p_1)(A_v/2)$
12. $V_f = 0.5(L\text{Sin}\theta)(L\text{Cos}\theta)w$
13. $F_y = (V_f)(\gamma)$
14. $F = (F_x^2 + F_y^2)^{0.5}$

where $p_1$ and $p_2$ are the respective pressures at the top (O) and bottom (A) of the submerged area of the object, $A_v$ is the vertical projected area of the *submerged* part of the object, $V_f$ is the volume of fluid above the submerged surface (the shaded area of Fig. 1-7), and $h_1$ and $h_2$ are the respective vertical distances from the free surface to the top and bottom of the submerged area. In this problem, $h_1 = 0$ and $h_2 = y$. Note that $F_y$, the vertical force acting on the submerged surface, is $(V_f)(\gamma)$, which is equal to the weight of the fluid above the submerged surface. The equations used in Methods 1 and 2 will be the same for an object of any shape (not necessarily a rectangular shape) with the exceptions of Eqs. 2, 10, 12, and 13, which will have to be rewritten for objects not having a rectangular shape.

## *Hint*

Method 1 is preferable, as it is simpler, but Eqs. 11 and 13 are the ones that are accessible to students in the fundamentals of engineering (FE) exam.

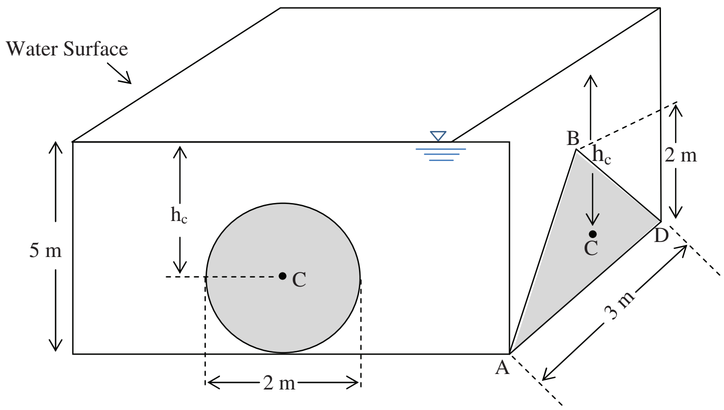
## *Solution Procedure*

The solution procedure is shown in Spreadsheet 1-2.

## *Solution*

Both methods determine a hydrostatic force of 499.4 kN.

***Fig. 1-8*** **Description of Problems 1-2-1 and 1-2-2**




*What-If Analyses*

A what-if analysis shows that

(a) $F_x$ does not change as θ increases to (say) 60 or 75 deg (why?),
(b) $F_y$ decreases as θ increases (why?), and
(c) $F$ increases as the depth of water ($y$) increases (why?).

*Assigned Problem 1-2-1*

Rewrite the equations and create a spreadsheet to determine, using Method 1, the values of $F$, $F_x$, and $F_y$ acting on the 2-m diameter circular gate shown in Fig. 1-8.

*Assigned Problem 1-2-2*

Rewrite the equations and create a spreadsheet to determine, using Method 1, the values of $F$, $F_x$, and $F_y$ acting on the triangular gate ABD shown in Fig. 1-8.

*Assigned Problem 1-2-3*

Determine the hydrostatic force and its $x$ and $y$ components, acting on the *submerged* portion, AJ, of the inclined rectangular gate, AB, shown in Fig. 1-7, using Methods 1 and 2. The vertical distance of J from the hinge is 4 m (Fig. 1-7). The gate is 2 m wide (perpendicular to the page) and is inclined at an angle, $\theta = 45$ deg. The unit weight of water is 9.81 kN/m$^3$.

# Problem 1-3: Calculation of the Location of the Hydrostatic Force on a Planar Triangular Surface

## *Problem Statement*

A resultant hydrostatic force, $F$, is acting on a planar triangular surface, AB, shown in Fig. 1-9. The triangular gate has a height, $h = 5$ m, and a width, $w = 4$ m (Fig. 1-9). The gate is tilted at an angle $\theta = 60$ deg (Fig. 1-9). The vertical distance, $h'$, between the top of the gate and the free surface is 6 m. The centroid of the gate is located at C. Determine (a) the distance between the centroid of the gate and the center of pressure (CP) and (b) the location of the resultant hydrostatic force from the bottom of the gate (Point B in Fig. 1-9).

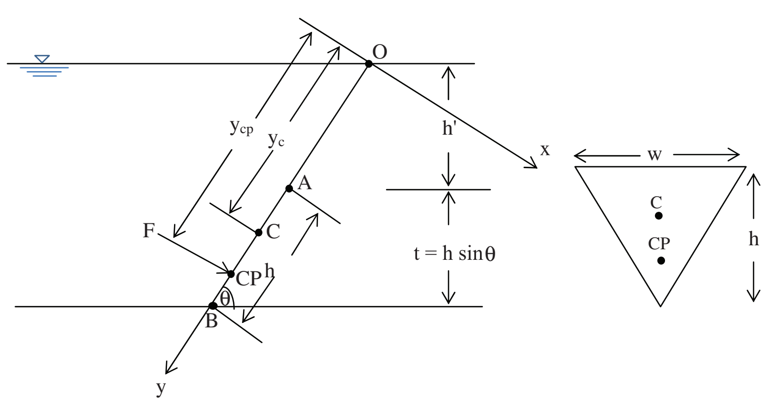
## *Background*

Engineers must know the location of the hydrostatic force for different types of analyses (for example, to determine the overturning moment on a dam). The point where the free surface intersects the plane of the submerged object is defined as the origin (Point O in Fig. 1-9). In these types of problems, the $y$ axis is defined as the axis along the plane of the submerged object and the $x$ axis is perpendicular to the $y$ axis but is coming out of the plane of the paper (Fig. 1-9).

## *Equations*

1. $A = 0.5wh$
2. $t = h\mathrm{Sin}\theta$
3. $y_c = [h' + (2/3)(t)]/\mathrm{Sin}\theta$
4. $I_{\mathrm{bar}} = (1/36)(wh^3)$
5. $y_{cp} = y_c + I_{\mathrm{bar}}/(y_c A)$
6. $D = y_{\mathrm{cp}} - y_c$
7. $\mathrm{BP} = [(h' + t)/\mathrm{Sin}\theta] - y_{cp}$

***Fig. 1-9*** **Description of Problem 1-3**



where $A$ is the area of the submerged gate, $t$ is the vertical component of $h$, $y_c$ is the distance between the centroid of the submerged object (in this case, the triangular gate) to the origin O, $y_{cp}$ is the distance between the center of pressure, CP, and the origin O, $I_{\text{bar}}$ is the moment of inertia of the submerged area about an axis that passes through its centroid and is parallel to the $x$ axis, and $D$ is the distance between CP and C.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 1-3.

### *Solution*

The distance between C and CP is 0.16 m, and the distance from B to CP = 3.17 m. Incidentally, it can be shown that $y_{cp}$ is always greater than $y_c$, which implies that the center of pressure is always lower than the centroid (unless the submerged surface is horizontal).

### *What-If Analyses*

Two interesting questions that come up are these:

(a) What happens to the distance between the center of pressure and centroid ($y_{cp} - y_c$) when the gate is rotated? To answer this question, determine the distance $y_{cp} - y_c$ for $\theta = 30$ deg and 5 deg; all other parameters remain the same as before. (Answers: 0.10 m, 0.02 m)

(b) What happens to the distance between the center of pressure and centroid ($y_{cp} - y_c$) when water height above the gate is increased; does CP get closer to the centroid or does it go farther away? Determine the difference between $y_{cp}$ and $y_c$ for $h' = 8$ m and 20 m. All other parameters remain the same as before ($\theta = 60$ deg). (Answers: 0.13 m, 0.06 m)

The analysis shows that the center of pressure moves closer to the centroid as the inclined surface becomes flatter and as the water height increases.

### *Assigned Problem 1-3-1*

Rewrite the equations and create a spreadsheet to determine the location of the resultant hydrostatic force if the triangular gate shown in Fig. 1-9 was replaced by a 4 m diameter circular gate inclined at an angle of $\theta = 60°$. Note that Eqs. 1, 3, and 4 will change.

# Problem 1-4: Calculation of Forces during Internal Flow–Application of the Momentum Equation in the Horizontal Plane

## *Problem Statement*

A pipe, bent in the horizontal plane as shown in Fig. 1-10 ($\theta = 120$ deg; $\alpha = 60$ deg), discharges water into the air at atmospheric pressure. The pipe diameters at Sections 1 and 2 are $d_1 = 0.6$ m and $d_2 = 0.3$ m. The flowrate, $Q$, in the pipe is $0.7\ \text{m}^3/\text{s}$. The upstream pressure is $p_1 = 47\ \text{kN/m}^2$. Determine the $x$ and $y$ components ($F_{wx}$ and $F_{wy}$) of the force exerted by the water on the pipe. The density of water, $\rho$, is $1{,}000\ \text{kg/m}^3$.

## *Background*

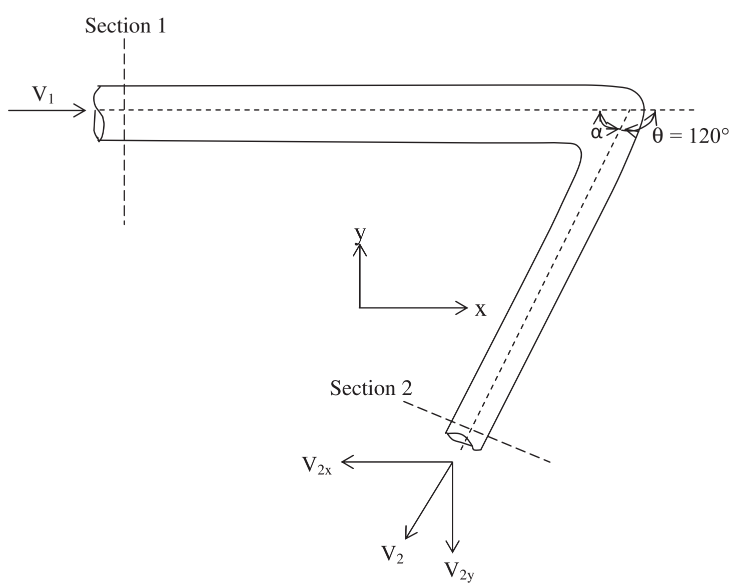
Because the flowing fluid is constrained by a pipe, it will apply an *internal force* on the pipe. To determine $F_{wx}$ and $F_{wy}$, one needs to use the scalar momentum equations in the $x$ and $y$ directions. See Eqs. 15 and 16.

## *Equations*

The complete set of equations for calculating $F_{wx}$ and $F_{wy}$ are

1. $A_1 = \pi d_1^2/4$
2. $A_2 = \pi d_2^2/4$
3. $V_1 = Q/A_1$
4. $V_2 = Q/A_2$

***Fig. 1-10*** **Description of Problem 1-4**



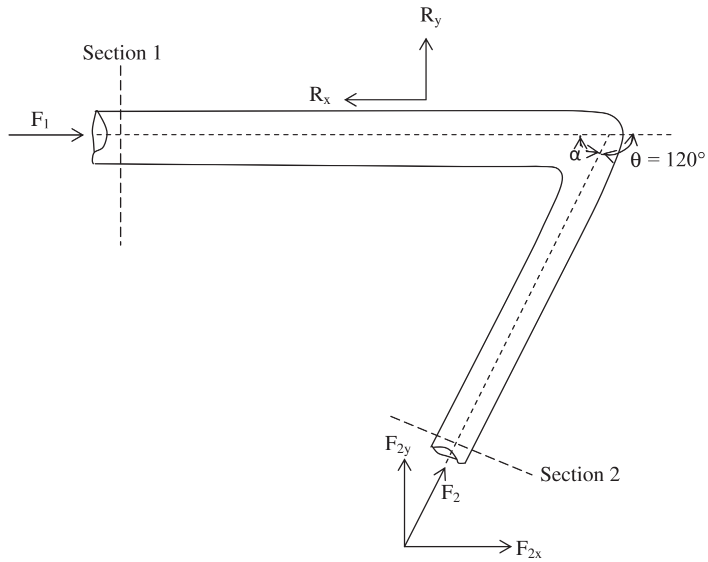
5. $V_{1x} = V_1$
6. $V_{2x} = V_2 \text{Cos}\alpha$
7. $V_{1y} = 0$
8. $V_{2y} = V_2 \text{Sin}\alpha$
9. $F_1 = p_1 A_1$
10. $F_2 = p_2 A_2$
11. $F_{1x} = F_1$
12. $F_{2x} = F_2 \text{Cos}\theta$
13. $F_{1y} = 0$
14. $F_{2y} = F_2 \text{Sin}\alpha$
15. $R_x = F_{1x} + F_{2x} + (V_{1x} Q\rho + V_{2x} Q\rho)/1{,}000$
16. $R_y = -F_{1y} - F_{2y} - (V_{1y} Q\rho + V_{2y} Q\rho)/1{,}000$
17. $F_{wx} = -R_x$
18. $F_{wy} = -R_y$

where $V_1$, $V_2$, and $p_1$, $p_2$, are the respective velocities and pressures at Sections 1 and 2. $F_1$ and $F_2$ are the forces applied by the adjacent water on the fluid inside the bend at Sections 1 and 2, $F_{1x}$, $F_{1y}$, and $F_{2y}$ are the $x$ and $y$ components of $F_1$ and $F_2$, respectively. $R_x$ and $R_y$ are the forces applied *by the pipe on the fluid*. The forces applied by the water on the pipe ($F_{wx}$ and $F_{wy}$) will be equal in magnitude but opposite in direction to the forces applied by the pipe on the water as indicated in Eqs. 17 and 18. Eqs. 15 and 16 were derived by writing the momentum equations in the $x$ and $y$ directions using the free body diagram (FBD) of the *fluid* inside the bent pipe, which is shown in Fig. 1-11. Notice that forces $F_{wx}$ and $F_{wy}$ are not shown in the FBD, because they represent forces that are acting *on the pipe* by the fluid (one should show only those forces that are acting on the fluid, as this is a FBD of the fluid). In this problem, it is arbitrarily assumed that $R_x$ is *directed to the left* and $R_y$ is *directed upward*. It is possible to assume that $R_x$ and $R_y$ are directed in different directions (e.g., $R_x$ can be assumed to be directed to the right). By definition, the directions of $F_{wx}$ and $F_{wy}$ have to be *opposite* to the assumed directions for $R_x$ and $R_y$. Therefore, once the directions of $R_x$ and $R_y$ are selected as shown in Fig. 1-10, $F_{wx}$ *must* be directed to the right and $F_{wy}$ *must* be directed downward. The directions of the positive $x$ and $y$ axes are also shown in Fig. 1-11, which are also selected in an arbitrary manner. When expanding the momentum equation, one needs to be careful in assigning the correct sign to the scalar terms $V_{1x}$, $V_{2x}$, $V_{1y}$, $V_{2y}$, $F_{1x}$, $F_{2x}$, $F_{1y}$, and $F_{2y}$. These terms should be positive if oriented toward the positive $x$ and $y$ axes and negative otherwise. For example, $V_{2x}$ and $V_{2y}$ should be assigned a negative sign as they are oriented *opposite* to the positive $x$ and $y$ axes. In Eqs. 15 and 16, the division by 1,000 is done to convert the force units to kN.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 1-4.

***Fig. 1-11* Free body diagram of the fluid between Section 1 and 2 inside the bent pipe**




*Hint*

One has to first calculate $R_x$ and $R_y$ and *then* calculate $F_{wx}$ and $F_{wy}$.

*Solution*

The solution is shown in Spreadsheet 1-4; $F_{wx} = -18.49$ kN, $F_{wy} = 6$ kN. Because $F_{wx}$ is negative and $F_{wy}$ is positive, $F_{wx}$ is acting to the left and $F_{wy}$ is acting downward.

*What-If Analyses*

Determine the effect of θ on the force applied by the water on the pipe if

(a) The pipe has no bend, i.e., $\theta = 0$ ($\alpha = 180$);
(b) The pipe makes a U-turn bend, i.e., $\theta = 180$ ($\alpha = 0$); and
(c) The pipe makes a 90-deg bend, i.e., $\theta = 90$ ($\alpha = 90$).

Calculate $F_{wx}$ and $F_{wy}$ for all three cases.

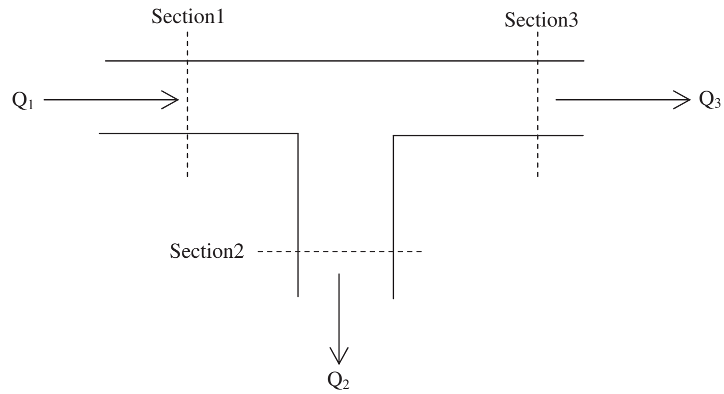
*Assigned Problem 1-4-1*

Why is $F_{wy} = 0$ for (a) and (b) in the what-if analysis?

*Hint*

It has something to do with change of momentum.

***Fig. 1-12*** **Description of Problem 1-4-4**




### *Assigned Problem 1-4-2*

Determine the values of θ at which $F_{wx}$ will be (i) minimum or (ii) maximum. What do the answers have to do with change of momentum?

### *Assigned Problem 1-4-3*

Determine the values of θ at which $F_{wy}$ will be maximum. What does the answer have to do with change of momentum?

### *Assigned Problem 1-4-4*

Water is flowing through a horizontal T-section as shown in Fig. 1-12. $Q_1 = 0.25\ \text{m}^3/\text{s}$, $Q_2 = 0.10\ \text{m}^3/\text{s}$, $p_1 = 100\ \text{kPa}$, $p_2 = 70\ \text{kPa}$, $p_3 = 80\ \text{kPa}$, $d_1 = 15\ \text{cm}$, $d_2 = 7\ \text{cm}$, and $d_3 = 15\ \text{cm}$. Determine the forces applied by the fluid on the T-section.

### *Hints*

Eqs. 15 and 16 need to be modified, as there are two exits rather than just one exit as in Problem 1-4. Also, the continuity equation will be needed to estimate $Q_3$.

# Problem 1-5: Calculation of Forces during Internal Flow–Application of Momentum Equation in the Vertical Plane

### *Problem Statement*

A pipe, bent at 60 deg from the $z$-axis in the vertical plane (Fig. 1-13), discharges water into the air at atmospheric pressure. The pipe diameters at Sections 1 and 2 are $d_1 = 0.6\,\text{m}$ and $d_2 = 0.2\,\text{m}$. The flowrate, $Q$, in the pipe is $0.8\,\text{m}^3/\text{s}$. The upstream pressure, $p_1 = 50\,\text{kN/m}^2$. Determine the $z$ component ($F_{wz}$) of the force exerted by the water on the bent pipe. The density, ρ, of water is $1{,}000\,\text{kg/m}^3$. The weight of the bend, $W_b$, is 5 kN and the volume of water, $V_w$, in the bend is $1.5\,\text{m}^3$.

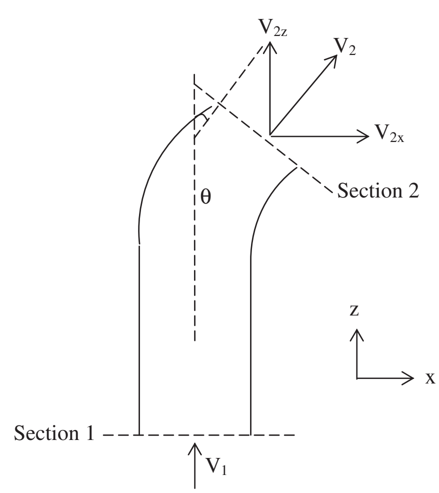
### *Background*

The main difference between this problem and the previous problem is that because the desired force component is in the vertical plane, you must expand the momentum equation in the $z$ direction, must include the weight of the bent pipe, and must include the weight of the fluid within the bent pipe in computing the desired force. Therefore, all equations are similar to Problem 1-4 except for Eq. 11, which is the scalar momentum equation in the $z$ direction for this problem.

### *Equations*

1. $A_1 = \pi d_1^2/4$
2. $A_2 = \pi d_2^2/4$
3. $V_1 = Q/A_1$

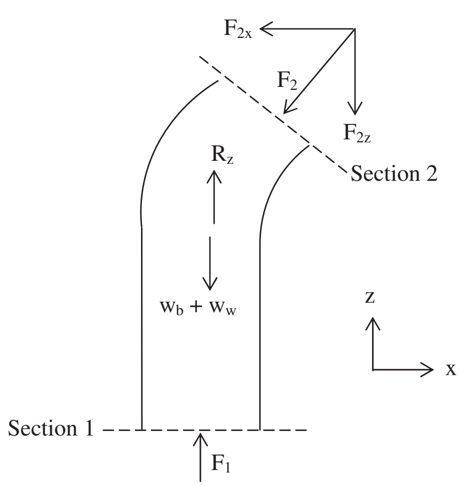
***Fig. 1-13*** **Description of Problem 1-5**



4. $V_2 = Q/A_2$
5. $V_{1z} = V_1$
6. $V_{2z} = V_2\text{Cos}\theta$
7. $F_1 = p_1 A_1$
8. $F_2 = p_2 A_2$
9. $F_{1z} = F_1$
10. $F_{2z} = F_2\text{Cos}\theta$
11. $R_z = -F_{1z} + F_{2z} + W_b + V_w \rho g/1{,}000 - (V_{1z}Q\rho - V_{2z}Q\rho)/1{,}000$
12. $F_{wz} = -R_z$

where $V_1$, $V_2$, and $p_1$, $p_2$, are the respective velocities and pressures at Sections 1 and 2. $F_1$ and $F_2$ are the forces applied by the adjacent water on the fluid inside the bend at Sections 1 and 2, $F_{1z}$ and $F_{2z}$ are the $z$ components of $F_1$ and $F_2$, respectively. Because $R_z$ is the force applied *by the pipe on the fluid*, the force applied by the water on the pipe, $F_{wz}$, will be equal in magnitude but opposite in direction to $R_z$ as indicated in Eq. 12. Eq. 11 was derived by writing the momentum equation in the $z$ direction using the free body diagram (FBD) of the *fluid* inside the bent pipe, which is shown in Fig. 1-14. In this problem, it is arbitrarily assumed that $R_z$ is directed *upward*. It is possible to assume that $R_z$ is acting downward. By definition, the direction of $F_{wz}$ will have to be *opposite* to the assumed direction for $R_z$. Therefore, once the direction of $R_z$ is selected upward, $F_{wz}$ *must* be directed in the downward direction. The direction of the positive $z$ axis also is shown in Fig. 1-14. This direction also is selected in an arbitrary manner. In other words, it could have been assumed to be acting in the downward direction. When expanding the momentum equation, one needs to be careful in assigning the correct sign to the scalar terms $V_{1z}$, $F_{1z}$, $W_b$, and $W_w$. These terms should be positive if oriented toward the positive $z$ axis and negative otherwise. For example, $V_{1z}$ should be assigned a positive sign, whereas $V_{2z}$ should be assigned a negative sign. In Eq. 11, the division by 1,000 is done to convert the force units to kN.

***Fig. 1-14*** **Free body diagram of the fluid inside the bent pipe**



### *Solution Procedure*

The solution procedure is shown in Spreadsheet 1-5.

### *Solution*

$F_{wz} = -16.44$ kN, and, because $F_{wz}$ is negative, it is acting in the upward direction.

### *What-If Analyses*

Determine the effect of the following:

(a) The flowrate on $F_{wz}$ by plotting a graph of $F_{wz}$ versus $Q$; $Q$ should have a range of 0.4 $m^3/s$ to 1.6 $m^3/s$. All other parameters remain the same as in Problem 1-5.

(b) The diameter $d_2$ on $F_{wz}$ by plotting a graph of $F_{wz}$ versus $d_2$; the range of $d_2$ is from 0.1 m to 0.2 m. Why does the magnitude of $F_{wz}$ increase as $d_2$ decreases?

### *Assigned Problem 1-5-1*

Determine the force $F_{wz}$ if (a) $\theta = 90$ deg (the pipe makes a 90-deg bend); (b) $\theta = 180$ deg (the pipe makes a U-bend); and (c) $\theta = 0$ deg (there is no bend). Why is the magnitude of $F_{wz}$ greater for $\theta = 180$ deg as compared to $\theta = 0$ deg?

# Problem 1-6: Calculation of Forces for External Flows

### *Problem Statement*

Determine the force exerted on a 30 m long 0.02 m diameter pipe, which is vertically submerged in a river (Fig. 1-15). The current velocity, $V$, perpendicular to the pipe, is $4\,\mathrm{m/s}$. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are $999.1\,\mathrm{kg/m^3}$ and $1.138\times10^{-3}\,\mathrm{kgs^{-1}/m}$, respectively.

### *Background*

Because the river flow is not constrained by a conduit, this problem is an external flow problem and the drag force equation (Eq. 3) is required.

### *Equations*

1. $A = Ld$
2. $\mathrm{R}_e = Vd\rho/\mu$
3. $F_d = (C_D)A\rho V^2/2$

where $A$ is the *projected* area, $d$ is the diameter, and $L$ is the length of the pipe. The value of $C_D$ can be obtained from Fig. 1-5.

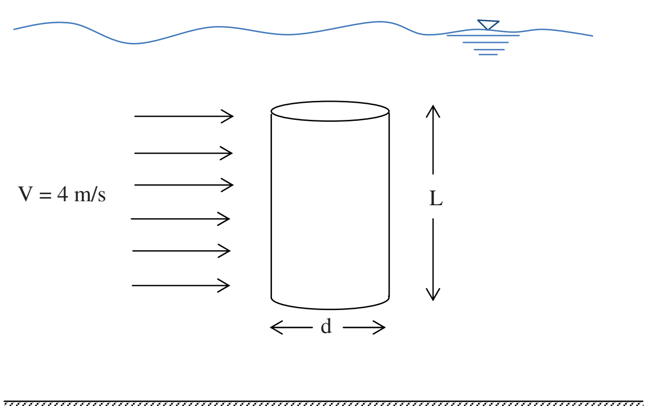
### *Solution Procedure*

The solution is shown in Spreadsheet 1-6.

### *Solution*

The resulting force is $F_d = 5{,}275\,\mathrm{N}$.

***Fig. 1-15*** **Description of Problem 1-6**



### *What-If Analyses*

How is the force $F_d$ affected if the diameter of the pipe is changed? Combining Eqs. 1 and 3 indicates that $F_d$ will be directly proportional to $d$. However, if the value of the diameter is doubled, the value of $F_d$ is 11,606 N, which is slightly more than the previously calculated value. Why? The reason is because an increase in diameter also causes a slight increase in the value of $C_D$.

### *Assigned Problem 1-6-1*

Determine the force exerted on a 30 m long 0.02 m diameter pipe, which is *horizontally* submerged in a river (see Fig. 1-2). The current velocity, $V$, perpendicular to the circular surface of the pipe, is 4 m/s. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are 999.1 kg/m$^3$ and $1.138 \times 10^{-3}$ kg/m-s, respectively.

# Problem 1-7: Calculation of Friction Factor for Transitional Flow

## *Problem Statement*

Determine the friction factor, $f$, in a 20 cm cast iron pipe that has a roughness height, $k_s = 0.12$ mm, if the flow velocity, $V$, is 1.59 m/s, the fluid density, $\rho$, is 1,000 kg/m$^3$, and the dynamic viscosity, $\mu = 0.001$ Ns/m$^2$, using (a) the Colebrook-White, (b) Swamee and Jain, and (c) Haaland equations.

## *Background*

The major head loss through a pipe can be calculated using the Darcy-Weisbach equation, which is given as

$$h_f = f(L/d)[V^2/(2g)]$$

in which $h_f$ is the major head loss, $L$ is the length of the pipe, $d$ is the pipe diameter, $V$ is the average fluid velocity, and $g$ is the acceleration due to gravity. The value of the friction factor depends on the Reynolds number, $\mathrm{R}_e$, and the pipe roughness, $k_s$. The Reynolds number, in turn, depends on the type of flow, which can be laminar, transitional, or fully rough. Both transitional and fully rough flows are considered to be turbulent flows. The value of $f$ can be obtained by three different methods: (a) the Colebrook-White equation, (b) the Swamee and Jain equation, or (c) the Haaland equation. You can also determine the value of $f$ by simply reading the Moody diagram, which was created to avoid solving the abovementioned three equations. (The Moody diagram is not discussed in this book.) The Colebrook-White equation for transitional flows is given by

$$1/f^{0.5} = -2.0\{\log_{10}[k_s/(3.7d) + 2.51/(\mathrm{R}_e f^{0.5})]\} \qquad (1\text{-}6)$$

## *Equations*

The equations for estimating $f$ for transitional flows are

1. $\mathrm{R}_e = Vd\rho/\mu$
2. $R_r = k_s/d$
3. $\mathrm{LHS} = 1/f^{0.5}$
4. $\mathrm{RHS} = -2.0\{\log_{10}[k_s/(3.7d) + 2.51/(\mathrm{R}_e f^{0.5})]\}$
5. $f = 0.25/\{\log_{10}[k_s/(3.7d)] + [5.74/(\mathrm{R}_e^{0.9})]\}^2$
6. $f = 1/\{-1.8\log_{10}[k_s/(3.7d)^{1.1} + 6.9/\mathrm{R}_e]\}^2$

where $R_r$ is a ratio known as the relative roughness, and LHS and RHS stand for left-hand side and right-hand side of the Colebrook-White equation. Eq. 5 is the Swamee and Jain equation, and Eq. 6 is the Haaland equation.

### *Solution Procedure*

The Colebrook-White equation needs to be solved by trial and error, because $f$ occurs on both sides of the equation. The technique is to assume a value of $f$ and to keep changing that value until the LHS = RHS. In 1976, Swamee and Jain introduced a simpler equation to calculate the friction factor (Eq. 5). Their equation usually predicts values within 3% of the Colebrook-White equation. The Haaland equation (Eq. 6) was developed in 1983, and the results from this equation are usually within 2% of the Colebrook-White equation. The Swamee and Jain and Haaland equations are approximate solutions of the Colebrook-White equation, but are simpler to solve, as $f$ occurs only on one side of the equation. The advantage of using these equations is that they do not require an iterative solution.

### *Solution*

As shown in Spreadsheet 1-7, the $f$ values calculated by the Colebrook-White, Swamee and Jain, and the Haaland equations are 0.0187, 0.0188, and 0.0188, respectively.

### *What-If Analyses*

Solutions using all three equations are shown in Spreadsheet 1-7. Conduct a what-if analysis to

(a) Plot a graph of $f$ versus $\mu$ and determine if $f$ increases or decreases with $\mu$.
(b) Plot a graph of $f$ versus $d$ and determine if $f$ increases or decreases with $d$.

### *Assigned Problem 1-7-1*

Prove that for very smooth pipes ($k_s = 0$), the Colebrook-White equation is transformed to

$$1/f^{0.5} = -2.0[\log_{10}(Rf^{0.5})] - 0.8 \tag{1-7}$$

Eq. (1-7) is also known as the Prandtl equation.

### *Assigned Problem 1-7-2*

Prove that for fully rough flows (i.e., relatively high $R$ values that imply, in turn, that $1/R$ can be neglected), the Colebrook-White equation is transformed to

$$f = \{1/[1.14 + 2\log_{10}(d/k_s)]\}^2 \tag{1-8}$$

### *Assigned Problem 1-7-3*

Determine the value of $f$ by Eq. (1-8) for $\mu = 0.0001\ \mathrm{Ns/m^2}$, $\rho = 1{,}000\ \mathrm{kg/m^3}$, $k_s = 1\ \mathrm{mm}$, $V = 1\ \mathrm{m/s}$, and $d = 1\ \mathrm{m}$ and compare the value from that obtained from the Moody diagram.

### *Associated Problem 1-7-4*

Determine the value of $f$ by Eq. (1-7) for a smooth pipe ($k_s = 0$) if $\mu = 0.0001$ Ns/m$^2$, $\rho = 1{,}000$ kg/m$^3$, $V = 1$ m/s and $d = 1$ m and compare the value with that obtained from the Moody diagram.

# Problem 1-8: Flowrate Estimation between Two Reservoirs Using the Darcy-Weisbach and Colebrook-White Equations Assuming Fully Rough Turbulent Flow and No Minor Losses



### *Problem Statement*

Using the Darcy-Weisbach and Colebrook-White equations, determine the flowrate from Reservoir $A$ to Reservoir $B$ (Fig. 1-16). The water level elevations $Z_1$ and $Z_2$ in the two reservoirs are 100 m and 84 m, respectively. A 300 m long 1 m diameter steel pipe (surface roughness, $k_s = 0.046$ mm) connects the two reservoirs. The water density (ρ) and the coefficient of dynamic viscosity (μ) are 1,000 kg/m$^3$ and 0.00114 Ns/m$^2$, respectively.

### *Background*

Because the flow in the pipe is assumed to be fully rough, this problem uses the Colebrook-White equation for fully rough turbulent flows, i.e., Eq. (1-8) of Problem 1-7.

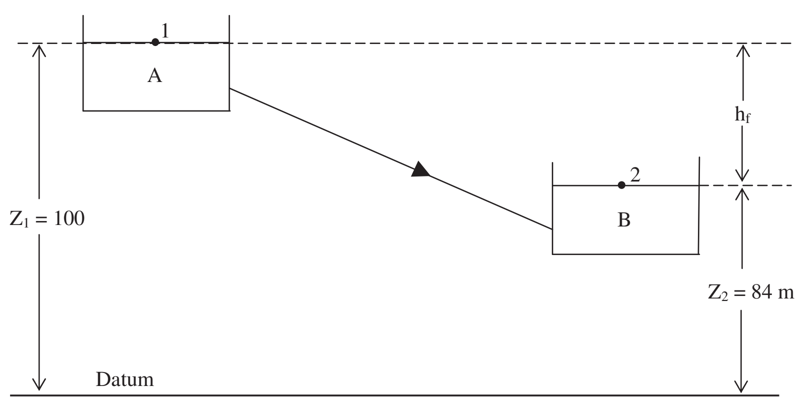
### *Assumptions*

(1) Fully rough turbulent flow, (2) no minor losses.

### *Unknowns and Available Equations in this Problem*

The unknowns and available equations are described in Appendix Table A-1. There are three unknowns in this problem: the major head loss, $h_f$, the friction factor, $f$, and the velocity, $V$ (or flowrate, $Q$) in the pipe. The velocity and flowrate are treated as a single unknown, as it is relatively easy to calculate either parameter if the other is known using the equation $Q = VA$. As there is only a single pipe in this problem, the number of available equations are one energy equation, one friction factor equation, and one major loss equation.

***Fig. 1-16*** **Description of Problem 1-8**



### *Equations*

The equations for estimating the flowrate are

1. $h_f = Z_1 - Z_2$
2. $R_r = k_s/d$
3. $f = 1/[1.14 + 2\log_{10}(d/k_s)]^2$
4. $V = [2gdhf/(fL)]^{0.5}$
5. $Q = (V)(\pi d^2/4)$

where $Z_1 - Z_2$ are the respective water levels in Reservoirs 1 and 2, and $R_r$ is the relative roughness.

### *Solution Procedure*

As indicated in Appendix Table A-1, the Colebrook-White equation can be used to calculate $f$. The energy equation can be modified to

$$h_f = Z_1 - Z_2$$

to calculate $h_f$, and a modified version of the Darcy-Weisbach equation,

$$V = [2gdhf/(fL)]^{0.5} \tag{1-9}$$

can be used to calculate the velocity. No continuity equations are required in this problem, because there are no junctions. Similarly, no minor loss equations are required, as minor losses are neglected.

### *Solution*

As shown in Spreadsheet 1-8, the velocity, $V$, in the pipe is 10.04 m/s, and the flowrate is $Q = 7.89\ \text{m}^3/\text{s}$. An important concept to learn in this problem is that the pipe slope never enters the equations, i.e., the flowrate is independent of the pipe slope. However, the flowrate is highly dependent on the slope of the energy grade line (EGL).

### *What-If Analyses*

Conduct a what-if analysis to

(a) Determine the flowrate if a pipe of diameter 1.25 m is used instead of 1 m.
(b) Determine the flowrate if the elevation $Z_1$ is 105 m instead of 100 m; all other parameters are as in the original problem.
(c) Determine the flowrate if the pipe length is 500 m instead of 300 m; all other parameters are as in the original problem.
(d) Decide whether the increases and/or decreases in the flowrates are as you expected.

### *Assigned Problem 1-8-1*

Draw the EGL and the HGL between the two reservoirs.

# Problem 1-9: Flowrate Estimation between Two Reservoirs Using the Darcy-Weisbach and Swamee and Jain Equations Assuming No Minor Losses

### *Problem Statement*

Using the Darcy-Weisbach and Swamee and Jain equations, determine the flowrate from Reservoir A to Reservoir B (Fig. 1-16). The water level elevations in the two reservoirs are 100 m and 84 m, respectively. A 300 m long 1 m diameter steel pipe (surface roughness, $k_s = 0.046$ mm) connects the two reservoirs. The water density (ρ) and the coefficient of dynamic viscosity (μ) are 1,000 kg/m$^3$ and 0.00114 Ns/m$^2$, respectively.

### *Background*

The difference between this problem and the previous one is that we are no longer making the assumption of fully rough turbulent flow, as we will be using the Swamee and Jain equation for calculating the friction factor.

### *Assumption*

There are no minor losses.

### *Unknowns and Available Equations in this Problem*

The unknowns and available equations are described in Appendix Table A-1. Because we are using the Swamee and Jain equation, we have one more unknown, the Reynolds number ($R_e$), in this problem compared to Problem 1-8. Four equations are available for this problem: the three equations listed for Problem 1-8 and the Reynolds number equation ($R_e = Vd\rho/\mu$).

### *Equations*

The equations for estimating the flowrate are

1. $h_f = Z_1 - Z_2$
2. $R_e = Vd\rho/\mu$
3. $R_r = k_s/d$
4. $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/(R_e^{0.9})]\}^2$
5. $h_f = f(L/d)[V^2/(2g)]$
6. $Q = (V)(\pi d^2/4)$

where all symbols have been defined in previous problems.

### Solution Procedure

A study of the available equations (Appendix Table A-1) shows that although $h_f$ can be calculated using the modified energy equation, the other three equations have more than one unknown. In this case, a trial-and-error solution is applicable. The system of equations can be solved in many ways, and two of the methods are presented.

*Method 1–Using Manual Iterations:* A trial-and-error solution (Method 1, Spreadsheet 1-9) is used to determine the flowrate. The value of $V$ is assumed and changed until $h_f$, calculated by Eqs. 1 and 5 are equal. Spreadsheet 1-9 shows four possible iterations. The first two iterations assume $V = 5$ m/s and 7 m/s. The calculated $h_f$ values using Eq. 5 are 4.27 m and 8.22 m, respectively, and both values are *less* than $h_f$ calculated by Eq. 1 (16 m). The third iteration assumes a value of $V = 10$ m/s. Using this value, the calculated value of $h_f = 16.56$ m, which is *greater* than 16 m. This indicates that the correct value of $V$ is somewhere between 7 m and 10 m, and more trial iterations (all iterations are not shown) indicated that a value of $V = 9.83$ m/s (iteration 4) yielded a $h_f$ value of 16 m (from Eq. 5), which is equal to the value computed by Eq. 1. Thus, the correct answers are $V = 9.83$ m/s and $Q = 7.72\ \text{m}^3/\text{s}$.

*Method 2–Using Goal Seek:* This method is also a trial-and-error solution (Method 2, Spreadsheet 1-9). However, it uses a spreadsheet feature known as "Goal Seek." Because we desire $h_f$ calculated in Cell G1 to be equal to the $h_f$ calculated in Cell B1, Cell G1 is designated as the Goal Seek cell. In most spreadsheets you can find Goal Seek under "Data" and "What-If" Analysis. The spreadsheet will ask for the value desired in this cell. The goal is $h_f = 16$ m, so you should enter a value of 16. Next, you must enter the cell number whose value has to be changed to achieve the desired goal. Because we wish to change the value of $V$ until $h_f = 16$ m, we designate this cell to be Cell C1. The spreadsheet will then automatically find the value of $V$ such that $h_f = 16$ m.

### Solution

The velocity, $V$ in the pipe is 9.83 m/s and the flowrate, $Q = 7.72\ \text{m}^3/\text{s}$. The percentage difference in the calculated flowrate using the Swamee and Jain equation (this problem) versus the Colebrook-White equation used in Problem 1-8 is 2.1%.

### What-If Analyses

Conduct a what-if analysis to determine the effect of the selected pipe by changing the pipe from steel to cast iron ($k_s = 0.26$ mm). (Answer: $V = 8.45$ m/s, $Q = 6.64\ \text{m}^3/\text{s}$). Is the decrease in the flowrate as expected?

# Problem 1-10: Flowrate Estimation between Two Reservoirs Using the Hazen-Williams Equation Assuming No Minor Losses

### *Problem Statement*

Using the Hazen-Williams equation, determine the flowrate from Reservoir A to Reservoir B (Fig. 1-16). The water level elevations in the two reservoirs are 100 m and 84 m, respectively. A 300 m long 1 m diameter steel pipe (roughness coefficient, $C_H = 110$) connects the two reservoirs. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are 1,000 kg/m$^3$ and 0.00114 Ns/m$^2$, respectively.

### *Background*

The difference between this problem and Problem 1-8 is that the major head loss will be calculated by the Hazen-Williams equation instead of the Darcy-Weisbach equation.

### *Assumptions*

(1) Flow is fully rough turbulent because the Hazen-Williams equation is applicable only for fully rough turbulent flow; (2) there are no minor losses.

### *Unknowns and Available Equations in This Problem*

The unknowns and available equations are described in Appendix Table A-1. Because we are using the Hazen-Williams equation, we have a total of four unknowns: major head loss, $h_f$, the hydraulic radius, $R$, the slope of the energy grade line, $S$, and the flowrate, $Q$. Four equations are also available.

### *Equations*

The equations for estimating the flowrate are

1. $h_f = Z_1 - Z_2$
2. $R = d/4$
3. $S = h_f/L$
4. $A = \pi d^2/4$
5. $Q = 0.85 C_H A R^{0.63} S^{0.54}$

where $C_H$ is the Hazen-Williams constant. Other symbols have been defined previously.

### *Solution Procedure*

A study of the equations shows that the equations can be solved in the sequence that has been given.

### *Solution*

As shown in Spreadsheet 1-10, the flowrate is $Q = 6.30\ \mathrm{m}^3/\mathrm{s}$.

### *What-If Analyses*

Note that the flowrates computed by the Darcy-Weisbach equation (Problem 1-8) and the Hazen-Williams equation are similar but not exact. Conduct a what-if analysis to determine the effect of the selected pipe by changing the pipe from steel pipe to a cast iron, wrought plain pipe ($C_H = 100$). (Answer: $Q = 5.73\ \mathrm{m}^3/\mathrm{s}$). Is the decrease in the flowrate as expected?

### *Note*

$C_H$ values can be obtained at http://www.engineeringtoolbox.com/hazen-williams-coefficients-d_798.html.

# Problem 1-11: Flowrate Estimation between Two Reservoirs Using the Darcy-Weisbach and Swamee and Jain Equations Including Minor Losses

## *Problem Statement*

Using the Darcy-Weisbach and Swamee and Jain equations, determine the flowrate from Reservoir A to Reservoir B (Fig. 1-16). The water level elevations in the two reservoirs are 100 m and 84 m, respectively. A 300 m long 1 m diameter steel pipe (surface roughness, $k_s = 0.046$ mm), with an inward projected entrance (entrance loss coefficient $K_e = 1$) connects the two reservoirs. The exit loss coefficient $K_{ex}$ is also equal to 1. The water density (ρ) and the coefficient of dynamic viscosity (μ) are 1,000 kg/m$^3$ and 0.00114 Ns/m$^2$, respectively. Include minor losses.

## *Background*

The difference between this problem and Problem 1-9 is that now we are including the effects of entrance and exit losses known as minor losses. Other types of minor losses may also be possible in other problems. It should be noted that the exit loss coefficient, $K_{ex}$, is always also equal to one when water is discharged into a reservoir.

## *Unknowns and Available Equations in this Problem*

The unknowns and available equations are described in Appendix Table A-1. We have added two more unknowns to those shown in Problem 1-9. These are the entrance and exit losses, also known as minor losses. We have also added two minor loss equations—an entrance loss and an exit loss equation—to those shown in Problem 1-9.

## *Equations*

The sequence of equations for estimating the flowrate can be

1. $h_t = Z_1 - Z_2$
2. $R_e = Vd\rho/\mu$
3. $R_r = k_s/d$
4. $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/(\mathrm{R}_e^{0.9})]\}^2$
5. $h_e = K_e[V^2/(2g)]$
6. $h_{ex} = K_{ex}[V^2/(2g)]$
7. $h_{mi} = h_e + h_{ex}$

8. $h_f = f(L/d)[V^2/(2g)]$
9. $h_t = h_f + h_{mi}$
10. $Q = (V)(\pi d^2/4)$

where $h_e$, $h_{\text{exit}}$, and $h_{mi}$ are the entrance, exit, and combined minor losses, and $h_t$ is the total head loss (sum of the major and minor head losses). All other symbols have been defined in previous problems. Mainly for convenience in writing cell equations, some equations have been added to those shown in Appendix Table A-1.

### *Solution Procedure*

A trial-and-error solution will be convenient because, after calculating $h_t$ using Eq. 1, we find that all other available equations have at least two unknowns.

### *Solution*

A trial-and-error solution is shown in Spreadsheet 1-11. The value of $V$ is changed until $h_t$, calculated by Eqs. 1 and 9 are equal. The solution will be quicker to find using Goal Seek. The velocity, $V$, in the pipe is 7.71 m/s and the flowrate, $Q = 6.06\ \text{m}^3/\text{s}$. The percentage difference in the calculated flowrate including minor losses (this problem) versus the flowrate calculated in Problem 1-9, in which minor losses were not included, is 21.5%.

### *What-If Analyses*

Conduct a what-if analysis to determine the effect of different entrances as opposed to an inward projected entrance. Determine the velocity, flowrate, and the increase in flowrate if (a) the entrance is square edged ($K_e = 0.5$), (b) the entrance is chamfered ($K_e = 0.25$), and (c) the entrance is rounded ($K_e = 0.09$). The answers are (a) $V = 8.11$ m/s, $Q = 6.37\ \text{m}^3/\text{s}$, % increase 5.1; (b) $V = 8.33$ m/s, $Q = 6.55\ \text{m}^3/\text{s}$, % increase 8.1; and (c) $V = 8.49$ m/s, $Q = 6.67\ \text{m}^3/\text{s}$, % increase 10.1.

### *Note*

The minor loss coefficients can be obtained from http://udel.edu/~inamdar/EGTE215/ Minor_loss.pdf.

### *Assigned Problem 1-11-1*

Draw the EGL and the HGL between the two reservoirs. Note that this problem is different from Assigned Problem 1-8-1 because in this problem you must include minor losses.

# Problem 1-12: Pump Head, Power Requirement, and Cost for Flow between Two Reservoirs (Include Minor Losses)

## *Problem Statement*

Using the Darcy-Weisbach and Swamee and Jain equations, determine the pump head and pump power requirement if water is being pumped up from Reservoir B (elevation 84 m) to Reservoir A (elevation 100 m) as shown in Fig. 1-17. The required flowrate is 5 $m^3/s$, and the reservoirs are connected by a 300 m long 1 m diameter steel pipe (surface roughness, $k_s = 0.046$ mm), with a well-rounded entrance (entrance loss coefficient $K_e = 0.04$). Also, compute the monthly electricity cost if the pump efficiency ($\eta$) is 70%, and the electricity cost is \$0.085 per kWh. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are 1,000 $kg/m^3$ and 0.00114 $Ns/m^2$, respectively. Include minor losses.

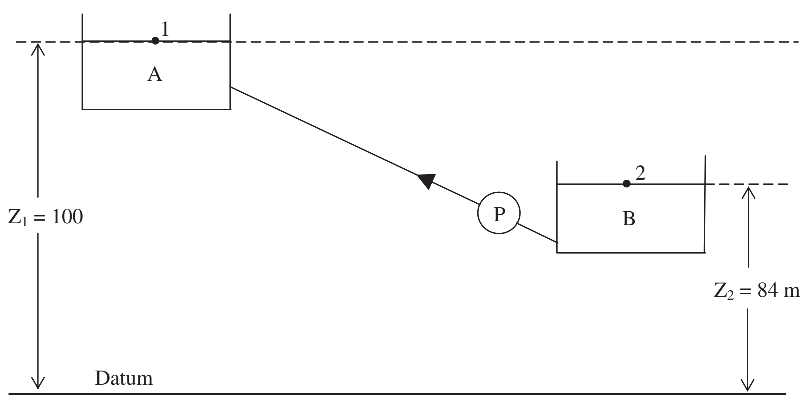
## *Background*

A main difference between this problem and Problem 1-11 is that the flow from the two reservoirs is reversed and the flow is *from the lower to the higher reservoir*. A pump will be required to overcome the effects of gravity and the energy lost to major and minor head losses. A pump is a device used to add pressure energy into the system.

## *Unknowns and Available Equations in this Problem*

The unknowns and available equations are described in Appendix Table A-2. This problem has the same number of unknowns as Problem 1-11 with the exception that $Q$ has been replaced by $h_p$, which is the head provided by the pump. Also, we have added three other unknowns: the mechanical power, $P$; the electric power, EP; and the cost of the electric power, for a total of nine unknowns. The same six equations used in Problem 1-11 are available in this case. The two additional equations, for the additional unknowns, are the power equation (Eq. 12 in the equation set following), Eq. 13, which describes that the

***Fig. 1-17*** **Description of Problem 1-12**



efficiency of a pump, $\eta = P/\text{EP}$, and an equation for calculating the cost of the electric power.

### Equations

The equations for estimating the pump head, pump power requirement, and the monthly electric cost are

1. $V = 4Q/(\pi d^2)$
2. $\text{R}_e = Vd\rho/\mu$
3. $R_r = k_s/d$
4. $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/(\text{R}_e^{0.9})]\}$
5. $h_e = K_e[V^2/(2g)]$
6. $h_{ex} = K_{ex}[V^2/(2g)]$
7. $h_{mi} = h_e + h_{ex}$
8. $h_f = f(L/d)[V^2/(2g)]$
9. $h_t = h_f + h_{mi}$
10. $h_p = h_t + Z_1 - Z_2$
11. $P = \rho g h_p Q/1{,}000$
12. $\text{EP} = P/\eta$
13. $\text{Cost} = (E)(30\ \text{days/month})(24\ \text{hr/day})(0.085)$

where $P$ and EP are in kW. The equation set has 13 equations instead of nine, because in addition to Eq. 1, we have added Eqs. 3, 7, and 9 for convenience.

### Solution Procedure

The solution is shown in Spreadsheet 1-12. As $Q$, and therefore $V$, is given, the solution does not require a trial-and-error solution provided the equations are solved in a proper sequence. The aforementioned equation set presents one such sequence (other sequences are also possible).

### Solution

The head provided by the pump is 24.98 m, and the power requirement is 1,225 kN-m/s (or 1,225 kW). The monthly cost of operating the pump is USD $107,123.

### What-If Analyses

Determine the effect of the desired flowrate, $Q$, on the monthly costs by plotting a graph of monthly cost versus $Q$. Why is this relationship nonlinear?

### Assigned Problem 1-12-1

Draw the EGL and the HGL between the two reservoirs.

# Problem 1-13: Estimation of Pipe Diameter for Flow between Two Reservoirs Using the Darcy-Weisbach and Swamee and Jain Equations with No Minor Losses

## *Problem Statement*

Water flows from Reservoir A to Reservoir B (Fig. 1-16). Using the Darcy-Weisbach and Swamee and Jain equations, determine the pipe diameter necessary to carry a flowrate of 7.5 $m^3/s$ between the two reservoirs, which are 300 m apart, and in which the water level elevations are 100 m and 84 m, respectively. The desired pipe is a steel pipe (surface roughness, $k_s = 0.046$ mm), and minor losses can be neglected. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are 1,000 $kg/m^3$ and 0.00114 $Ns/m^2$, respectively. Ignore minor losses.

## *Background*

Usually minor losses are neglected when designing long pipes, but neglecting minor losses can lead to significant errors if the pipe lengths are relatively short or if minor losses are large. The main difference between this problem and Problem 1-9 is that the flowrate is known, whereas the pipe diameter is not known.

## *Assumption*

Minor losses are neglected.

## *Unknowns and Available Equations in this Problem*

The unknowns and available equations are described in Appendix Table A-2. This problem has the same number of unknowns and available equations as Problem 1-9 with the exception that $Q$ has been replaced by $d$.

## *Equations*

The equations for estimating the pipe diameter are

1. $h_f = Z_1 – Z_2$
2. $K = [8f/(g\pi^2)]^{0.2}$
3. $d = K(LQ^2/h_f)^{0.2}$
4. $R_r = k_s/d$
5. $A = \pi d^2/4$
6. $V = Q/A$
7. $\mathrm{R}_e = Vd\rho/\mu$
8. $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/(\mathrm{R}_e^{0.9})]\}^2$

where $K$ is a constant. Eqs. 2 and 3 are obtained by modifying the Darcy-Weisbach equation.

### Solution Procedure

A study of the available equations (Appendix Table A-2) shows that although $h_f$ can be calculated using the energy equation (Eq. 1), the remaining three equations have at least two unknowns. As a result, a trial-and-error solution is utilized. Two methods are provided to calculate the pipe diameter using an iterative solution procedure.

*Method 1–Using Manual Iterations and Assuming* f*:* A trial-and-error solution (Method 1, Spreadsheet 1-13) is used to determine the pipe diameter. The value of $f$ is assumed and changed until the assumed value is equal to the value calculated by Eq. 8. A fast technique for converging to the correct value of $f$ and to avoid multiple iterations the new assumed value is to set equal to the previously calculated value. This is illustrated in Method 1, Spreadsheet 1-13. An initial value of $f = 0.001$ is assumed (iteration 1), which results in a calculated value of 0.0116 by Eq. 8. The next assumed value for $f$ is 0.0116, which results in a calculated value of $f$ equal to 0.0108. This process is repeated, and we can see that convergence to the correct value ($f = 0.0109$) is achieved fairly rapidly over four iterations. Note that in this problem we desired an accuracy of up to four decimal figures.

*Method 2–Using Circulation Cells and Assuming* f*:* The circulation cell method is also illustrated in Spreadsheet 1-13. This method is applicable when one wants two cells to converge to the same value. In this case, we would want Cells C1 and J1 to have the same value. This is accomplished by designating Cell J1 as a *circulation cell*. This can be done as follows: (1) click on Cell J1, (2) click on File/Options/Formulas, (3) set the Workbook Calculation on "Automatic," (4) activate "Enable Iterative Calculation," (5) set the number of "Maximum Iterations," and (6) set the number of "Maximum Change" (this was set at 0.0001 for this problem). After designating Cell J1 as a circulation cell, type "= Cell J1" in Cell C1. The spreadsheet will automatically calculate a value of $f = 0.0109$.

### Solution

The solution is shown in Spreadsheet 1-13. The estimated pipe diameter is 0.99 m, so the selected pipe size will be 1 m or the next higher pipe size that is available.

### What-If Analyses

Determine the effect of the desired flowrate, $Q$, on the pipe diameter, $d$, by plotting a graph of $d$ versus $Q$ (using a range of $Q$ from $5\,\text{m}^3/\text{s}$ to $10\,\text{m}^3/\text{s}$). All other parameters are the same as in Problem 1-13.

### *Assigned Problem 1-13-1*

Solve Problem 1-13 using Goal Seek.

### *Assigned Problem 1-13-2*

What pipe diameter would be needed if a concrete pipe ($k_s = 1.5$ mm) was used instead of a steel pipe? All other parameters are the same as in Problem 1-13.

# Problem 1-14: Estimation of Pipe Diameter for Flow between Two Reservoirs Using the Darcy-Weisbach and Swamee and Jain Equations and Include Minor Losses



## *Problem Statement*

Using the Darcy-Weisbach and Swamee and Jain equations, determine the pipe diameter necessary to carry a flowrate of $7.5\,\text{m}^3/\text{s}$ between two reservoirs, which are 300 m apart and in which the water level elevations are 100 m and 84 m, respectively. The desired pipe is a steel pipe (surface roughness, $k_s = 0.046\,\text{mm}$). The entrance and exit loss coefficients are 0.09 and 1.0, respectively. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are $1{,}000\,\text{kg/m}^3$ and $0.00114\,\text{Ns/m}^2$, respectively. Include minor losses.

## *Background*

The main difference between this problem and Problem 1-13 is that minor losses are not neglected in this problem. Neglecting minor losses can lead to estimating a smaller pipe size than what is actually needed to carry the desired flowrate.

## *Unknowns and Available Equations in this Problem*

The unknowns and available equations are described in Appendix Table A-2. This problem has the same number of unknowns and available equations as Problem 1-13, with the exception that two minor losses [the entrance loss ($h_e$) and exit loss ($h_{ex}$)] are no longer neglected. Because the entrance and exit losses can be calculated by Eqs. 5 and 6 of Problem 1-12, we have a total of six unknowns and six equations. A study of the equations shows that the solution procedure used in Problem 1-13 is no longer applicable, because $h_f$ can no longer be calculated using Eq. 1 of Problem 1-13, and therefore, $d$ can no longer be calculated using Eqs. 2 and 3 of Problem 1-13 even after assuming $f$.

A possible sequence in which equations can be solved for estimating the pipe diameter is

## *Equations*

1. $h_t = Z_1 - Z_2$
2. $A = \pi d^2/4$
3. $V = Q/A$
4. $\mathrm{R}_e = Vd\rho/\mu$
5. $R_r = k_s/d$
6. $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/(\mathrm{R}_e^{0.9})]\}^2$

7. $h_f = f(L/d)[V^2/(2g)]$
8. $h_e = K_e[V^2/(2g)]$
9. $h_{ex} = K_{ex}[V^2/(2g)]$
10. $h_t = h_e - h_{ex} + h_f$

where $h_t$ is the total head loss.

### *Solution Procedure*

A study of the available equations (Appendix Table A-2) shows that all equations have at least two unknowns. The variable $h_t$ has been introduced mainly for convenience so that the energy equation can be broken into two equations: Eqs. 1 and 10. The feature Goal Seek is used by specifying that the value of $h_t$, calculated by Eq. 10, must be equal to that calculated by Eq. 1.

### *Solution*

The calculated value of $d$ (Spreadsheet 1-14) is equal to 1.05 m. Thus, the required pipe diameter is 1.05 m or the next higher available size. Note that this is slightly higher than the pipe size calculated in Spreadsheet 1-13 (why?).

### *What-If Analyses*

Determine the effect of the pipe length, $L$, on the pipe diameter, $d$, by plotting a graph of $d$ versus $L$ (using a range of $L$ from 300 m to 3,000 m). All other parameters are the same as in Spreadsheet 1-14.

### *Assigned Problem 1-14-1*

What pipe diameter would be needed if there were four bends in the pipe and the pipe had a wide-open globe valve? The loss coefficient for each of the bends is 0.19, whereas the loss coefficient for the globe valve is 10. All other parameters are the same as in Spreadsheet 1-14.

# Problem 1-15: Flowrate Estimation between Two Reservoirs When the Water Is Being Pumped Up and Include Minor Losses

## *Problem Statement*

Using the Darcy-Weisbach and Swamee and Jain equations, determine the flowrate and the head provided by the pump if water is being pumped from a lower reservoir (elevation 0 m) to a higher reservoir (elevation 10 m) as shown in Fig. 1-18. The reservoirs are connected by a 30 m long 0.15 m diameter steel pipe (surface roughness, $k_s = 0.046$ mm). There are three bends in the pipe ($K_b = 0.19$) and an open gate valve ($K_v = 0.2$). The entrance is rounded, $K_e = 0.12$. The pump performance curve is given by $h_p = 20 - 4713Q^2$, where the head provided by the pump is in m, and $Q$ is the flowrate in $m^3/s$. The water density ($\rho$) and the coefficient of dynamic viscosity ($\mu$) are 1,000 $kg/m^3$ and 0.00114 $Ns/m^2$, respectively. Include minor losses.

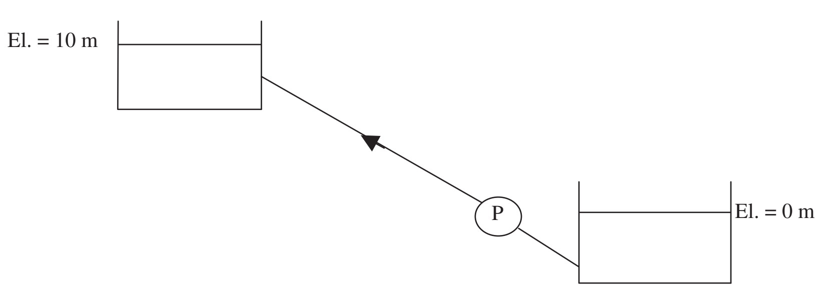
## *Background*

This problem is slightly more complicated than Problem 1-12 because both the head provided by the pump *and* the flowrate are unknown. Therefore, an extra equation is needed to solve the problem. This extra equation is provided by the pump manufacturer either in the form of a graph known as the "pump characteristic" or "pump performance" curve or in the form of an equation (as in this problem). For solution purposes, converting the pump performance curve, if presented as a graph, to an equation using the trendline technique is more convenient when using a spreadsheet. In addition, we also added the minor head losses because of bends in the pipe ($h_b$) and the presence of a gate valve ($h_v$). The calculation of $h_b$ and $h_v$ requires two additional equations, which are Eqs. 8 and 9 in the equation list following.

## *Unknowns and Available Equations in this Problem*

As described in Appendix Table A-3, this problem has nine unknowns. There is only a single pipe in this equation, so the available equations are one energy equation, one friction factor equation (Swamee and Jain), one Reynolds number equation, one major loss equation (Darcy-Weisbach), four minor loss equations, and the pump performance equation.

***Fig. 1-18* Description of Problem 1-15**



### Equations

The equations for estimating the flowrate are

1. $H = Z_1 - Z_2$
2. $Q = V\pi d^2/4$
3. $\mathrm{R}_e = Vd\rho/\mu$
4. $R_r = k_s/d$
5. $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/(\mathrm{R}_e^{0.9})]\}^2$
6. $h_e = K_e[V^2/(2g)]$
7. $h_{ex} = K_{ex}[V^2/(2g)]$
8. $h_b = K_b[V^2/(2g)]$
9. $h_v = K_v[V^2/(2g)]$
10. $h_{mi} = h_e + h_{ex} + h_b + h_v$
11. $h_f = f(L/d)[V^2/(2g)]$
12. $h_t = h_f + h_{mi}$
13. $h_p = h_t + H$
14. $Q = [(20 - \mathrm{hp})/4713]^{0.5}$

where $K_b$ and $K_v$ are the bending and valve loss coefficients, and $H$ is the elevation difference between the two reservoirs. All other symbols have been defined in previous problems.

### Solution Procedure

Several solution procedures are possible; however, two trial-and-error methods are shown in Spreadsheet 1-15. In Method 1, the value of $V$ is changed until the flowrate calculated by Eqs. 2 and 14 are equal. In Method 2, the value of $h_p$ is changed until the pump head calculated by Eq. 13 is equal to the assumed head. Note that Eq. 2 has to be rearranged in Method 2 to $V = 4Q/\pi d^2$. Manual iterations were used in both procedures.

### Solution

Both solution procedures lead to the same answers: $Q = 0.042\ \mathrm{m^3/s}$ and $h_p = 11.50\ \mathrm{m}$.

### What-If Analyses

Determine the effect of the number of pipe bends on the pipe diameter, $d$, by plotting a graph of $d$ versus number of bends. The number of bends range from 1 to 5. All other parameters are the same as in Problem 1-15.

### Assigned Problem 1-15-1

Find the flowrate, $Q$, for the aforementioned problem using Method 1, but use the Goal Seek feature instead of manual iterations.

### Assigned Problem 1-15-2

Find the flowrate, $Q$, for the aforementioned problem using Method 2, but use the circulation cell feature instead of manual iterations.

# Problem 1-16: Flowrate Estimation between Three Reservoirs: The Classical Three Reservoir Problem

## *Problem Statement*

Three reservoirs (A, B, and C) are connected through a junction, J, as shown in Fig. 1-19. Using the Darcy-Weisbach method, determine the flowrates in the three pipes and the energy head, $E_j$, at the junction. The water levels in Reservoirs A, B, and C are 160 m, 100 m, and 80 m, respectively. The pipe lengths diameters and friction factors are given in Spreadsheet 1-16.

## *Assumptions*

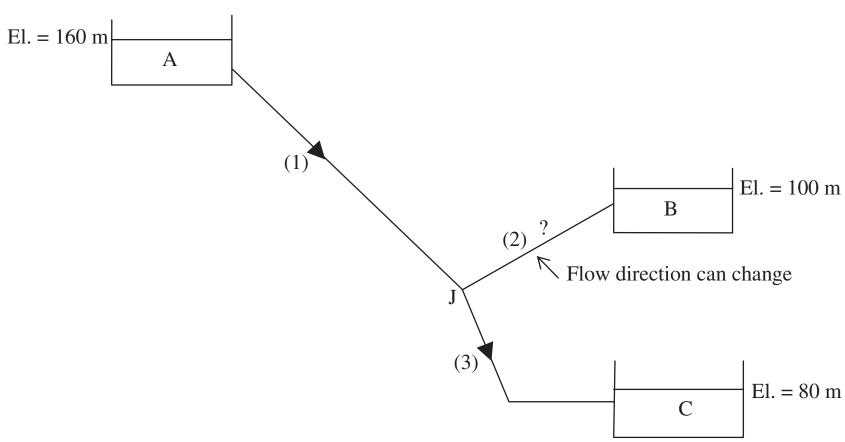
(1) Minor losses are negligible; (2) friction factors in all pipes are 0.03.

## *Background*

Four concepts are usually applied when solving networking problems: (1) hypothetical reservoirs, (2) flow directions, (3) number of unknowns, and (4) number of available equations.

If Reservoir C in this problem represents a city, the water level (ME) in Reservoir C will change based on the city's water demand at any given time of the day. The water level in Reservoir C (80 m) represents the city's water demand. Under low-demand conditions for the city, the flow will be from the junction to Reservoir B, but under high-demand conditions, the flow will be from Reservoir B to the junction as both Reservoirs A and B will be needed to meet the city's water demand.

***Fig. 1-19*** **Description of Problem 1-16**



### *Flow Direction*

The flow directions in the pipes are usually known; however, that may not always be the case. For example, it is easy to see that the flow direction is from Reservoir *A* to the junction in Pipe 1, and from the junction to Reservoir C in Pipe 3. However, we do not know the direction of flow in Pipe 2. If the ME at the junction is greater than 100 m, the flow will be toward Reservoir B; otherwise it will be in the other direction.

### *Unknowns and Available Equations*

The unknowns and available equations are shown in Appendix Table A-4. This problem has seven unknowns. As there are three pipes and one junction in the given network, there are seven available equations in this problem: three energy equations, three major loss equations, and one continuity equation.

### *Equations*

1. $h_{f1} = E_A - E_j$
2. $h_{f2} = E_B - E_j$ (if $E_j < E_B$)
   $h_{f2} = E_j - E_B$ (if $E_j > E_B$)
3. $h_{f3} = E_j - E_C$
4. $V_1 = [2gd_1h_{f1}/(f_1L_1)]^{0.5}$
5. $Q_1 = V_1\pi d_1^2/4$
6. $V_2 = [2gd_2h_{f2}/(f_2L_2)]^{0.5}$
7. $Q_2 = V_2\pi d_2^2/4$
8. $V_3 = [2gd_3h_{f3}/(f_3L_3)]^{0.5}$
9. $Q_3 = V_3\pi d_3^2/4$
10. $\mathrm{Net}Q = Q_1 + Q_2 - Q_3$ (if $E_j < E_B$)
    $\mathrm{Net}Q = Q_1 - Q_2 - Q_3$ (if $E_j > E_B$)

where Net$Q$ is the algebraic sum of the inflow and outflow from the junction, and $E_A$, $E_B$, and $E_C$ are the mechanical energies in Reservoirs A, B, and C, respectively. Eqs. 1, 2, and 3 are the three energy equations for Pipes 1 to 3, respectively. Eqs. 4, 6, and 8 are the respective Darcy-Weisbach equations for Pipes 1 to 3, and Eq. 10 is the continuity equation at the junction.

### *Solution Procedure*

We decided to write the three energy equations across the three pipes: 1 to J, 2 to J or J to 2, and J to 3. However, it would be valid to write any three energy equations. For example, the third energy equation could be from 1 to 3 instead of from J to 3. As stated before, the number of useful energy equations will be equal to the number of pipes; the remaining equation or equations will be redundant.

Thus, if you are using the energy equations from 1 to J and 2 to J, you can use either the energy equation from J to 3 or the energy equation from 1 to 3 but not both equations. A trial-and-error procedure is used because all available equations have at least two unknowns. In problems with junctions, it is usually convenient to assume the ME at the junction ($E_j$). The solution procedure is shown in Spreadsheet 1-16. An $E_j$ value of 90 m, representing high-demand conditions, was used in Iteration 1 and resulted in positive Net$Q = 0.191\ \mathrm{m^3/s}$. An $E_j$ value of 99.8 m, also representing high-demand conditions, was used in Iteration 2 and resulted in a negative Net$Q = 0.019\ \mathrm{m^3/s}$. The change of sign in Net$Q$ indicates that the correct value of $E_j$ would be somewhere between 90 m and 99.8. Further iterations (Iteration 3) provided a Net$Q = 0$ for $E_j = 99.45$ m.

### *Solution*

The flowrates in Pipes 1 to 3 are $0.242\ \mathrm{m^3/s}$, $0.042\ \mathrm{m^3/s}$, and $0.285\ \mathrm{m^3/s}$, whereas $E_j = 99.45$ m. As $E_j$ is less than $E_B$, the flow is from the Reservoir B to the junction and represents a high-demand situation.

### *What-If Analyses*

Determine the flowrates in the pipes and the value of $E_j$ when $E_B = 90$ m (instead of 100 m), i.e., the water level in the reservoir has depleted from 100 m to 90 m because of supplying water to the city. Under this condition, the respective flowrates in Pipes 1 to 3 are $0.262\ \mathrm{m^3/s}$, $0.013\ \mathrm{m^3/s}$, and $0.249\ \mathrm{m^3/s}$, whereas $E_j = 90.05$ m. As $E_j$ is greater than $E_B$, the flow is from the junction to Reservoir B and indicative of low-demand conditions.

### *Assigned Problem 1-16-1*

The city of Problem 1-16 desires a flowrate $Q_3 = 0.4\ \mathrm{m^3/s}$ under high-demand conditions ($E_B = 100$ m). All parameters are the same as in Problem 1-16 with two exceptions: (1) the diameter of Pipe 3 ($d_3$) is not known, whereas the desired flowrate to the city, $Q_3 = 0.4\ \mathrm{m^3/s}$. Determine the pipe diameter $d_3$ needed to provide the desired flowrate.

### *Hint*

The number of unknowns are still seven as before.

### *Assigned Problem 1-16-2*

Solve Problem 1-16 using Goal Seek.

### *Assigned Problem 1-16-3*

The city of Problem 1-16 desires $Q_3 = 0.28\ \mathrm{m^3/s}$ and $Q_1 = 0.3\ \mathrm{m^3/s}$ under high-demand conditions ($E_B = 100$ m). All parameters are the same as in Problem 1-16 with two exceptions: (1) the diameters of Pipes 1 and 2 ($d_1$ and

$d_2$) are not known, whereas the desired flowrates are $Q_1 = 0.0.25\,\mathrm{m}^3/\mathrm{s}$ and $Q_3 = 0.0.28\,\mathrm{m}^3/\mathrm{s}$. Determine the pipe diameters $d_1$ and $d_2$ needed to provide the desired flowrate.

### *Hints*

The number of unknowns are still seven as before. The problem will be much easier to solve using Goal Seek.

# Problem 1-17: Estimation of Pumped Flowrate in a Pipe Network

## *Problem Statement*

Water from Reservoir A is being pumped to Reservoirs B and C through two junctions, $J_1$ and $J_2$, as shown in Fig. 1-20. The pump is located in Pipe 1, and the pump characteristic equation for the pump is $h_p = 60 - 10Q_1^2$, where $h_p$ is the head provided by the pump in m and $Q_1$ is the flowrate in Pipe 1 in $m^3/s$. Using the Darcy-Weisbach method, determine the flowrates in the five pipes and the energy heads, $E_{j1}$ and $E_{j2}$, at the two junctions. Also determine the head provided by the pump ($h_p$). The water levels in Reservoirs A, B, and C are 0 m, 50 m, and 48 m, respectively. The pipe lengths, diameters and friction factors are given in Spreadsheet 1-17.

## *Assumptions*

(1) Minor losses are negligible, (2) friction factor in all pipes is 0.02.

## *Background*

A brief discussion on the unknowns and available equations is provided following. The three reservoirs may not be actual reservoirs.

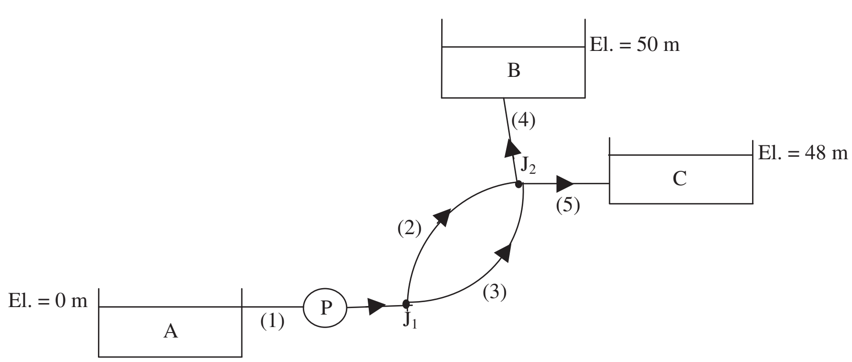
## *Unknowns and Available Equations*

The unknowns and available equations are shown in Appendix Table A-5. This problem has 13 unknowns. As there are five pipes and two junctions in the given network, there are 13 available equations: five energy equations, five major loss equations, two continuity equations, and the pump characteristic equation.

## *Equations*

1. $h_{f4} = E_{J2} - E_B$
2. $h_{f5} = E_{J2} - E_C$

***Fig. 1-20*** **Description of Problem 1-17**



3. $V_4 = [2gd_4h_{f4}/(f_4L_4)]^{0.5}$
4. $Q_4 = V_4\pi d_4^2/4$
5. $V_5 = [2gd_5h_{f5}/(f_5L_5)]^{0.5}$
6. $Q_5 = V_5\pi d_5^2/4$
7. $Q_1 = Q_4 + Q_5$
8. $V_1 = Q_1/(\pi d_1^2/4)$
9. $h_{f1} = f_1L_1V_1^2/(2gd_1)$
10. $h_p = 60 - 10Q_1^2$
11. $E_{J1} = E_A + h_p - h_{f1}$
12. $h_{f2} = E_{J1} - E_{J2}$
13. $V_2 = [2gd_2h_{f2}/(f_2L_2)]^{0.5}$
14. $Q_2 = (\pi d_2^2/4)(V_2)$
15. $h_{f3} = E_{J1} - E_{J2}$
16. $V_3 = [2gd_3h_{f3}/(f_3L_3)]^{0.5}$
17. $Q_3 = (\pi d_3^2/4)(V_3)$
18. $Q_1 = Q_2 + Q_3$

### *Solution Procedure*

After studying the equations, one realizes that no equation has a single unknown. Therefore, a trial-and-error solution would be appropriate. As stated earlier, it is usually convenient to assume the mechanical energy at a junction or the head provided by the pump. In this problem, a convenient solution procedure can be outlined after one assumes the mechanical energy at Junction 2 ($E_{J2}$). The 18 equations are written in the exact order in which the equations are solved after assuming $E_{J2}$. The solution procedure is demonstrated in Spreadsheet 1-17. The value of $E_{J2}$ is changed until the $Q_1$ value calculated by Eqs. 7 and 18 are equal. The assumed value of $E_{J2}$ should be greater than $E_B$ and $E_c$, because the flow is from $J_2$ to Reservoirs B and C. Although the problem has been solved by manual iterations in Spreadsheet 1-17, it can be solved more conveniently using Goal Seek.

### *Solution*

The flowrates in Pipes 1 to 5 are 0.064 $m^3/s$, 0.035 $m^3/s$, 0.029 $m^3/s$, 0.028 $m^3/s$, and 0.036 $m^3/s$; $E_{j1} = 59.83$ m; $E_{j2} = 53.35$ m; and $h_p = 59.96$ m. There is more than one way to solve this problem. For example, a solution procedure can also be written if you assume $Q_4$ (or $Q_5$) instead of $E_{J2}$.

### *What-If Analyses*

(a) Determine the flowrates in the five pipes, the energy heads, $E_{j1}$ and $E_{j2}$, and $h_p$ if the water level in Reservoir B is 40 m instead of 50 m. The flowrates in Pipes 1 to 5 are 0.076 $m^3/s$, 0.042 $m^3/s$, 0.034 $m^3/s$, 0.051 $m^3/s$, and

$0.025\,m^3/s$; $E_{j1} = 59.76\,m$; $E_{j2} = 50.76\,m$; and $h_p = 59.94\,m$. As expected, a lower water level in Reservoir B results in a higher flowrate through Pipe 4 ($0.051\,m^3/s$ as opposed to $0.028\,m^3/s$) and a higher total water supply to both reservoirs ($0.076\,m^3/s$ as opposed to $0.064\,m^3/s$).

(b) Determine the flowrates in the five pipes, the energy heads, $E_{j1}$ and $E_{j2}$, and $h_p$ if the friction factor in all pipes is 0.03 (instead of 0.02) indicating older pipes. The water level in Reservoir B is 50 m. The flowrates in Pipes 1 to 5 are $0.053\,m^3/s$, $0.029\,m^3/s$, $0.024\,m^3/s$, $0.023\,m^3/s$, and $0.029\,m^3/s$; $E_{j1} = 59.84\,m$; $E_{j2} = 53.35\,m$; and $h_p = 59.97\,m$. As expected, a higher friction factor in the pipes results in a lower total water supply to both reservoirs ($0.053\,m^3/s$ as opposed to $0.064\,m^3/s$) and a lower flowrate in all five pipes.

### *Assigned Problem 1-17-1*

Assume that there is no pump in the system and there is gravity flow from Reservoirs B and C to Reservoir A. Using the Darcy-Weisbach method, determine the flowrates in the five pipes and the energy heads, $E_{j1}$ and $E_{j2}$, at the two junctions.

### *Hints*

The number of unknowns are now 12 because $h_p$ is no longer an unknown. The available equations are also 12 because the pump characteristic equation is no longer required. The problem will be much easier to solve using Goal Seek.

# Problem 1-18: Estimation of Pumped Flowrate in a Pipe Network

### *Problem Statement*

Water from Reservoir A is being pumped to Reservoirs B and C through two junctions, Junctions $J_1$ and $J_2$, as shown in Fig. 1-20. The pump is located in Pipe 1 and the pump characteristic equation for the pump is $h_p = 70 - 10Q_1^2$, where $h_p$ is the head provided by the pump in m and $Q_1$ is the flowrate in Pipe 1 in $m^3/s$. Using the Darcy-Weisbach method, determine the pipe diameter for Pipe 4 if the desired flowrate to Reservoir B is $0.08\ m^3/s$. Also determine the head provided by the pump ($h_p$). The water levels in Reservoirs A, B, and C are 0 m, 50 m, and 48 m, respectively. The pipe lengths diameters and friction factors are given in Spreadsheet 1-18.

### *Assumptions*

(1) Minor losses are negligible; (2) friction factors in all pipes are 0.02.

### *Background*

A brief discussion on the unknowns and available equations is provided following.

### *Unknowns and Available Equations*

The unknowns and available equations are shown in Appendix Table A-6. Like Spreadsheet 1-17, this problem also has 13 unknowns. The difference between this problem and Spreadsheet 1-17 is that $d_4$ is an unknown instead of $Q_4$. The same 13 equations available for Spreadsheet 1-17 are also available in this problem. The pump characteristic equation is also so different from the one used in Spreadsheet 1-17.

### *Equations*

1. $V_4 = Q_4/\pi d_4^2/4$
2. $h_{f4} = f_4 L_4 V_4^2/(2gd_4)$
3. $E_{J2} = h_{f4} + E_B$
4. $h_{f5} = E_{j2} - E_C$
5. $V_5 = [2gd_5 h_{f5}/(f_5 L_5)]^{0.5}$
6. $Q_5 = (\pi d_5^2/4)(V_5)$
7. $Q_1 = Q_4 + Q_5$
8. $V_1 = Q_1/(\pi d_1^2/4)$
9. $h_{f1} = f_1 L_1 V_1^2/(2gd_1)$
10. $h_p = 60 - 10Q_1^2$

11. $E_{J1} = E_A + h_p - h_{f1}$
12. $h_{f2} = E_{J1} - E_{J2}$
13. $V_2 = [2gd_2h_{f2}/(f_2L_2)]^{0.5}$
14. $Q_2 = (\pi d_2^2/4)(V_2)$
15. $h_{f3} = E_{J1} - E_{J2}$
16. $V_3 = [2gd_3h_{f3}/(f_3L_3)]^{0.5}$
17. $Q_3 = (\pi d_3^2/4)(V_3)$
18. $Q_1 = Q_2 + Q_3$

### *Solution Procedure*

Because $Q_4$ is known and $d_4$ is unknown, this problem requires a slightly different solution procedure than Problem 1-17. A trial-and-error solution is again appropriate, as no equation has a single unknown. In this problem, it is more convenient to assume $d_4$ instead of $E_{J2}$. Although the same 18 equations used in Problem 1-17 are used in this problem, some of the equations have been reorganized because the unknown in the equation is different compared with Problem 1-17. For example, whereas the Darcy-Weisbach equation for Pipe 4 was used to calculate $V_4$ in Problem 1-17 (Eq. 3), in this problem, it is used to calculate the major head loss $h_{f4}$. The value of $d_4$ is changed until the $Q_1$ value calculated by Eqs. 7 and 18 are equal.

### *Solution*

The required diameter for Pipe 4, $d_4$, was found to be 0.364 m (364 mm).

### *What-If Analyses*

Determine the diameter, $d_4$, if the desired flowrate to Reservoir B ($Q_4$) is $0.06\,\mathrm{m}^3/\mathrm{s}$ (instead of $0.08\,\mathrm{m}^3/\mathrm{s}$). As expected, a larger pipe diameter will be required, as the desired flowrate is higher (Answer: 0.258 m).

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# 2

# *Hydraulics*

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# Background

This chapter covers the following topics: estimation of flow rate and normal depth in prismatic channels (Problems 2-1 through 2-5), construction of the specific energy diagram (SED) and the estimation of critical depth (Problems 2-6 through 2-8), channel transitions (Problems 2-9 through 2-14), hydraulic jumps (Problems 2-15), and channel design (Problems 2-16 through 2-19). A brief background is provided for the material covered in these five areas.

# Open Channel Flow

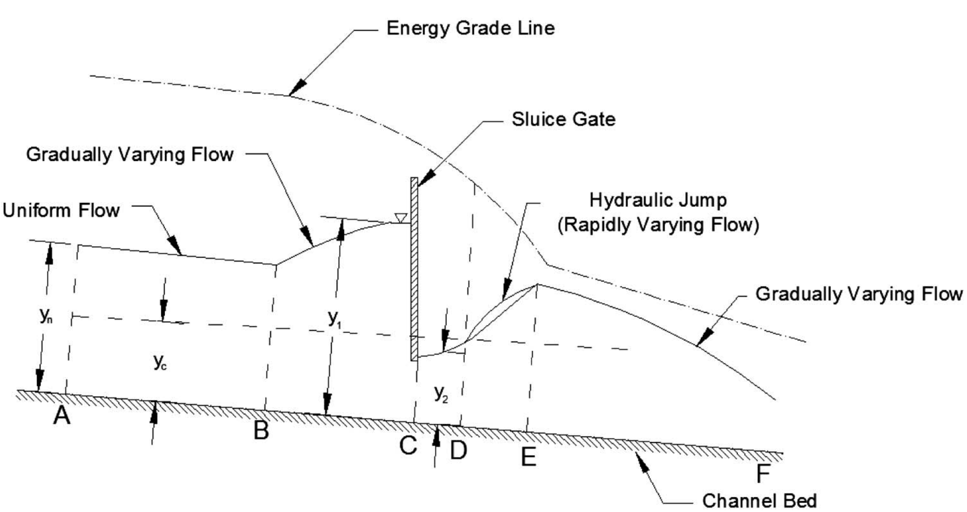
An "open" channel is any conduit that has a "free" surface. The pressure at the free surface is always zero gauge. So irrigation canals, roadside drainage ditches, roadside gutters, and even roads are considered open channels. Pipes that are not flowing full are also open channels.

### *Uniform and Nonuniform Flows*

The different types of flows that can occur in an open channel are shown in Fig. 2-1. This figure shows water flowing underneath a sluice gate located at C. A sluice gate is used to discharge water in a controlled manner by raising or lowering the gate (also see Problem 2-14). Open channel flows are classified as uniform in open channel flows if the water depth does not change. Thus, as shown in Fig. 2-1, the flow in the section from A to B is uniform flow. It is proper to assume that the pressure distribution in uniform flows is hydrostatic perpendicular to the direction of flow, i.e., one can use the equation $p = \gamma h$ to calculate the pressure at any cross section.

If the water depth does change in any section of the channel, the flow is called "nonuniform." Nonuniform flows can be gradually varying or rapidly varying, as shown in Fig. 2-1. If the depth change is gradual with distance, as in Sections

***Fig. 2-1* Description of flow in open channels**



BC, DE, and EF, the flow is termed "gradually varying." Conversely, if the water depth changes rapidly, as during a hydraulic jump, which occurs in Section DE of Fig. 2-1, the flow is termed "rapidly varying." A hydraulic jump, discussed in more detail in Problem 2-15, is characterized by a rapid change in water level associated with significant energy loss, which is why there is a significant drop in the energy grade line (EGL) shown in Fig. 2-1 at Section DE. A key difference between gradually and rapidly varying flows is that one can assume that the pressure distribution, perpendicular to the direction of flow, in gradually varying flows is hydrostatic without introducing significant error. Therefore, the equation $p = \gamma h$ is applicable for gradually varying flows. However, this assumption cannot be applied to rapidly varying flows without adding a significant error.

### *Depth and Normal Depth*

The depth in an open channel flow is the vertical depth from the water (free) surface to the lowest point in the channel bed. The depth associated with uniform flow is termed "normal" depth. The normal depth is shown as $y_n$ in Fig. 2-1. The normal depth is usually calculated by Manning's $n$ equation:

$$Q = (C_m/n)(AR^{2/3})(S^{1/2}) \tag{2-1}$$

where $Q$ is flowrate, $n$ is Manning's $n$ or resistance coefficient, $A$ is the cross-sectional area, $R$ is the hydraulic radius, and $S$ is the channel slope. $C_m$ is a constant that is equal to 1 when using SI units, or 1.49 when using British units. When using SI units, $Q$ must have units of $m^3/s$, $A$ must be in $m^2$, and $R$ must be in m. When using the British system, $Q$ must have units of $ft^3/s$, $A$ must be in $ft^2$, and $R$ must be in ft. When calculating the water depth under gradually varying nonuniform flows, $S$ *must* be replaced by $S_f$, which is the slope of the energy grade line (EGL). For example, when calculating the depth at Section CD of Fig. 2-1, one should use $S_f$ and not $S$ in Manning's $n$ equation. Note that under uniform flow conditions, the channel bed is parallel to the EGL and $S = S_f$.

### *Manning's* n

Some typical Manning's $n$ values are shown in Spreadsheet 2-1. Manning's $n$ values are relatively small for channels with smooth surfaces and larger for rougher surfaces. The reason for assigning different values to $C_m$ under the two unit systems is to allow using the same value of $n$ regardless of the unit system.

## Concepts in Gradually and Rapidly Varying Flows

Some of the important concepts involved in gradually and rapidly varying flows are discussed subsequently.

### *Specific Energy; Specific Energy Diagram; and Critical, Subcritical, and Supercritical Depths*

The value of the potential energy—and therefore the mechanical energy—at any point in an open channel depends on the location of the datum. If the arbitrary nature of the datum is removed by establishing the datum at the bottom of the channel, then the mechanical energy is known as specific energy ($E$). The specific energy is defined as

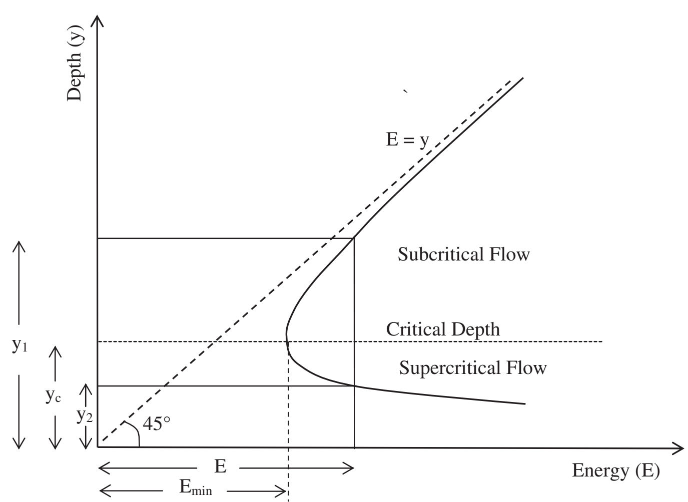
$$E = y + V^2/(2g) \tag{2-2}$$

where $y$ is the channel depth and $V$ is velocity. Eq. (2-2) is a cubic equation, and there are three values of $y$ that will satisfy this equation. However, only two of the values are real, whereas the third is imaginary. A plot of $y$ versus $E$ is known as the specific energy diagram (SED), and a typical plot of the SED is shown in Fig. 2-2. As shown in Fig. 2-2, the depth at which $E$ has a minimum value ($E_{\min}$) is termed the "critical depth" ($y_c$), and the corresponding flow is known as "critical flow." As there are two real roots of Eq. (2-2), water can flow at two different depths for any given value of $E$ greater than $E_{\min}$. If the water flows at a depth greater than $y_c$ ($y_1$ in Fig. 2-2), then the depth is termed "subcritical depth," and the corresponding flow is termed "subcritical flow." If the water flows at a depth less than $y_c$ ($y_2$ in Fig. 2-2), then the depth is termed "supercritical depth," and the corresponding flow is termed "supercritical flow." The subcritical depth is known as the alternate depth of the supercritical depth and vice versa. By definition, the specific energies at the two alternative depths are equal.

The minimum specific energy can be calculated by substituting $y_c$ for $y$ in Eq. (2-2). Therefore,

$$E_{2\min} = y_c + V_c^2/(2g) \tag{2-3}$$

***Fig. 2-2*** **A typical specific energy diagram**



where $V_c$ is the velocity at critical depth, because

$$V_c = Q/A_c \tag{2-4}$$

in which $A_c$ is the cross-sectional area at critical depth. For example,

$$A_c = by_c \tag{2-5}$$

for rectangular cross sections. Substituting Eq. (2-4) in Eq. (2-3) yields

$$E_{2\,\min} = y_c + Q^2/(2gA_c^2) \tag{2-6}$$

### *Estimation of Critical Depth*

The proof that the Froude number ($\mathrm{F}_r$) is equal to 1 if the depth is equal to the critical depth can be found in almost all standard textbooks. The Froude number is defined as

$$\mathrm{F}_r = V/(gD)^{0.5} \tag{2-7}$$

in which $D$ is known as the hydraulic depth. The hydraulic depth, in turn, is defined as

$$D = A/T \tag{2-8}$$

where $T$ is the top width, or the channel width at the water surface. Because $\mathrm{F}_r = 1$ at critical depth, combining Eqs. (2-7) and (2-8), substituting $V = Q/A$ and $A = A_c$, and squaring both sides of the equation, one can show that at critical depth the term

$$Q^2T/(gA_c^3) = 1 \tag{2-9}$$

Also, by substituting $T = b$ in Eq. (2-9) and Eq. (2-5) into Eq. (2-9), one can show that the critical depth for rectangular cross sections can be determined by

$$y_c = q^2/(g)^{(1/3)} \tag{2-10}$$

Finally, substituting Eqs. (2-5) and (2-10) into Eq. (2-6), one can show that for rectangular cross sections

$$E_{\min} = 1.5y_c \tag{2-11}$$

### *Channel Transitions*

A channel transition causes a change in the flow depth. Examples of five types of channel transitions are provided in this book. These examples show the calculations of the changes in flow depth because of (a) sudden change in channel bed elevation (Problems 2-9 and 2-10); (b) change in channel geometry (e.g., channel constriction; Problems 2-11 and 2-12); (c) change in the slope of the channel bed (Problem 2-13); (d) presence of a sluice gate (Problem 2-14); and (e) hydraulic jump (Problem 2-15). The first four transitions (a through d) occur as gradually varying flow, whereas the hydraulic jump is an example of rapidly varying flow. The reasons for changes in flow depths because of the transitions occurring in gradually varying flows can be explained with the help of an SED.

# Open Channel Design

Channel design is generally divided into two categories: (a) unlined or erodible channels or (b) lined or nonerodible channels. Channels with channel beds consisting of natural earthen material, such as sand, silt, volcanic ash, gravel, grass, cobbles, etc., are considered to be erodible channels. Nonerodible materials can be either "rigid" or "flexible." Materials used to form the lining of a rigid channel are usually cast-in-place concrete, grouted riprap, stone masonry, or a soil-cement combination. Flexible linings include vegetation, riprap, and gravel. Rigid channels can carry water at much higher velocities and, therefore, have smaller cross sections and use less land space; this could be an important criterion if land is expensive. However, rigid boundary channels are subject to failure from structural instability. Flexible channels, however, cannot carry water at a very high velocity, as erosion can be a concern. The advantage of using flexible channels is that they are less expensive.

### *Design of Erodible (Unlined) Channels*

Erodible channels are usually designed by two methods: (i) the "permissible velocity method" and (ii) the "tractive force method." An example of the permissible velocity method is described in Problem 2-16. The permissible velocity, $V$, is the maximum possible velocity that would not erode the channel bottom or sides. Typically, the value of $V$ depends on the type of soil. Relatively higher velocities are allowed for clay soils (1.8 m/s) as opposed to sandy soils (0.6 m/s). However, there are additional factors, such as whether the water is transporting colloidal silts or if the channel is straight or curved, that can factor in the selection of allowable velocity. Similarly, the minimum possible value of the side slope ($m$) for a trapezoidal channel depends on the type of soil. For example, $m = 1$ can be used for clay soils, but a minimum value of 3 should be used for sandy loams. More information about permissible velocities and side slopes can be obtained from Chow (1959) and Chaudhry (1993). Under some circumstances, the permissible velocity method will not provide a suitable design. In that case, one could apply the modified permissible velocity method, which is discussed in Problem 2-17.

### *Design of Nonerodible (Lined) Rigid Channels*

Lined channels are usually trapezoidal in shape and are designed by the "most efficient hydraulic section method" also known as the "best hydraulic section method." Two possibilities exist within the most efficient hydraulic section method for trapezoidal channels: (a) the side slope ($m$) must be 0.5774 ($1/\sqrt{3}$), or (b) the side slope value has a value other than 0.5774 and is specified by a regulatory agency or by local code. The U.S. Bureau of Reclamation, for example, recommends a side slope value of 1 vertical to 1.5 horizontal

($m = 1.5$). Because channel lining is expensive, the goal of the most efficient hydraulic section method is to minimize the wetted parameter, $P$. For trapezoidal channels:

$$P = 2y(1 + m^2)^{0.5} + b \tag{2-12}$$

where $y$ is the depth, $b$ is the bottom width, and $m$ is used to define the side slope and is the ratio of the horizontal distance to the vertical distance. To obtain the minimum $P$, one must satisfy two equations: $dP/dy = 0$ and $dP/dm = 0$. If $m$ is specified, then $dP/dm$ is not required, and

$$dp/dy = 2y(1 + m^2)^{0.5} = 0 \tag{2-13}$$

does not yield anything meaningful. However, if $m$ is not specified, then a useful equation can be derived as described subsequently. For trapezoidal channels

$$A = my^2 + by \tag{2-14}$$

If one defines a parameter $q' = b/y$, then dividing Eq. (2-14) by $y^2$ and rearranging, the resulting equation will be

$$y = [A/(m + q')]^{0.5} \tag{2-15}$$

Rearranging Eq. (2-14) one obtains

$$b = (A - my^2)/y \tag{2-16}$$

Substituting Eq. (2-16) into Eq. (2-12) yields

$$P = 2y(1 + m^2)^{0.5} + (A - my^2)/y \tag{2-17}$$

Taking the derivative of $P$ with respect to $y$, setting $dP/dy$ equal to zero, and rearranging the equation, one obtains

$$b/y = 2y[(1 + m^2)^{0.5} - m] \tag{2-18}$$

Because $q' = b/y$, substituting for $b/y$ in Eq. (2-18), one obtains

$$q' = 2y[(1 + m^2)^{0.5} - m \tag{2-19}$$

The application of Eqs. (2-15) and (2-19) is shown in Problems 2-18 and 2-19, respectively.

# Problem 2-1: Estimation of Flowrate in a Composite Cross Section

## *Problem Statement*

Determine the flowrate in a compound cross section, shown in Fig. 2-3, if the bed slope, $S = 0.0006$, and the depth, $y$, in the main channel is 6 m. Manning's $n = 0.065$ for the overbank section and 0.015 for the main channel.

## *Background*

If Manning's $n$ values are different for the overbank areas and the main channel, then the composite cross section must be divided into subsections such that each subsection can be defined by a single Manning's $n$ value. The composite cross section of Fig. 2-3 should be divided into three subsections as shown in Fig. 2-3. The depth shown for each section (Spreadsheet 2-1) is from the channel bed. The total flowrate is the sum of the flowrates from each subsection.

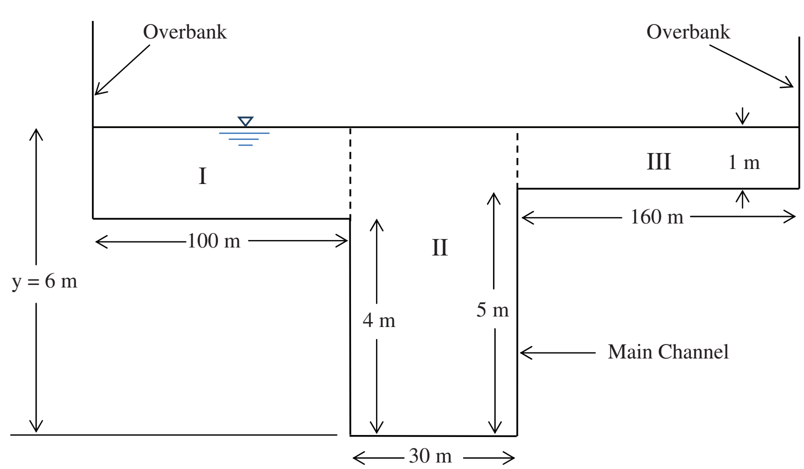
## *Assumption*

Uniform flow

## *Equations*

1. $A_1 = b_1 y_1$
2. $P_1 = b_1 + y_1$
3. $R_1 = A_1/P_1$
4. $Q_1 = (C_m/n)(A_1 R_1^{2/3})(S^{1/2})$
5. $A_2 = b_2 \times y_2$
6. $P_2 = (4 + b_2 + 5)m$
7. $R_2 = A_2/P_2$

***Fig. 2-3*** **Description of Problem 2-1**



8. $Q_2 = (C_m/n)(A_2 R_2^{2/3})(S^{1/2})$
9. $A_3 = b_3 \times y_3$
10. $P_3 = b_3 + y_3$
11. $R_3 = A_3/P_3$
12. $Q_3 = (C_m/n)(A_3 R_3^{2/3})(S^{1/2})$
13. $Q_T = Q_1 + Q_2 + Q_3$

where $P$ is the wetted perimeter, i.e., the length of the channel that is *in contact with the water*.

### *Solution Procedure*

The solution procedure required the calculation of parameters using Eqs. 1 to 13 in the order shown. Manning's $n$ value used in Eq. 4 is for the main channel, and the value used in Eqs. 8 and 12 are for the overbank section.

### *Solution*

The answer is 993 $m^3/s$ as shown in Spreadsheet 2-1.

### *What-If Analyses*

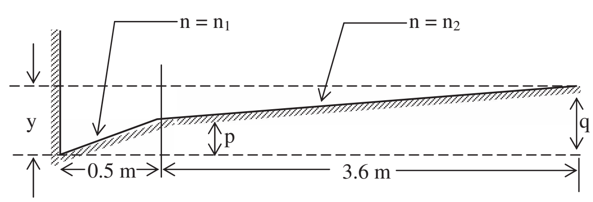
A what-if analysis shows that

(a) The flowrate decreases to 504 $m^3/s$ if the depth in Section 2 reduces from 6 m to 4.5 m. Note that in this case the flow will be only through Subsections 1 and 2.
(b) The flowrate decreases to 257 $m^3/s$ if the depth in Section 2 reduces from 6 m to 3 m. Note that in this case the flow will be only through Subsection 2.

### *Assigned Problem 2-1-1*

Determine the flowrate for the road and gutter cross section shown in Fig. 2-4 if $p = 0.15$ m, $q = 0.2$ m, $y = 0.3$ m, $n_1 = 0.015$, $n_2 = 0.013$, and $S = 0.0006$.

***Fig. 2-4* Road and gutter cross section**



# Problem 2-2: Estimation of Normal Depth in a Rectangular Cross Section

## *Problem Statement*

Determine the normal depth, $y_n$, for a rectangular cross section if the bed slope $S = 0.004$, the flowrate $Q = 25\ \mathrm{m}^3/\mathrm{s}$, Manning's $n = 0.012$, and the channel width $b = 4$ m.

## *Background*

The description of normal depth provided in Section 2.

## *Equations*

1. $A = by_n$
2. $P = b + 2y_n$
3. $R = A/P$
4. $Q = (C_m/n)(AR^{2/3})(S^{1/2})$

## *Solution Procedure*

Different values of the normal depth were assumed until the calculated flowrate, $Q = 25\ \mathrm{m}^3/\mathrm{s}$.

## *Solution*

The answer is 1.37 m as shown in Spreadsheet 2-2.

## *What-If Analyses*

A what-if analysis shows that

(a) The normal depth increases to 1.68 m if the channel is lined with rough asphalt ($n = 0.016$). This makes sense because a higher Manning's $n$ (0.016 compared with 0.012) implies that more resistance is provided to the flow. More resistance to flow will cause a lower flow velocity, which, in turn, will lead to a higher cross-sectional area being occupied by the water as $Q = VA$, and $Q$ is still $25\ \mathrm{m}^3/\mathrm{s}$. A higher cross-sectional area implies a higher normal depth, as $A = by_n$ and $b$ is still 4 m.

(b) The normal depth increases to 6.03 m if the channel bed is earthen with overgrown weeds ($n = 0.080$).

## *Assigned Problem 2-2-1*

Solve Problem 2-2 using Goal Seek.

# Problem 2-3: Estimation of Normal Depth in a Trapezoidal Cross Section

## *Problem Statement*

Determine the normal depth, $y_n$, for a trapezoidal cross section if the bed slope $S = 0.0019$, the flowrate $Q = 10\ \mathrm{m^3/s}$, Manning's $n = 0.013$, the channel width $b = 3\ \mathrm{m}$, and the side slope ratio, $m$, is 2 horizontal to 1 vertical $(2H:1V)$. Thus, $m = 2$ in this problem.

## *Background*

This problem is very similar to Problem 2-2 except that the equations for calculating $A$ and $P$ are different from those used in Problem 2-2.

## *Assumption*

Uniform flow

## *Equations*

1. $A = my_n^2 + by_n$
2. $P = 2(my_n^2 + y_n^2)^{0.5} + b$
3. $R = A/P$
4. $Q = (C_m/n)(AR^{2/3})(S^{1/2})$

## *Solution Procedure*

Different values of the normal depth were assumed until the calculated flowrate, $Q = 10\ \mathrm{m^3/s}$.

## *Solution*

The answer is 0.88 m as shown in Spreadsheet 2-3.

## *What-If Analyses*

A what-if analysis shows that the normal depth decreases to 0.774 m if the channel side slope becomes flatter and $m$ is increased to 4 (from 2).

## *Assigned Problem 2-3-1*

Determine the normal depth for a trapezoidal cross section with unequal side slope; the side slopes for the two sides are $m_1 = 2.5$ and $m_2 = 3.5$. Also $b = 4\ \mathrm{m}$, $Q = 30\ \mathrm{m^3/s}$, $n = 0.014$, and $S = 0.0004$.

# Problem 2-4: Estimation of Normal Depth in a Composite Cross Section

## *Problem Statement*

Determine the normal depth, $y_n$, for the composite cross section shown in Fig. 2-3 if the bed slope = 0.0006 and the flowrate $Q = 400\ \text{m}^3/\text{s}$. Manning's $n = 0.065$ for the overbank section, 0.015 for the main channel.

## *Background*

This problem is similar to Problem 2-1 as it has the same exact cross section. The difference is that in Problem 2-1 the normal depth was given and one had to find flowrate. In this problem, the flowrate is given and one has to find the normal depth, $y_n$. This problem is slightly trickier compared with Problem 2-1 because the normal depth could be in three different zones or criteria for this particular cross section: (i) $y_n < 4$ m, (ii) $4\ \text{m} < y_n < 5$ m, and (iii) $y_n > 5$ m. Each composite cross section will have its own set of possible criteria.

## *Assumption*

Uniform flow

## *Equations*

See equations for Problem 2-1.

## *Solution Procedure*

First, you need to find which of the three criteria will apply for this problem. One way to determine this is by assuming $y_2 = 4$ m and determining the flowrate. Under this condition, the flow will take place only in Section 2. The flowrate is $415\ \text{m}^3/\text{s}$ at $y_2 = 4$ m (Spreadsheet 2-4). Because the given flowrate $Q$ ($400\ \text{m}^3/\text{s}$), is less than $415\ \text{m}^3/\text{s}$, *the normal depth will be less than 4 m and the flow will be only in Subsection 2 if* $Q = 400\ \text{m}^3/\text{s}$.

## *Solution*

The answer is 3.92 m as shown in Spreadsheet 2-4.

## *What-If Analyses*

A what-if analysis shows that

(a) The normal depth increases to 4.84 m if the flowrate is increased to $600\ \text{m}^3/\text{s}$ (from $400\ \text{m}^3/\text{s}$). In this case, the flow will be only in Sections 1 and 2 (why?).

(b) The normal depth increases to 5.51 m if the flowrate is increased to $800\,m^3/s$ (from $400\,m^3/s$). Under these conditions, the flow will be in all three subsections.

### *Assigned Problem 2-4-1*

Determine the normal depth for the cross section shown in Fig. 2-4 if the flowrate is (a) $0.4\,m^3/s$ and (b) $0.1\,m^3/s$.

# Problem 2-5: Estimation of Normal Depth in a Circular Pipe Using Manning's $n$ Equation

## *Problem Statement*

Determine the normal depth in a circular pipe with a diameter $d = 0.25$ m, under uniform conditions, if the bed slope = 0.01, Manning's $n = 0.012$, and the flowrate (a) $Q = 0.02\ \mathrm{m^3/s}$, (b) $Q = 0.05\ \mathrm{m^3/s}$, and (c) $Q = 0.1\ \mathrm{m^3/s}$.

## *Background*

This is very similar to Problems 2-1 and 2-2 except the equations for calculating $A$ and $P$ will be different from those used in Problems 2-1 and 2-2. Also, a circular pipe can carry a maximum amount of flowrate before it becomes full. Manning's $n$ equation will *not* apply if the pipe is flowing full.

## *Assumption*

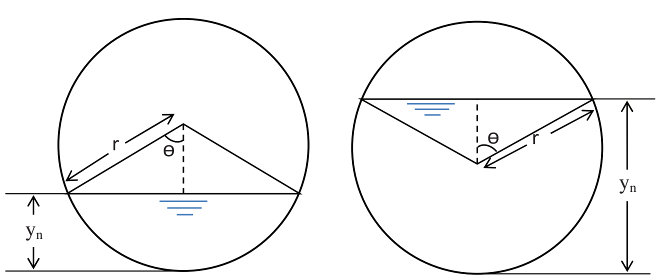
Uniform flow

## *Equations*

1. $\mathrm{Cos}\theta = \mathrm{ABS}[(y_n - d/2)/(d/2)]$
2. $\theta = \mathrm{Cos}^{-1}(\mathrm{Cos}\theta)$
3. $A = (d^2/4)(\theta - \mathrm{Sin}\theta\mathrm{Cos}\theta)$ (for $y_n < r$)
4. $A = (d^2/4)(\pi - \theta + \mathrm{Sin}\theta\mathrm{Cos}\theta)$ (for $y_n > r$)
5. $P = 2r\theta$ (for $y_n < r$)
6. $P = (2r)(\pi - \theta)$ (for $y_n > r$)
7. $R = A/P$
8. $Q = (C_m/n)(AR^{2/3})(S^{1/2})$

where $r$ is the radius of the pipe, and the angle $\theta$ is defined in Fig. 2-5 for both $y_n < r$ and $y_n > r$. The equations for estimating the cross-sectional area and the wetted perimeter are different if the normal depth is below (Eqs. 3 and 5) or above the center of the pipe (Eqs. 4 and 6).

***Fig. 2-5* Description of Problem 2-5**




$y_n < r$ $y_n > r$

### *Solution Procedure*

An “IF” statement is required when calculating $A$ and $P$ because two equations are possible depending on whether $y_n < r$ or $y_n > r$. Different values of $y_n$ are assumed, and the various parameters are calculated using the aforementioned equations until the value of the flowrate calculated in the last column is equal to a given flowrate. Spreadsheet 2-5 shows two iterations for Problem 2-5a ($y_n = 0.1$ m and $y_n = 0.095$ m) and two iterations for Problem 2-5b ($y_n = 0.15$ m and $y_n = 0.165$ m).

### *Solution*

The normal depths for Problems 2-5a and 2-5b are 0.095 m and 0.165 m, respectively. No solution is possible for Problem 2-5c, because the desired $y_n$ is greater than the pipe diameter.

### *What-If Analyses*

Conduct a what-if analysis to demonstrate that the normal depth will decrease if the pipe slope increases by plotting a graph of $y_n$ versus $S$, keeping all other parameters the same as those of Problem 2-5.

### *Assigned Problem 2-5-1*

Solve Problem 2-5 using Goal Seek.

### *Assigned Problem 2-5-2*

Using Fig. 2-5, derive Eqs. 3, 4, 5, and 6.

### *Assigned Problem 2-5-3*

Assume that pipes can be purchased in increments of 0.25 m (0.25 m, 0.5 m, etc.). At what minimum pipe diameter, $d$, can the pipe of Problem 2-5 carry a flowrate, $Q = 0.1\ \mathrm{m}^3/\mathrm{s}$? All other parameters, except diameter, are identical to Problem 2-5.

### *Assigned Problem 2-5-4*

Assume that pipes can be purchased in increments of 0.25 m (0.25 m, 0.5 m, etc.). At what minimum pipe diameter, $d$, can the pipe of Problem 2-5 carry a flowrate, $Q = 1.2\ \mathrm{m}^3/\mathrm{s}$. All other parameters, except diameter, are identical to Problem 2-5. What will be the normal depth in the pipe under these conditions?

### *Hint*

You can use the spreadsheet of Problem 2-5 to solve this problem.

### *Assigned Problem 2-5-5*

Prove that (i) $\mathrm{Cos}\theta = \mathrm{ABS}[(y_n - d/2)/(d/2)]$, (ii) $A = (d^2/4)(\theta - \mathrm{Sin}\theta\mathrm{Cos}\theta)$ if $y_n < r$, and (iii) $A = (d^2/4)(\pi - \theta + \mathrm{Sin}\theta\mathrm{Cos}\theta)$ if $y_n > r$.

# Problem 2-6: Plotting a Specific Energy Diagram

## *Problem Statement*

Plot the specific energy diagram (SED) for a trapezoidal channel with a width of $b = 3$ m and side slopes of 1 : 2.25 vertical to horizontal. Therefore, the vertical to horizontal ratio is $m = 2.25$. The flowrate is $Q = 40\ \text{m}^3/\text{s}$.

## *Background*

The SED is an important plot in the field of open channel flows. You need to know the channel shape, dimensions, and flowrate to plot the SED; *the values of Manning's* n *and bed slope are not required*. An SED becomes a particularly useful tool when you have to determine the change in channel depth because of channel transitions, such as change in channel width, change in channel geometry, or change in bed slope elevation. Although it is possible to quantitatively determine the depth changes because of channel transitions without using the SED, the SED is a useful tool to explain why these changes occur. This is demonstrated in subsequent problems.

## *Assumption*

Uniform flow

## *Equations*

1. $A = my^2 + by$
2. $V = Q/A$
3. $E = y + V^2/(2g)$

## *Solution Procedure*

A value of $y$ was assumed and the corresponding values of $E$ were computed using Eqs. 1 to 3, as shown in Spreadsheet 2-6. Values of $y$ should be selected so that the $E$ values are similar for the upper and lower limbs, as shown in Fig. 2-6. Plot $E$ on the $x$-axis and $y$ on the $y$-axis. Typically, the scale of the $x$ and $y$ axes should be identical.

## *Solution*

The SED is plotted in Fig. 2-6.

## *What-If Analyses*

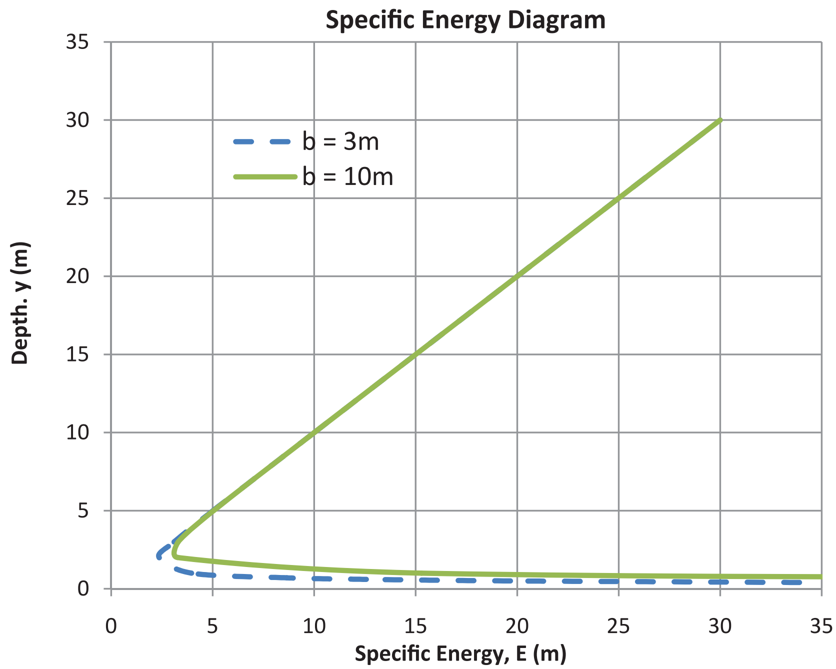
Conduct a what-if analysis by plotting the necessary SEDs to show that

(a) The SED moves to the left as $b$ increases to 10 m (from 3 m). The SED will always move to the left if the cross-sectional area $A$ increases. This is demonstrated in Fig. 2-6.

(b) The SED moves to the right as $b$ decreases to 0.5 m (from 3 m). The SED will always move to the right if the cross-sectional area $A$ decreases.

(c) The SED moves to the right as $Q$ increases to 100 $m^3/s$ (from 100 $m^3/s$; $b = 3$ m). The SED will always move to the right if the flowrate increases and all other parameters remain the same.

***Fig. 2-6* Specific energy diagrams for Problem 2-6**



# Problem 2-7: Determine the Critical Depth of a Trapezoidal Cross Section

## *Problem Statement*

Determine the critical depth, $y_c$, for a trapezoidal cross section if the flowrate $Q = 10\ \text{m}^3/\text{s}$, the channel width $b = 3$ m, and the side slope ratio is 4 horizontal to 1 vertical ($4H:1V$). The side slope is characterized by a parameter, $m$, which is the ratio of the horizontal distance to the vertical distance. Thus, $m = 4$ in this problem.

## *Background*

It can be shown that the Froude number is $\text{F}_r = 1$ at critical depth $y_c$. As shown in Section 2, this also implies that the parameter $Q^2T/(gA^3) = 1$ at $y = y_c$. The values of $\text{F}_r$ and $Q^2T/(gA^3)$ will be greater than 1 if the flow is supercritical and less than 1 if the flow is subcritical. This is another way of determining if the flow is subcritical or supercritical. The starting equation for determining critical depth can be either $\text{F}_r = 1$ or $Q^2T/(gA^3) = 1$, although it is more convenient to start with the latter equation.

## *Equations*

1. $A = my_c^2 + by_c$
2. $T = 2my_c + b$
3. $\text{Parameter} = (Q^2T)/(gA^3)$
4. $D = A/T$
5. $V = Q/A$
6. $\text{F}_r = V/(gD)^{0.5}$

## *Solution Procedure*

A value of $y_c$ was assumed (1 m) and the corresponding values of $Q^2T/(gA^3)$ and $\text{F}_r$ were calculated using Eqs. 1 to 5. The value of $y_c$ for which $Q^2T/(gA^3)$ or $F_r = 1$ is the critical depth. Eqs. 4 to 6 are really not needed, because it is sufficient to calculate the term $Q^2T/(gA^3)$. However, $\text{F}_r$ was calculated mainly to demonstrate that $\text{F}_r = 1$ when $Q^2T/(gA^3) = 1$.

## *Solution*

The critical depth $y_c = 0.75$ m as shown in Spreadsheet 2-7.

## *What-If Analyses*

A what-if analysis shows that

(a) The critical depth increases to 2.3 m if $Q$ increases from $10\ \text{m}^3/\text{s}$ to $20\ \text{m}^3/\text{s}$.
(b) The critical depth increases to 0.99 m if $b$ decreases from 3 m to 0.5 m ($Q = 10\ \text{m}^3/\text{s}$).

The analyses show that the critical depth increases when the flowrate is increased or the cross section area is decreased. An identical conclusion can be made from the SEDs of Problem 2-6.

### *Assigned Problem 2-7-1*

Determine the critical depth for a trapezoidal cross section with unequal side slope; the side slopes for the two sides are $m_1 = 2.5$ and $m_2 = 3.5$. Also $b = 4$ m, $Q = 30\ \mathrm{m^3/s}$, $n = 0.014$, and $S = 0.0004$.

# Problem 2-8: Determine the Critical Depth of a Composite Cross Section

## *Problem Statement*

Determine the critical depth, $y_c$, for the composite cross section of Fig. 2-3. The flowrate is $Q = 400\ \mathrm{m}^3/\mathrm{s}$.

## *Background*

Calculating the critical depth of a composite cross section is slightly tricky, because the equations for computing $y_c$ may change depending on the value of $y_c$. For example, the equations for this particular cross section will be different if (i) $y_c < 4\ \mathrm{m}$, (ii) $4\ \mathrm{m} < y_c < 5\ \mathrm{m}$, or (iii) $y_c > 5\ \mathrm{m}$.

## *Equations*

For $y_c < 4\ \mathrm{m}$

1. $A = b_2 y_c$
2. $T = b_2$

For $4\ \mathrm{m} < y_c < 5\ \mathrm{m}$

3. $A = b_1(y_c - 4) + b_2 y_c$
4. $T = b_1 + b_2$

For $y_c > 5\ \mathrm{m}$

5. $A = b_1(y_c - 4) + b_2 y_c + b_3(y_c - 5)$
6. $T = b_1 + b_2 + b_3$

In addition, one can calculate the flowrate, $Q$ by

7. $Q = (gA^3/T)^{0.5}$
8. $\text{Parameter} = (Q^2 T)/(gA^3)$

## *Solution Procedure*

Assume a value of $y_c$, and compute the corresponding values of $Q$ and $Q^2T/(gA^3)$ using a combination of the equations listed. For example, you should use Eqs. 3, 4, and 7 if the assumed $y_c$ is between 4 m and 5 m. The trial-and-error solution will indicate the correct range of value for $y_c$. For example, $Q = 752\ \text{m}^3/\text{s}$ when $y_c = 4$ m. Therefore, $y_c$ must be less than 4 m if the given $Q = 400\ \text{m}^3/\text{s}$ as in this problem.

### *Solution*

The critical depth $y_c = 2.63$ m as shown in Spreadsheet 2-8.

### *Assigned Problem 2-8-1*

Determine the critical depth, $y_c$, for the composite cross section of Fig. 2-3 if (a) the flowrate is $Q = 1{,}000\ \text{m}^3/\text{s}$ and (b) the flowrate, $Q = 2{,}000\ \text{m}^3/\text{s}$. What equations did you use in calculating the answers to Parts a and b?

# Problem 2-9: Channel Transition: Determine the Downstream Normal Depth if There Is a Sudden Rise or Drop in the Channel Bed (No-Choke Condition)

## *Problem Statement*

The flowrate, $Q$, in a rectangular channel is $20\,\mathrm{m^3/s}$. The channel width is $b = 5\,\mathrm{m}$. There is a plan to raise the channel bed by 0.2 m ($\Delta z = 0.2\,\mathrm{m}$). Assume that Sections 1 and 2 lie just upstream and downstream of the sudden rise, as shown in Fig. 2-7. The depth, $y_1$, at Section 1 is 2 m. Determine the depth, $y_2$, that will occur at Section 2 as a result of raising the channel bed.

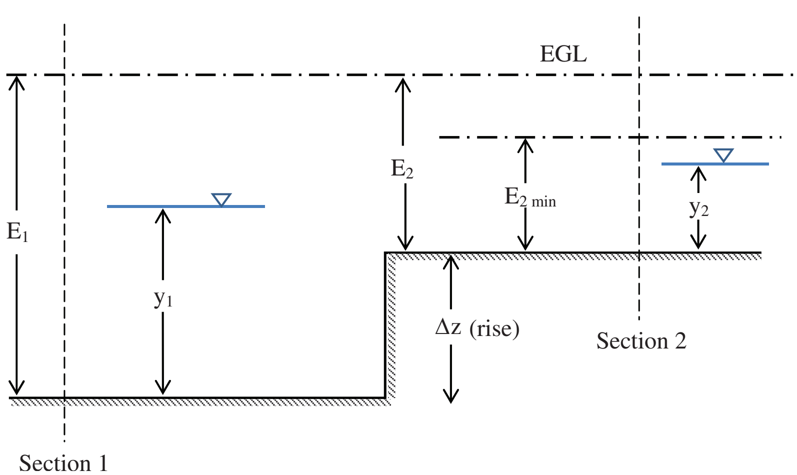
## *Background*

It is assumed that the channel bed is horizontal and that no energy loss occurs between Sections 1 and 2, i.e., the energy grade line is also horizontal (Fig. 2-7). A sudden rise ($\Delta z$) in the channel bed causes the specific energy at Section 2, $E_2$, to decrease compared with $E_1$. In this problem, the change in the channel bed elevation is not sufficiently large to cause $E_2$ to become less than $E_{2\,\mathrm{min}}$ (Fig. 2-7), which is the minimum specific energy required at Section 2. This is called the "no-choke condition." Under the no-choke condition, the flow type at Section 2 will be subcritical if the flow at Section 1 is subcritical and will be supercritical if the flow at Section 1 is supercritical.

## *Assumptions*

The channel bed is horizontal, and no energy loss occurs between Sections 1 and 2.

***Fig. 2-7*** **Description of Problem 2-9 (no-choke condition, i.e., $E_2 > E_{2\,\mathrm{min}}$)**



### *Equations*

1. $A_1 = b_1 y_1$
2. $V_1 = Q/A_1$
3. $E_1 = y_1 + V_1^2/(2g)$
4. $E_2 = E_1 - \Delta z$
5. $y_{1c} = [(Q^2/b_1^2)/(g)]^{(1/3)}$
6. $y_{2c} = [(Q^2/b_2^2)/(g)]^{(1/3)}$
7. $E_{2\,\min} = (1.5)(y_{2c})$
8. $A_2 = b_2 y_2$
9. $V_2 = Q/A_2$
10. $E_2 = y_2 + V_2^2/(2g)$

### *Solution Procedure*

The values of $E_2$ and $E_{2\,\min}$ are compared (Spreadsheet 2-9) to determine if a choke has occurred. In this case, a choke has not occurred, because $E_2$ (2.00 m) is greater than $E_{2\,\min}$ (1.77 m). Because a choke has not occurred, assume a value of $y_2$ (Spreadsheet 2-9) until $E_2 = 2.00$ m. Note that, as expected, there are two values of $y_2$ (1.73 m and 0.84 m) that provide $E_2 = 2$ m (Spreadsheet 2-9). These values are the two real roots of Eq. (2-2). However, because the depth at Section 1 is subcritical ($y_1 > y_{ic}$), the depth at Section 2 must also be subcritical, i.e., $y_2 > y_{2c}$, and the correct answer is 1.73 m. Note that Eq. 1 and Eqs. 5 through 8 are applicable only for rectangular sections.

### *Solution*

The depth $y_2$, at Section 2, is 1.73 m. It is interesting to note that, contrary to intuition, the depth at Section 2 *decreases compared with the depth at Section 1* as a result of the rise in channel bed elevation.

### *What-If Analyses*

A what-if analysis shows that

(a) $y_{2\text{new}}$ *decreases* to 1.56 m as a result of a 0.3 m rise in bed elevation (instead of a 0.2 m rise).
(b) $y_{2\text{new}}$ *increases* to 3.2 m as a result of a 1 m *drop* ($\Delta z = -1$ m) in bed elevation.

### *Assigned Problem 2-9-1*

Showing all steps, prove Eqs. (2-10) and (2-11).

### *Assigned Problem 2-9-2*

Determine the minimum value of $\Delta z$ at which a choke will form, i.e., $E_2$ will become equal to $E_{2\,\min}$.

### *Assigned Problem 2-9-3*

Using Goal Seek, determine the minimum value of $\Delta z$ at which a choke will form, i.e., $E_2$ will become equal to $E_{2\,\mathrm{min}}$, if $b_1$ and $b_2$ are equal to 4 m and all other parameters are the same as Problem 2-9.

### *Assigned Problem 2-9-4*

Determine the value of $y_2$ if the channel of Problem 2-9 was a trapezoidal channel with side slopes having horizontal to vertical ratios of 4 to 1. All other parameters are the same as Problem 2-9.

### *Hint*

Eq. 1 and Eqs. 5 through 8 will no longer apply, and $E_{2\,\mathrm{min}}$ will have to be calculated using Eq. (2-3).

# Problem 2-10: Channel Transition: Determine the Normal Depths if There Is a Sudden Rise or Drop in the Channel Bed (Choke Condition)

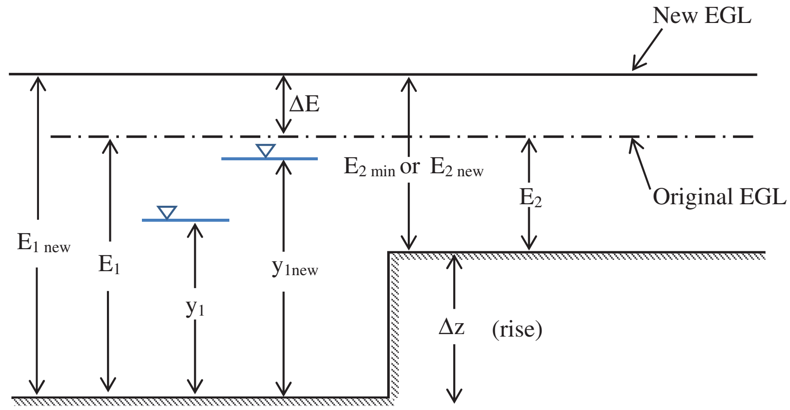
## *Problem Statement*

The flowrate, $Q$, in a rectangular channel is 20 $m^3/s$. The channel width is $b=5$ m. There is a plan to raise the channel bed by 2 m ($\Delta z=2$ m). Assume that Sections 1 and 2 lie just upstream and downstream of the sudden rise, as shown in Fig. 2-8. The depth, $y_1$, at Section 1, prior to change in bed elevation, is 2 m. Determine the depth, $y_2$, that will occur at Section 2 as a result of raising the channel bed. Also, determine the new channel depth, $y_{2\text{new}}$, as a result of the choke.

## *Background*

This problem demonstrates the calculations of $y_2$ and $y_{1\text{new}}$ if $\Delta z$ is large enough to create a choke at Section 2. The higher the rise in channel bed, the lower will be the drop in the specific energy at Section 2 ($E_2$). At some value of $\Delta z$, which is termed $\Delta z_{cr}$, $E_2$ will become equal to $E_{2\,\text{min}}$. A *choke* will form at Section 2 if the drop in specific energy is such that $E_2 \leq E_{2\ \text{min}}$, which is the minimum specific energy required at Section 2. When a choke forms, sufficient energy is added to the system owing to the rise in channel bed (or channel constriction as discussed in the next problem), such that $E_2$ becomes equal to $E_{2\,\text{min}}$. In effect, the added specific energy, $\Delta E = E_2 - E_{2\,\text{min}}$, leads to the formation of a new energy grade line, which is shown in Fig. 2-8. As a result, the specific energies at Sections 1 and 2, respectively, increase to $E_{i\text{new}}$ and $E_{2\text{new}}$ where $E_{2\text{new}} = E_{2\,\text{min}}$ and $E_{1\text{new}} = E_{2\ \text{min}} + \Delta z$, as shown in Fig. 2-8. Under choke conditions $y_2$ is equal to the critical depth at Section 2 ($y_{2c}$). Furthermore, the upstream depth, $y_1$, also *increases* to $y_{1\text{new}}$ under choke conditions (Fig. 2-8). The depth $y_{1\text{new}}$ *must be subcritical* after a choke occurs, irrespective of the initial flow condition at Section 1, i.e., irrespective of whether the flow was subcritical or supercritical at Section 1 prior to change in the bed elevation.

***Fig. 2-8*** **Description of Problem 2-10 (choke condition, i.e., $E_2 < E_{2\,\text{min}}$)**



### *Assumptions*

Assume that the channel bed is horizontal, and no energy loss occurs between Sections 1 and 2.

### *Equations*

1. $A_1 = b_1 y_1$
2. $V_1 = Q/A_1$
3. $E_1 = y_1 + V_1^2/(2g)$
4. $E_2 = E_1 - \Delta z$
5. $y_{1c} = [(Q^2/b_1^2)/(g)]^{(1/3)}$
6. $y_{2c} = [(Q^2/b_2^2)/(g)]^{(1/3)}$
7. $E_{2\,\mathrm{min}} = (1.5)(y_{2c})$
8. $E_{1\mathrm{new}} = E_1 + (E_{2\,\mathrm{min}} - E_2)$
9. $A_{1\mathrm{new}} = b_1 y_{1\mathrm{new}}$
10. $V_{1\mathrm{new}} = Q/A_{1\mathrm{new}}$
11. $E_{1\mathrm{new}} = y_{1\mathrm{new}} + V_{1\mathrm{new}}^2/(2g)$

where $A_{1\mathrm{new}}$ and $V_{1\mathrm{new}}$ are the respective area and velocity at Section 1 after bed construction.

### *Solution Procedure*

The values of $E_2$ and $E_{2\,\mathrm{min}}$ are compared (Spreadsheet 2-10) to determine if a choke has occurred. In this case a choke has occurred, because $E_2$ (0.2 m) is less than $E_{2\,\mathrm{min}}$ (1.77 m). Because a choke has occurred, $y_2 = y_{2c}$. Also, $y_{1\mathrm{new}}$ is determined by trial and error as shown in Spreadsheet 2-10a by assuming values of $y_{1\mathrm{new}}$ until $E_{1\mathrm{new}} = 3.77$ m. Note that $y_{1\mathrm{new}} = 3.71$ m and 0.5 m both provide an $E_{1\mathrm{new}} =$ 3.77 m. But the new depth at Section 1 *must* be subcritical depth, i.e., $y_{1\mathrm{new}}$ *must* be greater than $y_{1c}$. Therefore, the correct answer is 3.77 m and not 0.5 m.

### *Solution*

$y_{2\mathrm{new}} = 3.71$ m. The value of $y_{1\mathrm{new}}$ *increases* from 2 m to 3.71 m as a result of the 2 m rise in bed elevation.

### *What-If Analyses*

A what-if analysis shows that $y_{1\mathrm{new}}$ would *increase* from 2 m to 6.75 m as a result of a 5 m dam construction; a dam construction is similar to a change in bed elevation.

### *Assigned Problem 2-10-1*

Determine the value of $y_2$ and $y_{1\mathrm{new}}$ if $y_1$ were equal to 0.6 m instead of 2 m. All other parameters are the same as Problem 2-10.

# Problem 2-11: Channel Transition: Determine the Change in Depth because of Channel Constriction (or Expansion)

## *Problem Statement*

The flowrate, $Q$, in a rectangular channel is $20\ \mathrm{m}^3/\mathrm{s}$, and the channel width at Section 1 ($b_1$) is equal to 5 m. Encroachment in the channel (Fig. 2-9) causes the channel width at Section 2 ($b_2$) to be 4 m. The depth, $y_1$, at Section 1 is 2 m prior to the bridge construction. Determine the depth, $y_2$, which will occur at the constricted section.

## *Background*

As shown in Problem 2-6, a constriction in the channel geometry at Section 2 will shift the SED to the right. Thus, unlike Problems 2-9 and 2-10, there will be separate, specific energy diagrams for Sections 1 and 2. A constriction will result in an increase in the critical depth and in the minimum specific energy required to maintain flow. The shifting in the SED causes a change in the depth at Section 2 ($y_2$), even though the specific energies at Sections 1 and 2, $E_1$ and $E_2$, are equal. A choke will form at Section 2 if the minimum specific energy required to maintain flow in Section 2 ($E_{2\,\mathrm{min}}$) is less than $E_2$.

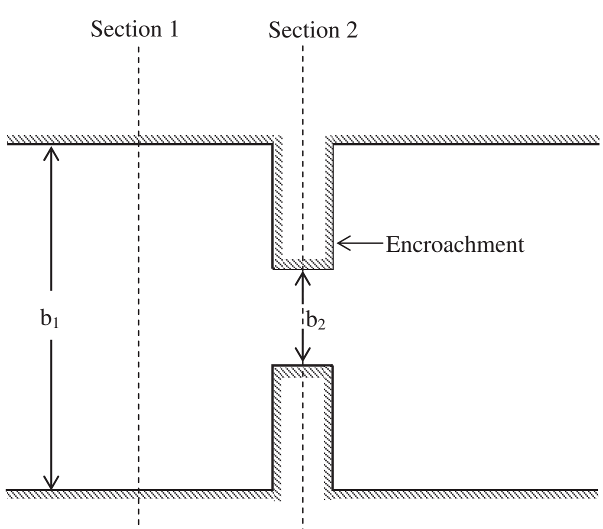
## *Assumptions*

Assume that the channel bed is horizontal, and no energy loss occurs between Sections 1 and 2.

## *Equations*

1. $A_1 = b_1 y_1$
2. $V_1 = Q/A_1$

***Fig. 2-9*** **Description of Problem 2-11**



3. $E_1 = y_1 + V_1^2/(2g)$
4. $E_2 = E_1$
5. $y_{1c} = [(Q^2/b_1^2)/(g)]^{(1/3)}$
6. $y_{2c} = [(Q^2/b_2^2)/(g)]^{(1/3)}$
7. $E_{2\,\min} = (1.5)(y_{2c})$
8. $A_2 = b_2 y_2$
9. $V_2 = Q/A_2$
10. $E_2 = y_2 + V_2^2/(2g)$

### *Solution Procedure*

The calculations are shown in Spreadsheet 2-11. A comparison of the values of $E_2$ and $E_{2\,\min}$ indicates that a choke has not occurred, because $E_2$ (2.2 m) is greater than $E_{2\,\min}$ (2.05 m). Because a choke has not occurred, assume a value of $y_{2\text{new}}$ (Spreadsheet 2-11a) until $E_2 = 2.2$ m.

### *Solution*

Two values of $y_2$ provide a value of $E_2 = 2.20$ m (Spreadsheet 2-11a); however, $y_2 = 1.81$ m is the correct value, because the flow at Section 1 was subcritical prior to the bridge construction ($y_1 > y_{1c}$).

### *What-If Analyses*

Using a what-if analysis and plotting a graph of $y_2$ versus $Q$ show that $y_2$ will decrease as $Q$ increases (all other parameters remain the same).

### *Assigned Problem 2-11-1*

At some critical value of $Q$, $y_2$ will become equal to $y_{2c}$, and a choke will occur. Determine this critical value of $Q$.

### *Assigned Problem 2-11-2*

Determine the value of $y_2$ and $y_{1new}$ if $b_2$ is equal to 3 m instead of 4 m. All other parameters are the same as Problem 2-11.

### *Hint*

Note that this will be a choke condition.

### *Assigned Problem 2-11-3*

Determine the value of $y_2$ if $y_1$ is equal to 0.7 m instead of 2 m. All other parameters are the same as Problem 2-11.

### *Hint*

The flow in Section 1 is now supercritical.

# Problem 2-12: Channel Transition: Determine the Change in Depth if There Is a Channel Constriction and a Change in Elevation Bed

## *Problem Statement*

The flowrate, $Q$, in a rectangular channel is $20\ m^3/s$. The channel constricts from a width of 5 m at Section 1 to 4 m at Section 2. In addition, the channel bed at Section 2 is elevated by 1 m ($\Delta z = 1$ m) compared to Section 1. The depth, $y_1$, at Section 1 is 2 m. Determine the depths $y_{1\text{new}}$ and $y_2$ at Sections 1 and 2, respectively.

## *Background*

This problem combines the effect of the bed elevation and channel constriction discussed in Problems 2-9 to 2-11.

## *Assumptions*

Assume that the channel bed is horizontal, and no energy loss occurs between Sections 1 and 2.

## *Equations*

1. $A_1 = b_1 y_1$
2. $V_1 = Q/A_1$
3. $E_1 = y_1 + V_1^2/(2g)$
4. $E_2 = E_1 - \Delta z$
5. $y_{1c} = [(Q^2/b_1^2)/(g)]^{(1/3)}$
6. $y_{2c} = [(Q^2/b_2^2)/(g)]^{(1/3)}$
7. $E_{2\min} = (1.5)(y_{2c})$
8. $E_{1\text{new}} = E_{2\min} + \Delta z$
9. $A_{1new} = b_1 y_{1\text{new}}$
10. $V_{1\text{new}} = Q/A_{1\text{new}}$
11. $E_{1\text{new}} = y_{1\text{new}} + V_{1\text{new}}^2/(2g)$

## *Solution Procedure*

The values of $E_2$ and $E_{2\min}$ are compared to determine if a choke has occurred. In this case a choke has occurred, because $E_2$ (1.20 m) is less than $E_{2\min}$ (2.05 m).

## *Solution*

Because a choke has occurred, $y_2$ will be equal to $y_{2c}$. Also, the value of $y_1$ will change to $y_{1\text{new}}$, which is calculated in Spreadsheet 2-12. The value of $y_{1\text{new}} = 3.04$ m.

*What-If Analyses*

Conduct a what-if analysis by plotting a graph between $y_{1new}$ and $b_2$, and show that the value of $y_{1new}$ will increase if the channel width is lowered to below 4 m.

*Assigned Problem 2-12-1*

Using Goal Seek, determine the highest possible value of $b_2$ ($b_{2crit}$) at which a choke will occur. All other parameters are the same as Problem 2-12.

# Problem 2-13: Calculate the Critical Slope for a Trapezoidal Cross Section and Determine if the Bed Slope Is Mild or Steep

### *Problem Statement*

The flowrate, $Q$, in a trapezoidal channel is 60 $m^3/s$. The channel width is $b=3$ m. The side slope, characterized by a parameter, $m$, the ratio of the horizontal distance to the vertical distance, is equal to 2 ($m=2$). The channel has a bed slope $S=0.0019$ and Manning's $n=0.013$. Determine if the slope will act as a mild (M) or steep (S) slope under uniform flow conditions.

### *Background*

The critical slope, $S_c$, is the bed slope at which the critical depth ($y_c$) is equal to the normal depth, $y_n$, under uniform flow conditions. A bed slope is considered mild if it is less than the critical slope and steep if it is greater than the critical slope. Under uniform flow conditions, the flow is always subcritical ($y_c < y_n$) on a mild slope and always supercritical ($y_c > y_n$) on a steep slope.

### *Assumption*

Uniform flow

### *Equations*

Using the equations of Problem 2-7, calculate the critical depth as shown in Spreadsheet 2-13a. The equations for Spreadsheet 2-13b are

1. $A=my_c^2+by_c$
2. $P=2(my_c^2+y_c^2)^{0.5}+b$
3. $R=A/P$
4. $S_c=[(Qn)/(C_m \mathrm{AR}^{2/3})]^2$

### *Solution Procedure*

As shown in Spreadsheet 2-13a, the value of $y_c$ is calculated using the equations and solution procedure for Problem 2-7. The value of $S_c$ is calculated in Spreadsheet 2-13b. The normal depth will be greater than the critical depth for the given conditions if the flow was uniform.

### *Solution*

The critical depth, $y_c=0.86$ m. Because $S$ (0.0019) is smaller than $S_c$ (0.0021), the bed slope is considered to be mild.

### *What-If Analyses*

Conduct a what-if analysis by plotting a graph of $S_c$ versus $Q$ and show that the critical slope decreases as $Q$ increases.

### *Assigned Problem 2-13-1*

Determine the value of $Q$ at which the slope of Problem 2-13 will start acting as a steep slope.

# Problem 2-14: Calculate the Alternate Depth for a Trapezoidal Cross Section

### *Problem Statement*

Determine the depth, $y_1$, just upstream of a sluice gate (Section 1) in a trapezoidal cross section if the flowrate $Q = 10\,\text{m}^3/\text{s}$, the channel width $b = 3\,\text{m}$, the side slope ratio is four horizontal to one vertical ($m = 4$), and the depth just downstream of the sluice gate (Section 2) $y_2 = 0.5\,\text{m}$. In this problem, the critical depth, $y_c = 0.75\,\text{m}$ (from Problem 2-7).

### *Background*

A sluice gate is raised or lowered to control the water level and flowrate in an open channel. As energy loss across a sluice gate is negligible, it is assumed that the specific energies on both sides of the gate (Sections 1 and 2), $E_1$ and $E_2$, are equal. Depths with equal specific energies are known as alternate depths as described in Section 2. Thus, the depths on either side of the gate may be alternate depths. For example, if the gate is lowered to a point where the depth downstream of the gate ($y_2$) is supercritical, i.e., less than $y_c$, as in this example, the depth upstream of the gate ($y_1$) will be subcritical. This situation is also shown in Fig. 2-1.

### *Assumption*

Uniform flow

### *Equations*

1. $A_2 = my_2^2 + by_2$
2. $A_1 = my_1^2 + by_1$
3. $V_2 = (Q)/(A_2)$
4. $V_1 = (Q)/(A_1)$
5. $E_2 = y_2 + V_2^2/(2g)$
6. $E_1 = y_1 + V_1^2/(2g)$

### *Solution Procedure*

Values of $y_1$ were assumed until $E_1 = E_1$ as shown in Spreadsheet 2-14.

### *Solution*

$y_2 = 1.27\,\text{m}$.

### *What-If Analyses*

Conduct a what-if analysis by plotting a graph of $y_1$ versus $y_2$ and show that $y_1$ increases as $y_2$ decreases, i.e., as the sluice gate is lowered. Is this finding consistent with what is indicated by the SED?

# Problem 2-15: Calculate the Sequent Depth and Energy Loss across a Hydraulic Jump for a Trapezoidal Cross Section



## *Problem Statement*

Determine the sequent depth, $y_2$, and the energy loss, $h_L$, in a trapezoidal cross section if the flowrate $Q = 10\,\text{m}^3/\text{s}$, the channel width is $b = 3\,\text{m}$, and the side slope ratio is four horizontal to one vertical ($4\,H:1\,V$); $m = 4$. The critical depth, $y_c = 0.75\,\text{m}$ (from Problem 2-7). The upstream depth, $y_1 = 0.5\,\text{m}$.

## *Background*

The only way that water can transition from supercritical flow to subcritical flow is through a hydraulic jump. If a hydraulic jump is formed between Sections 1 and 2, as shown in Fig. 2-10a, the depth $y_1$ prior to the hydraulic jump must be supercritical and the downstream depth $y_2$ after the jump must be subcritical. The flow across a hydraulic jump is highly turbulent, and therefore, a significant amount of energy loss always occurs across a hydraulic jump. The depths $y_1$ and $y_2$ on either side of the hydraulic jump are known as sequent or conjugate depths. Incidentally, sequent depths *cannot* be alternate depths because alternate depths, by definition, must have equal specific energy. In this case $E_1$ is much greater than $E_2$ as shown in Fig. 2-10a. If either of the sequent depths is known ($y_1$ or $y_2$), the other sequent depth, for any cross section, can be determined by combining the continuity equation with the momentum equation. The momentum equation is written for a control volume (CV) encompassing the hydraulic jump (Fig. 2-10b). In this figure, $F_1$ and $F_2$ are the pressure forces acting on the water inside the CV because of the surrounding water, and $F_f$ is the friction force exerted by the ground. The energy loss, $h_L$, across the hydraulic jump can be obtained by the energy equation.

## *Assumptions*

The assumptions are (a) the channel bed is horizontal and (b) the force owing to friction, $F_f$ (Fig. 2-10b) is negligible.

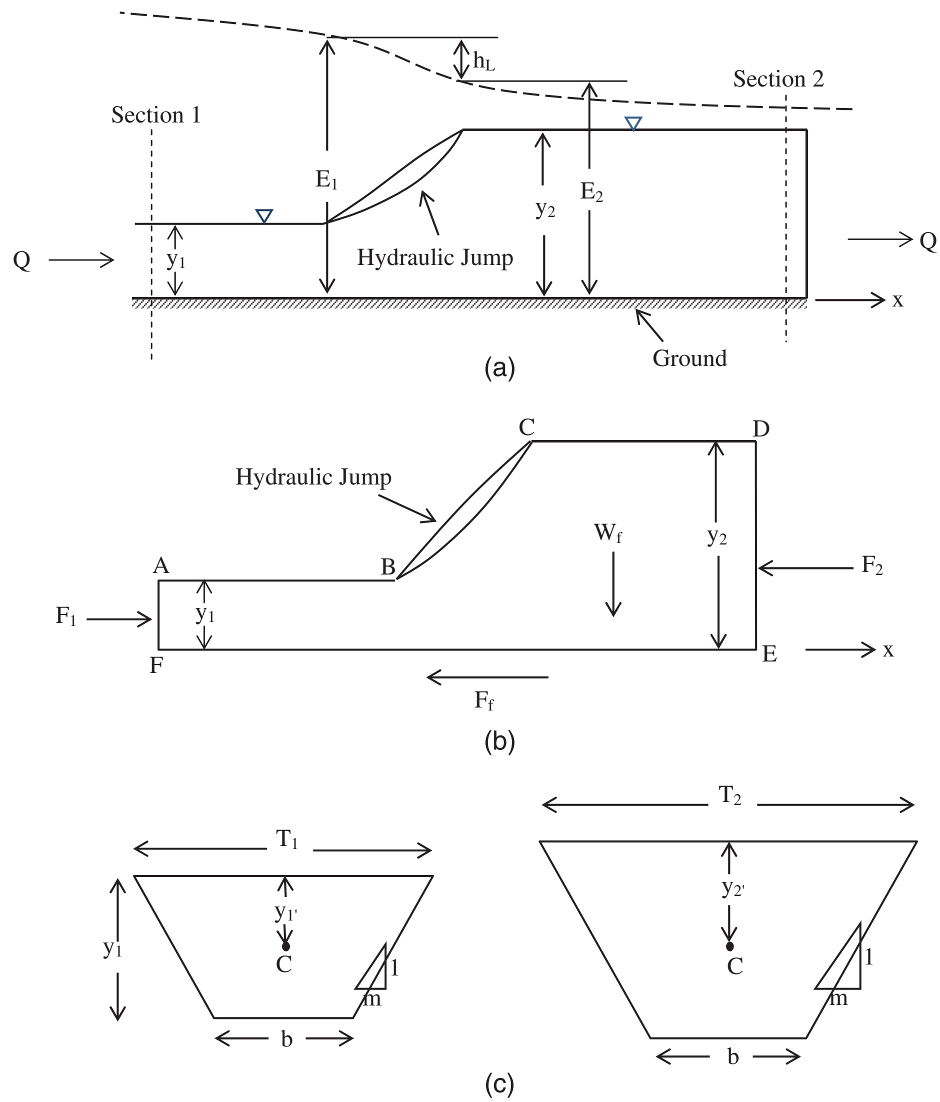
## *Equations*

The equations for estimating the sequent depth and the energy loss are given as follows. In these equations, the subscripts 1 and 2 refer to Sections 1 and 2 as described in Fig. 2-10b. The equations are

1. $A_1 = my_1^2 + by_1$
2. $T_1 = 2my_1 + b$
3. $y_1' = (y_1/3)[(2b + T_1)/(b + T_1)]$
4. $p_1 = (\gamma)(y_1')$
5. $V_1 = (Q)/(A_1)$
6. $F_1 = (p_1)(A_1)$

7. $\text{LHS} = F_1 + (\gamma/g)(V_1)(Q)$
8. $A_2 = my_2^2 + by_2$
9. $T_2 = 2my_2 + b$
10. $y_2' = (y_2/3)[(2b + T_2)/(b + T_2)]$
11. $p_2 = (\gamma)(y_2')$
12. $V_2 = (Q)/(A_2)$
13. $F_2 = (p_2)(A_2)$
14. $\text{RHS} = F_2 + (\gamma/g)(V_2)(Q)$
15. $E_1 = V_1^2/(2g)$

***Fig. 2-10*** **(a) Description of flow across a hydraulic jump, (b) free body diagram of the control volume around a hydraulic jump, (c) description of upstream and downstream channel geometry**



16. $E_2 = V_2^2/(2g)$
17. $h_L = E_1 - E_2$

where $A_1$ and $A_2$ are the respective areas at Sections 1 and 2, $T_1$ and $T_2$ are the respective top widths at Sections 1 and 2, $y_1'$ and $y_2'$ are heights of the water level from the centroids of Sections 1 and 2, $p_1$ and $p_2$ are the static pressures at the centroids, $V_1$ and $V_2$ are the velocities at Sections 1 and 2, $E_1$ and $E_2$ are the specific energies at Sections 1 and 2, and $F_1$ and $F_2$ are the forces acting on the water inside the control volume (CV) at Sections 1 and 2. Many of the terms used in these equations are shown in Fig. 2-10c. The terms LHS and RHS are abbreviated forms for "left-hand side" and "right-hand side" of the momentum equation in the $x$ direction.

### *Solution Procedure*

The LHS and the RHS of the momentum equation are calculated as shown in Spreadsheets 2-15a and 2-15b, respectively. The value of $y_2$ was determined using the Goal Seek feature. Essentially, this feature helps in assuming values of $y_2$ in Spreadsheet 2-15b until RHS = LHS. Because you know that $y_2$ must be subcritical depth, the assumed values of $y_2$ should be greater than $y_c = 0.75$ m. The value of $h_L$ was calculated as the difference in the specific energies on either side of the hydraulic jump (Spreadsheet 2-15c).

### *Solution*

$y_1 = 1.06$ m, $h_L = 0.26$ m

### *What-If Analyses*

A what-if analysis shows that if the upstream depth $y_1$ was 0.25 m (instead of 0.5 m), the sequent depth would increase from 1.31 m to 1.98 m and $h_L = 4.90$ m.

### *Assigned Problem 2-15-1*

What is the rationale for the sequent depth and head loss increasing as $y_1$ decreases?

### *Assigned Problem 2-15-2*

Determine the sequent depth if the flowrate were 40 $m^3/s$ instead of 10 $m^3/s$. All other parameters are the same as in Problem 2-15.

# Problem 2-16: Design of a Nonerodible Channel Using the Permissible Velocity Method

### *Problem Statement*

Design a trapezoidal channel with Manning's $n = 0.025$ with a bed slope of $S = 0.004$. The channel must carry a flowrate ($Q$) of $8\,\mathrm{m}^3/\mathrm{s}$. The maximum permissible velocity ($V$) is 1.8 m/s and the minimum required freeboard (FB) is 0.8 m. The minimum required side slope ratio is one horizontal to one vertical ($m = 1$). Also determine the total depth $y_t$ from the channel bottom to the ground surface and the top width ($T_g$) at the ground surface.

### *Background*

Several constraints are mentioned in this problem including maximum permissible velocity and minimum required side slope and freeboard. In some cases, there may be additional constraints as discussed in Problem 2-17. *Designing* a trapezoidal channel here implies finding the bottom width, $b$, and the depth at which water will flow ($y$).

### *Equations*

1. $A = Q/V$
2. $R = [(Vn)/(C_m S^{1/2})]^{1.5}$
3. $P = A/R$
4. $b = (A - my^2)/y$
5. $P = 2(my^2 + y^2)^{0.5} + b$
6. $y_t = y + \mathrm{FB}$
7. $T_g = 2my_t + b$

### *Solution Procedure*

After calculating $A$ and $P$ by Eqs. 1 to 3, the engineer is left with two unknowns, $y$ and $b$, and the following two equations: (i) $P = 2(my^2 + y^2)^{0.5} + b$ and (ii) $A = my^2 + by$. Eq. 4 is a modified form of this equation. Therefore, simple algebra can be used to calculate $b$ and $y$. However, using a spreadsheet can solve this problem by trial and error or by using the Goal Seek function. As shown in Spreadsheet 2-16, this problem was solved using the Goal Seek function by assuming a value of $y$ until the values of $P$, calculated by Eqs. 3 and 5, are equal.

### *Solution*

The values of $y$ and $b$ are 0.73 m and 5.34 m, respectively. The values of $y_t$ and $T_g$ are 1.53 m and 8.40 m, respectively.

### *What-If Analyses*

Plotting graphs of $y$ versus $Q$ and $b$ versus $Q$ shows that $y$ decreases and $b$ increases as $Q$ increases.

### *Assigned Problem 2-16-1*

Design the channel of Problem 2-16, but use $m = 4$ (instead of $m = 1$). All other values are the same as in Problem 2-16.

### *Hint*

You will find that a solution is not possible; can you explain why?

# Problem 2-17: Design of a Nonerodible Channel Using the Modified Permissible Velocity Method

## *Problem Statement*

Design the trapezoidal grassy channel of Assigned Problem 2-16-1 with Manning's $n = 0.025$ with a bed slope $S = 0.004$. The maximum permissible velocity is 1.8 m/s and the minimum required freeboard is 0.8 m. The minimum required side slope ratio is four horizontal to one vertical ($m = 4$). The maximum allowable top width, $T_g$, at the ground surface is 16.6 m, and the allowable maximum possible distance of the channel bottom from the ground surface, $y_t$, is 1.6 m.

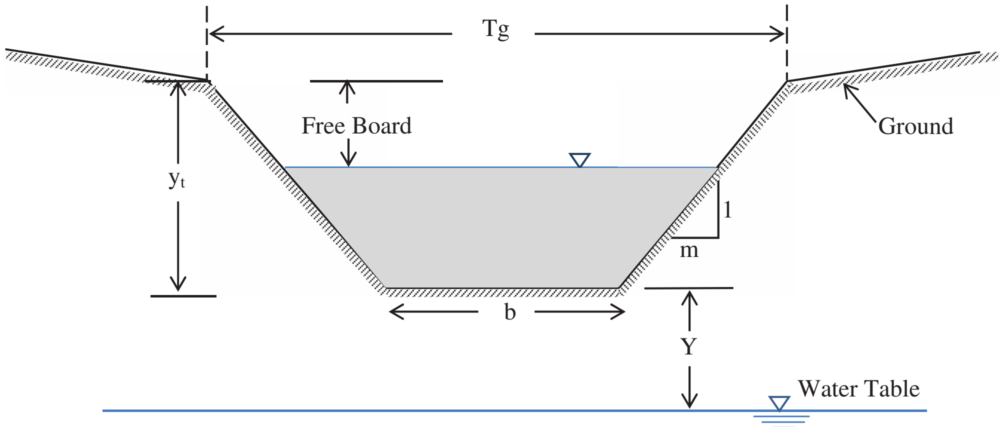
## *Background*

Sometimes the permissible velocity method, used in Problem 2-16, will not yield a design because the channel width, $b$, and the depth, $y$, are correlated by Eqs. 4 and 5 of the previous problem. For example, a solution of Problem 2-16-1 is not possible, because no possible combination of $y$ and $b$ can meet the design constraints and also satisfy Eqs. 4 and 5 (of the previous problem). This problem demonstrates the application of the modified permissible velocity method, a more flexible method, that can be used to design the channel of Assigned Problem 2-16-1. The modified permissible velocity method will be applicable even if we add two other constraints in addition to the constraints specified in Assigned Problem 2-16-1. These two additional constraints occur owing to right-of-way-related space constraints and also because of high water table conditions as described in Fig. 2-11. Right-of-way-constraints may provide a limit on the top width of the channel at the ground level, $T_g$ (Fig. 2-11). Moreover, client specifications or city codes may require that the channel bottom be at least a distance $Y$ above the water table (Fig. 2-11). As a result, the presence of a high water table can put a constraint on the depth of the channel bed from the ground, $y_t$ (Fig. 2-11), if the engineer does not wish to raise the ground elevation. Raising the ground elevation of a site can be expensive, as it requires purchasing and bringing fill to the site.

## *Equations*

1. $A = my^2 + by$
2. $P = 2(my^2 + y^2)^{0.5} + b$
3. $R = A/P$
4. $Q = (C_m/n)(AR^{2/3})(S^{1/2})$
5. $V = Q/A$
6. $y_t = y + \mathrm{FB}$
7. $T_g = 2my_t + b$

***Fig. 2-11*** **Description of channel**




### *Solution Procedure*

Six possible designs are shown in Spreadsheet 2-17, along with the design constraints. The design process consisted of selecting the channel width, $b$, and then calculating $A$, $P$, $R$, and $Q$ using Eqs. 1 to 4, respectively. The range of values selected for $b$ was from 1 m to 6 m. Although the value of $m = 4$ is specified in this problem, the design engineer could select a value greater than $m$, perhaps for aesthetic reasons, because the prescribed $m$ value should be considered to be a *minimum* specification. The various design parameters were calculated using Eqs. 1 to 7 after assuming a value for depth, $y$. The Goal Seek feature was used to determine the value of $y$ that yielded a $Q$, calculated by Eq. 4, to the specified $Q$, which is $8\ m^3/s$. The validity of each design was checked by ensuring that the calculated values of $V$, $y_t$, and $T_g$ were within the design constraints.

### *Solution*

A total of six designs are shown in the spreadsheet. Designs 1 and 2 are not acceptable, because the total depth is greater than the allowable depth of 1.6 m, whereas designs 5 and 6 are not acceptable because the top width, $T_g$, is greater than the allowable value 16.5 m. However, both designs 3 and 4 are acceptable.

### *Assigned Problem 2-17-1*

Come up with two possible designs that meet all the design constraints of Problem 2-17 with the exception that $m = 4.25$ instead of 4.0.

### *Assigned Problem 2-17-2*

Design a trapezoidal grassy channel with Manning's $n = 0.03$ and a bed slope of $S = 0.001$. The maximum permissible velocity is 1 m/s, and the minimum required freeboard is 0.5 m. The *minimum* (higher values can be selected, and are preferred, owing to aesthetic reasons) required side slope ratio is four horizontal to one vertical ($m = 4$). The maximum allowable top width, $T_g$, at the ground surface is 17.25 m, and the allowable maximum possible depth, from the ground surface is 1.5 m. Try to come up with at least two possible designs. The design range for the bottom width, $b$, is from 2 m to 4 m.

# Problem 2-18: Design of a Most Efficient Trapezoidal Lined, Rigid Channel

## Problem Statement

Design a most efficient lined trapezoidal channel with a Manning's $n = 0.015$. The channel must carry a flowrate $Q = 30\ \mathrm{m^3/s}$, whereas the minimum permissible velocity ($V_{\min}$) is 1.5 m/s. The side slope must have a ratio of 0.5774 horizontal to 1 vertical ($m = 0.5774$). Also, determine the freeboard (FB) using the U.S. Bureau of Reclamation equation with a $C$ value of 1.2 and the minimum bed slope $S$ required to carry the specified discharge.

## Background

There are several design constraints when designing a lined channel. First, lining a channel is expensive, which is why it is important to minimize the wetted perimeter, $P$. A channel with the least possible wetted perimeter is called the "most efficient channel." It can be shown that the side slope for the most efficient trapezoidal section is 1 vertical and 0.5774 horizontal ($m = 0.5774$). Also, it can be shown that the ratio, $q'$, of the bottom width of $b$ to the normal depth $y$ must be 1.155 ($q' = b/y = 1.155$) when $m = 0.5774$. Second, lined channels have a minimum velocity requirement to prevent the growth of weeds and to prevent sedimentation. Third, an appropriate FB must be provided for safety reasons. The U.S. Bureau of Reclamation suggests that the FB for lined channels be calculated by the equation $\mathrm{FB} = (C_y)^{0.5}$ in which $C$ is a coefficient varying from 0.8 for a flow capacity of $0.5\ \mathrm{m^3/s}$ to 1.4 for a flow capacity of $85\ \mathrm{m^3/s}$.

## Equations

1. $A = Q/V_{\min}$
2. $y = [A/(m + q')]^{0.5}$
3. $b = q'y$
4. $P = 2(my^2 + y^2)^{0.5} + b$
5. $R = A/P$
6. $S = (C_m Qn)/(AR^{2/3})^2$
7. $\mathrm{FB} = (C_y)^{0.5}$

Eq. 2 was derived in Section 2, Eq. (2-15).

## Solution Procedure

The solution procedure is straightforward and requires solving Eqs. 1 through 7, as shown in Spreadsheet 2-18.

*Solution*

The designed channel has a bottom width of 3.92 m, side slopes of 1*V* to 0.5774 *H*, a bed slope of 0.0002, and a wetted perimeter of 11.77 m. The depth and FB at the design flowrate will be 3.40 m and 2.02 m, respectively.

*What-If Analysis*

Conduct the following analyses:

(a) Plot a graph between $y$ versus $V_{min}$, and $b$ versus $V_{min}$ to determine how the values of $y$ and $b$ change as $V_{min}$ changes. Assume that $V_{min}$ varies from 1 m/s to 3 m/s.
(b) Show that a higher allowable $V_{min}$ will result in a smaller wetted perimeter and, therefore, a lesser cost.
(c) Show that a higher allowable $V_{min}$ will result in requiring a steeper channel bed.

*Assigned Problem 2-18-1*

The bed slope was not specified in Problem 2-18. However, sometimes the bed slope is specified based on the land topography. Design the channel of Problem 2-18 if the required bed slope is 0.0015.

*Hint*

This simply requires the calculation of the normal depth under the specified conditions.

*Assigned Problem 2-18-2*

Prove that $m = 1/\sqrt{3}$ (or 0.5774) for the most efficient channel.

# Problem 2-19: Design of an Efficient Trapezoidal Lined Channel with Prescribed Side Slopes

## *Problem Statement*

Design an efficient lined trapezoidal channel with Manning's $n = 0.015$. The channel must carry a flowrate $Q = 30\ \mathrm{m^3/s}$, whereas the minimum permissible velocity ($V_{\min}$) is 1.5 m/s. The side slope must have a ratio of two horizontal to one vertical ($m = 2$). Also, determine the FB using the U.S. Bureau of Reclamation equation with a $C$ value of 1.2 and the minimum bed slope $S$ required to carry the specified discharge.

## *Background*

The only difference between Problems 2-18 and 2-19 is the value of $m$. Sometimes a slope of 1 vertical and 0.5774 horizontal ($m = 0.5774$) may be considered to be too steep, and an engineer may decide to use a different value for $m$. In other cases, the local codes or the regulatory agency may require a value of $m$ other than 0.5774. Even in this case, the goal is to keep the wetted perimeter as small as possible. When $m$ has a value different than 0.5774, the ratio $q' = b/y$ is not 1.155 but must be calculated by a different equation (Eq. 1, following).

## *Equations*

1. $q' = 2[(1 + m^2)^{0.5} - m]$
2. $A = Q/V_{\min}$
3. $y = [A/(m + q')]^{0.5}$
4. $b = q'y$
5. $P = 2(my^2 + y^2)^{0.5} + b$
6. $R = A/P$
7. $S = (C_m Qn)/(AR^{2/3})^2$
8. $\mathrm{FB} = (Cy)^{0.5}$

Eqs. 1 and 3 were derived in Section 2, Eq. (2-19) and Eq. (2-15).

## *Solution Procedure*

The solution procedure is straightforward and requires solving Eqs. 1 through 7, as shown in Spreadsheet 2-19.

### *Solution*

The designed channel has a bottom width of 1.34 m, side slopes of $1V$ to $2\ H$, and a bed slope $S$ of 0.0003. The depth and FB at the design flowrate will be 2.84 m and 1.19 m, respectively. The wetted perimeter is 14.06 m. Although this is the least possible wetted parameter for $m = 2$, it is higher than 11.77 m for $m = 0.5774$ in Problem 2-18.

### *What-If Analysis*

Plot a graph of $P$ versus $m$ to show that the smallest $P$ occurs when $m = 0.5774$.

### *Assigned Problem 2-19-1*

The bed slope in Problem 2-19 was not specified. Design the channel of Problem 2-19 if the required bed slope is 0.0015.

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# 3

# *Hydrology*

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# Background

This chapter covers the topics of lake evaporation, direct runoff hydrograph (DRH), unit hydrograph, channel routing using the Muskingum method, and reservoir routing using the level pool routing method.

# Lake Evaporation

Several methods are available for estimating lake evaporation. Some of the most commonly used methods are (a) water budget method, (b) energy budget or energy balance method, (c) mass transfer or aerodynamic methods, (d) combination methods, and (e) pan method. The selection of the method to estimate lake evaporation depends on cost constraints, accuracy requirements, and the size of the lake or reservoir. A common base for mass transfer methods is Dalton's equation, proposed in 1802, which states that

$$E_d = B(e_s - e_a) \tag{3-1}$$

where $E_d$ is the daily evaporation from the lake surface, $B$ is a vapor transfer coefficient, and $e_s$ and $e_a$ are the respective saturation and actual vapor pressure. Thornthwaite and Holzman (1939) provided the following equation to calculate $B$:

$$B = 0.622k^2\rho_a u_2/\{p\rho_w[\ln(z_2/z_0)]^2\} \tag{3-2}$$

where $k$ is von Kármán's constant, $\rho_a$ is the air density, $u_2$ is the wind velocity at an elevation of $z_2$ meters above the ground surface, $p$ is atmospheric pressure, $\rho_w$ is the water density, $z_0$ is the roughness height, and $z_2$ is the elevation at which $u_2$ is measured. The roughness height varies from 0.01 for open water to 70 cm for trees (Chow et al. 1988, Table 2-8-2). The von Kármán constant is dimensionless and is used in describing the velocity profile of a turbulent fluid near a boundary with a no-slip condition. A "no-slip condition" in this case implies that the wind velocity is zero at the roughness height. The value of the von Kármán constant is typically equal to 0.4. The estimation of daily lake evaporation using the Thornthwaite and Holzman equation is described in Problem 3-1.

# Direct Runoff Hydrographs (DRH)

## *Overland Flow, Interflow, Ground Water Flow, and Direct Runoff*

A typical watershed consists of a natural ridge as a boundary and a single outlet as shown in Fig. 3-1. The watershed slopes toward the outlet. Some of the rainfall that falls on a watershed is abstracted by hydrologic processes (such as evaporation, infiltration, and transpiration), and the remaining rainfall is termed "net rainfall" or "excess rainfall." All net rainfall reaches the outlet of the watershed as either "overland flow" or "interflow," as shown in Fig. 3-2. Overland flow occurs above the ground surface and is also

***Fig. 3-1*** **Description of a watershed**

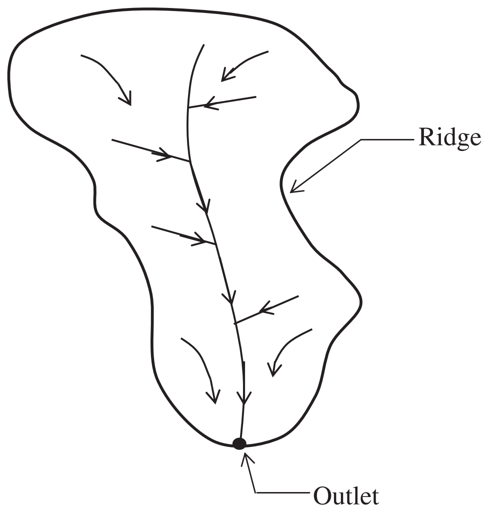
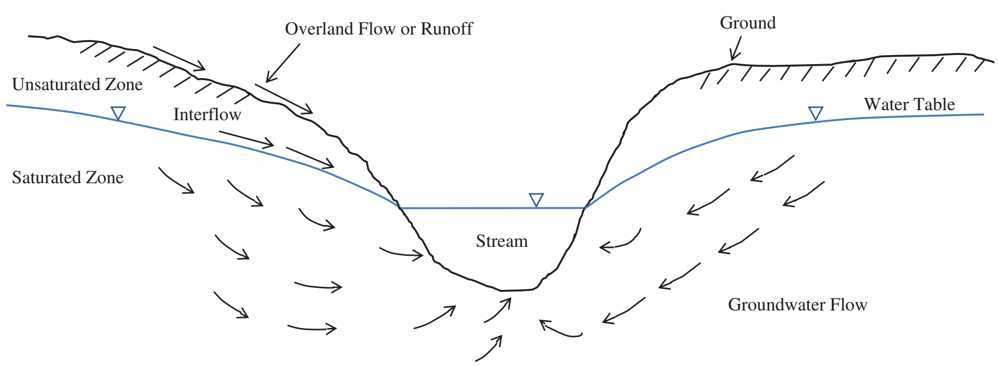



***Fig. 3-2*** **Description of water flowing to a stream as overland flow, interflow, and groundwater flow**




commonly referred to as "surface runoff." Interflow occurs in the unsaturated zone between the ground surface and the water table (Fig. 3-2). The sum of overland and interflow is called "direct runoff" (DRO). In addition, groundwater flow also occurs in the saturated zone below the water table, also shown in Fig. 3-2. Water travels much faster as overland flow and interflow compared with groundwater flow and, therefore, has a much quicker and critical effect on the flowrate at the outlet. As a result, engineers usually are interested in knowing the DRO and use a process called "baseflow separation" to determine the effect of just the DRO. The hydrograph produced as a result of the DRO is called the "direct runoff hydrograph" (DRH). A key objective in most engineering designs is to determine the DRH at the outlet of the watershed.

### *Convolution and Deconvolution*

The process of determining the DRH from a unit hydrograph is called "convolution," whereas the reverse process of determining the unit hydrograph from a DRH is termed "deconvolution." Two methods are used for determining the DRH: convolution equation method (Problem 3-2) and lagging method (Problem 3-3). Both these methods use the concept of the "unit hydrograph," which is defined as the DRH that is produced at the watershed outlet by a *constant intensity*, *net rainfall of one unit* (1 cm or 1 in.). If the net rainfall has a duration of "$t$" $h$, then the unit hydrograph is termed a "$t$-$h$ unit hydrograph." Therefore, theoretically a watershed can have an infinite number of unit hydrographs, because the value of $t$ is variable.

### *S-Hydrograph Method for Unit Hydrograph Conversion*

Unit hydrographs are developed based on streamflow data measured at the watershed outlet during rainfall events. Ideally, these rainfall events should be uniformly distributed across the entire watershed. The average rainfall intensity and depth are also measured during these events. The streamflow data are converted to DRH ordinates by baseflow separation. The rainfall data are converted to net rainfall data by a process called the "phi-index method," although other methods also can be used. The duration of the unit hydrograph determined from this process is equal to the duration of the *net rainfall*. It is not practical and cost effective for an engineer to develop a large number of unit hydrographs of different durations by field measurements, as this would require measuring streamflow and rainfall data multiple times. Instead, the engineer can use the $S$-curve method to convert an existing $R$-$h$ unit hydrograph to a $P$-$h$ unit hydrograph if a $P$-$h$ unit hydrograph is required for convolution. The $S$-curve method is discussed in Problem 3-4.

## Routing

Routing is the method used to determine the hydrograph at a downstream location of a channel or reservoir if the hydrograph at an upstream location is known. Broadly, routing can be classified as (a) channel routing and (b) reservoir routing. In both channel and reservoir routing, the inflow hydrograph at the upstream location, Section 1, is known, and the hydrograph at Section 2 needs to be determined as shown in Figs. 3-3 and 3-4. The storage inside the channel reach consists of both wedge and prism storage. As shown in Fig. 3-3, a channel will have both prism storage ($S_p$) and wedge storage ($S_w$) if the flow is nonuniform between Sections 1 and 2. The channel reach will have only prism storage ($S_p$) if the flow is uniform. A reservoir is assumed to have an even water surface profile, and the storage within the reservoir consists only of prism storage (Fig. 3-4). Thus, the storage, $S$, within a channel reach is the sum of wedge and prism storage ($S = S_p + S_w$), whereas the storage within a reservoir is simply equal to prism storage ($S = S_p$). Two phenomena occur as the hydrograph at the upstream section (Section 1) traverses downstream through the channel or reservoir to the downstream section, and these phenomena are shown in Fig. 3-5. First, the peak flowrate of the downstream hydrograph $(Q_p)_2$ is always

***Fig. 3-3* Prism and wedge storage within a channel reach**

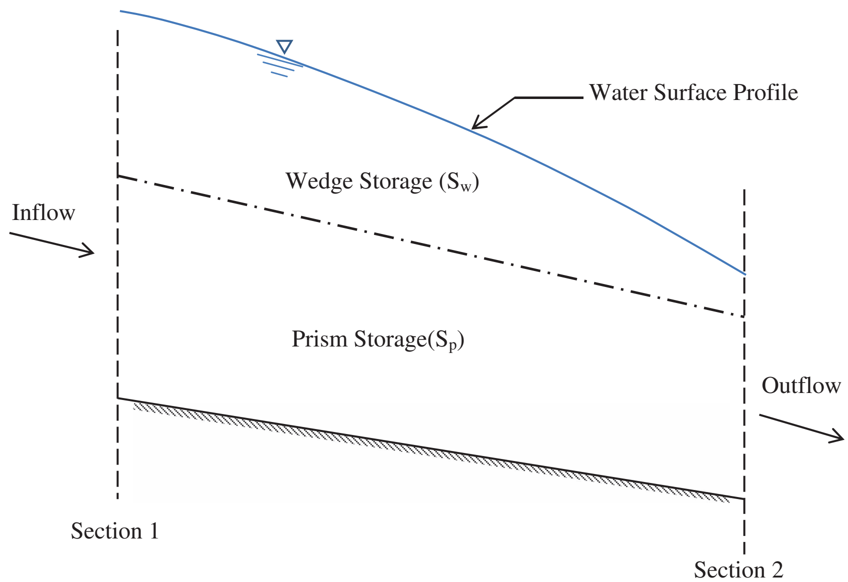
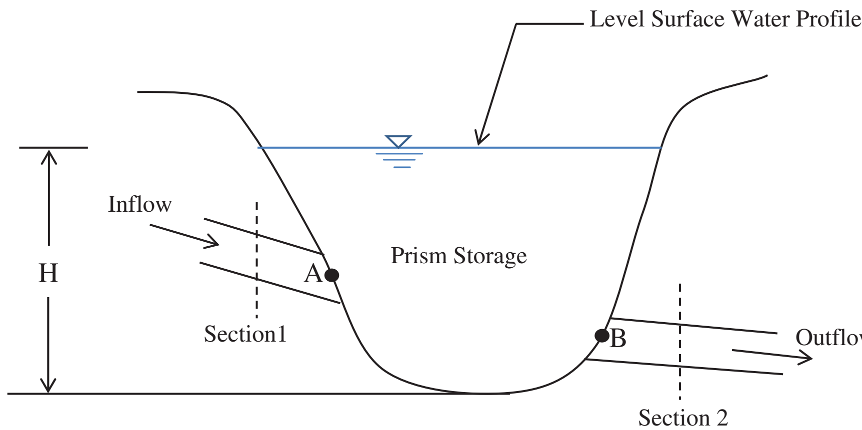



***Fig. 3-4* A reservoir has only prism storage**

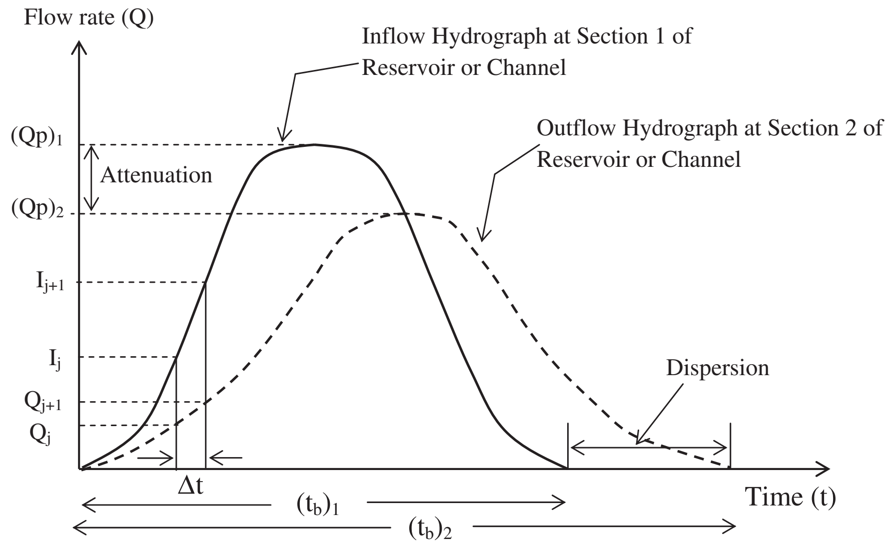



lower than the peak flowrate of the upstream hydrograph $(Q_p)_1$. Second, the time base of the hydrograph at the downstream section $(t_b)_2$ is always greater than the time base of the upstream hydrograph $(t_b)_1$. The lowering of the peak flowrate is known as "attenuation," whereas the increase in the time base is known as "dispersion."

### *Known and Unknown Parameters in a Routing Problem*

In every routing problem the inflow hydrograph at Section 1 is known. Because this is a time-dependent problem (as the outflow at Section 2 is time variant), the

***Fig. 3-5*** **Description of attenuation and dispersion that occurs in the inflow hydrograph as it travels through a channel reach or reservoir**




engineer must know, or guess, the initial outflow at Section 2. The three unknown parameters that are usually calculated using the routing process are (a) the flowrate, $Q$, at the downstream location (Section 2); (b) the storage, $S$, within the channel reach or the reservoir; and (c) the water depth, $H$ (Fig. 3-4).

## *Hydraulic and Hydrologic Routing Methods*

Three equations are required to conduct routing if the engineer wishes to calculate all three unknown parameters ($Q$, $S$, and $H$). The available equations are (a) continuity equation, (b) momentum equation, and (c) empirical equations. Routing methods are classified as (i) hydraulic routing methods and (ii) hydrologic routing methods depending on the equations selected for the purpose of routing. Hydraulic routing methods use the continuity equation, momentum equation, and an empirical equation. Hydrologic routing methods use the continuity equation and two empirical equations. Only hydrologic routing methods are covered in this book. Hydrologic routing methods can be further classified as channel routing methods and reservoir routing methods. A commonly used hydrologic channel routing method known as the Muskingum method is described in Problems 3-5 and 3-6. The book also discusses level pool routing (Problem 3-7), which is commonly used for hydrologic reservoir routing.

## *Muskingum Method*

The Muskingum method uses the continuity equation and an empirical equation known as the Muskingum equation to determine the outflow hydrograph and the storage within the reach. A third equation in the form of a stage-discharge curve also can be used if the water level also needs to be determined.

The continuity equation for both the Muskingum and level pool routing methods is written in the following form:

$$2S_{j+1}/\Delta t + Q_{j+1} = (2S_j/\Delta t - Q_j) + I_j + I_{j+1} \tag{3-3}$$

where $S$ is the storage within the channel reach, $I$ is inflow, and $Q$ is outflow. The subscripts $j$ and $j+1$ denote the times before and after the routing period, $\Delta t$. The routing period is selected by the engineer based on desired accuracy; smaller $\Delta t$ values lead to more accurate answers. Selection of very large routing periods can also lead to numerical instability.

The Muskingum equation provides the following relationship between $S$, $I$, and $Q$:

$$S = KQ + KX(I - Q) \tag{3-4}$$

where $K$ is a proportionality constant also known as the "storage time constant," and $X$ is a weighting factor. The constant $K$ has units of time. It can be shown that $K$ is the time that a wave takes to travel from the upstream to the downstream section. In Eq. (3-4), the term $KQ$ represents the prism storage, whereas $KX(I - Q)$ is the wedge storage. $X = 0$ implies that the channel behaves as a reservoir and provides only prism storage, whereas $X = 1$ implies that the channel reach consists of only wedge storage. In practice, the value of $X$ lies between 0 and 0.5. By adding proper subscripts, Eq. (3-4) can be modified to

$$S_j = KQ_j + KX(I_j - Q_j) \tag{3-5}$$

and

$$S_{j+1} = KQ_{j+1} + KX(I_{j+1} - Q_{j+1}) \tag{3-6}$$

Substituting Eqs. (3-5) and (3-6) in Eq. (3-3) and rearranging yields

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \tag{3-7}$$

where

$$C_1 = (\Delta t - 2KX)/[2K(1 - X) + \Delta t] \tag{3-8}$$

$$C_2 = (\Delta t + 2KX)/[2K(1 - X) + \Delta t] \tag{3-9}$$

$$C_3 = [2K(1 - X) - \Delta t]/[2K(1 - X) + \Delta t] \tag{3-10}$$

It turns out that

$$C_1 + C_2 + C_3 = 1 \tag{3-11}$$

Eq. (3-7) is used to calculate the outflow in the Muskingum method (Problem 3-5). Verifying that Eq. (3-11) is satisfied is a good way to check if the calculations are accurate.

By substituting Eqs. (3-5) and (3-6) in Eq. (3-3) and rearranging, it can be shown that

$$K = N/D \tag{3-12}$$

where

$$N = [(I_j + I_{j+1}) - (Q_j + Q_{j+1})](0.5\Delta t) \tag{3-13}$$

$$D = (X)(I_{j+1} - I_j) + (1 - X)(Q_{j+1} - Q_j)] \tag{3-14}$$

Eq. (3-12) indicates that a plot between $N$ and $D$ should plot as a straight line with a slope equal to $K$. This graph is useful in estimating $K$ and $X$ for a channel reach as demonstrated in Problem 3-6.

### *Level Pool Routing Method*

The level pool routing method also utilizes the continuity equation, Eq. (3-3), and two empirical equations. The two empirical equations usually are provided by the engineer who designed the reservoir (or pond). One of the empirical equations is a relationship between $Q$ and $S$. It is more convenient in the calculation process if the relationship between $Q$ and $S$ is transformed to an equation between $Q$ and $2S/\Delta t + Q$, because the term $2S/\Delta t + Q$ occurs on the left-hand side of the continuity equation. The second empirical equation is between $H$ and $S$ and also should be provided by the design engineer. The method for using these three equations to conduct level pool routing is described in Problem 3-7.

# Problem 3-1: Estimation of Evaporation Rates for a Lake or Open Surface Body by the Aerodynamic Method

## *Problem Statement*

Determine the evaporation rate from a lake in mm/day using the aerodynamic method. Given are air temperature $T = 25°\text{C}$, relative humidity $R_h = 40\%$, and atmospheric pressure $p = 101.3\,\text{kPa}$. The roughness height $z_0$ and the wind velocity $u_2$, measured at a height $z_2 = 2\,\text{m}$ above the water surface are 0.03 cm and 3 m/s, respectively. The gas constant for dry air is $R_d = 287\,\text{Nm/kg} - K$, and the von Kármán constant is $k = 0.4$.

## *Background*

The relative humidity of air is the ratio of the actual vapor pressure and the saturated vapor pressure. A high relative humidity implies more moisture in the air. A body of air with a high relative humidity and, therefore, a higher amount of water vapor will not have space to absorb more moisture and will inhibit evaporation from the lake surface. However, if the air temperature increases, the saturated air pressure increases, and therefore the relative humidity decreases. In essence, a higher air temperature allows more moisture to be absorbed by the air and should increase the evaporation rate. Also, a high wind velocity should increase the evaporation rate, because it will quickly remove the moist air from above the lake surface and replace it with dry air. The method using Eqs. (3-1) and (3-2) to calculate daily lake and reservoir evaporation is also known as the aerodynamic method. This method incorporates the effects of air temperature, relative humidity, and wind velocity. The measurements required to use this method are air temperature, relative humidity, and the wind velocity at a specific height $z_2$, which in this problem is 2 m.

## *Assumptions*

(1) The air is saturated at the ground elevation, and (2) the wind velocity is zero at the roughness height.

## *Equations*

1. $\rho_a = (8)(10^{-6})T - (4.173)(10^{-3})T + 1.286281$
2. $\rho_w = -0.0036T^2 - 0.0658T + 1000.6$
3. $e_s = 0.1613T^3 - 8.5928T^2 + 262.33T - 226.7$
4. $e_a = (R_h)(e_s)$
5. $B = 0.622k^2\rho_a u_2/\{p\rho_w[\ln(z_2/z_0)]^2\}$
6. Unit conversion
7. Unit conversion
8. $E_d = B(e_s - e_a)$

Eq. 1 was obtained by solving Assigned Problem 1-1-1, whereas Eqs. 2 and 3 are Eqs. 1 and 5 of Problem 1-1. All three equations were determined using trend lines. Eqs. 6 and 7 are simply unit conversion equations.

### *Solution Procedure*

A straightforward solution procedure, solving Eqs. 1 through 8 in order, is shown in Spreadsheet 3-1; it is important to convert the units of $B$ to mm/(day-Pa) to obtain the answer in mm/day.

### *Solution*

The evaporation rate is 7.44 mm/d as shown in Spreadsheet 3-1.

### *What-If Analysis*

Conduct what-if analyses by

(a) Plotting a graph of $E_d$ versus air temperature to show that the evaporation rate increases as the air temperature increases
(b) Plotting a graph of $E_d$ versus $u_2$ to show that the evaporation rate increases as the wind velocity increases
(c) Plotting a graph of $E_d$ versus $R_h$ to show that the evaporation rate decreases as the air temperature increases.

### *Assigned Problem 3-1-1*

Determine the relationship between the daily evaporation rate and the atmospheric pressure. Can you provide an explanation for that relationship?

# Problem 3-2: Estimation of the Direct Runoff Hydrograph by the Convolution Equation

### *Problem Statement*

Using the convolution equation, determine the ordinates of the direct runoff hydrograph (DRH) at the outlet of a watershed due to a 4 h rainfall event during which the *constant intensity*, net hourly rainfall depth pulse ($P$) values were as follows: $P_1 = 0.8$ cm, $P_2 = 2.4$ cm, $P_3 = 4.6$ cm, $P_4 = 0.7$ cm. The 1 h unit hydrograph ordinates at 1 h intervals are $U_1 = 10\,\mathrm{m^3/s/cm}$, $U_2 = 100\,\mathrm{m^3/s/cm}$, $U_3 = 200\,\mathrm{m^3/s/cm}$, $U_4 = 150\,\mathrm{m^3/s/cm}$, $U_5 = 100\,\mathrm{m^3/s/cm}$, and $U_6 = 50\,\mathrm{m^3/s/cm}$. Also determine the area of the watershed.

### *Background*

The convolution equation is given as

$$Q_n = \sum P_m U_{n-m+1} \quad \text{for } m = 1 \text{ to } n \le M; \quad \text{and} \quad n = 1 \text{ to } N \tag{3-15}$$

where $Q$ are the DRH ordinates, $N$ is the number of nonzero DRO ordinates, and $M$ is equal to the number of net rainfall pulses. In this problem, $M = 4$, as there are four nonzero pulses. Moreover, if $X$ = number of nonzero unit hydrograph ordinates, then it can be shown that $X = N - M + 1$ or $N = X + M + 1$. Therefore, $N = 6 + 4 - 1 = 9$ in this problem.

The direct runoff volume ($V_d$) can be determined by the following equation:

$$V_d = (\Delta t) \sum Q_i \quad \text{for } i = 1 \text{ to } N \tag{3-16}$$

where $\Delta t$ is the time interval between the UH ordinates and the DRH ordinates. The DRH depth ($R_d$) can be obtained by

$$R_d = \sum P_i \quad \text{for } i = 1 \text{ to } M \tag{3-17}$$

Finally, the area of the watershed, $A$, can be determined by

$$A = V_d / R_d \tag{3-18}$$

The ratio of the duration of a pulse, $\Delta t$, to the duration $t$ of the unit hydrograph (1 h in this problem) must be equal to an integer. This rule helps in selecting hydrographs with the proper duration.

### *Assumptions*

(1) The time interval between the unit hydrograph ordinates, $\Delta t$, must be equal to the duration of the rainfall pulses and the duration of the unit hydrograph. In this problem, all three are equal to 1 h. (2) All rainfall pulses are of constant or

uniform intensity. Therefore, the rainfall intensity during $P_1$ is 0.8 cm/h. (3) The duration of all pulses are equal.

### *Equations*

Substituting appropriate values of $m$ and $n$ in the convolution equation yields

1. $Q_1 = P_1U_1$
2. $Q_2 = P_1U_2 + P_2U_1$
3. $Q_3 = P_1U_3 + P_2U_2 + P_3U_1$
4. $Q_4 = P_1U_4 + P_2U_3 + P_3U_2 + P_4U_1$
5. $Q_5 = P_1U_5 + P_2U_4 + P_3U_3 + P_4U_2$
6. $Q_6 = P_1U_6 + P_2U_5 + P_3U_4 + P_4U_3$
7. $Q_7 = P_2U_6 + P_3U_5 + P_4U_4$
8. $Q_8 = P_3U_6 + P_4U_5$
9. $Q_9 = P_4U_6$

### *Solution Procedure*

The solution procedure is straightforward and shown in Spreadsheets 3-2a and 3-2b. As expected, the DRH has nine nonzero ordinates.

### *Solution*

As shown in Spreadsheet 3-2, the DRH ordinates are $Q_1 = 8\ \text{m}^3/\text{s}$, $Q_2 = 104\ \text{m}^3/\text{s}$, $Q_3 = 446\ \text{m}^3/\text{s}$, $Q_4 = 1{,}067\ \text{m}^3/\text{s}$, $Q_5 = 1{,}430\ \text{m}^3/\text{s}$, $Q_6 = 1{,}110\ \text{m}^3/\text{s}$, $Q_7 = 635\ \text{m}^3/\text{s}$, $Q_8 = 300\ \text{m}^3/\text{s}$, and $Q_9 = 35\ \text{m}^3/\text{s}$. The watershed area is 21,960 ha.

### *What-If Analysis*

Conduct a what-if analysis by comparing DRH hydrographs (draw figures) if the pulse occurred in a different order:

(a) $P_1 = 0.7$ cm, $P_2 = 0.8$ cm, $P_3 = 2.4$ cm, $P_4 = 4.6$ cm
(b) $P_1 = 0.7$ cm, $P_2 = 4.6$ cm, $P_3 = 2.4$ cm, $P_4 = 0.8$ cm

Which order of the pulses gives the highest peak flow rate?

### *Assigned Problem 3-2-1*

The ratio between the sum of the DRH ordinates (5,185 $\text{m}^3/\text{s}$) and the sum of the UH ordinates (610 $\text{m}^3/\text{s/cm}$) is 8.5. What does this indicate?

### *Hint*

This verifies one of the important assumptions of the unit hydrograph theory.

*Assigned Problem 3-2-2*

Determine the ordinates ($Q$ values) of the DRO hydrograph at the outlet of a watershed due to the following 4 h rainfall event during which the net hourly rainfall depth pulse ($P$) values were as follows: $P_1 = 1$ cm, $P_2 = 3$ cm, $P_3 = 0$ cm, and $P_4 = 2$ cm. The unit hydrograph ordinates at 1 h intervals are $U_1 = 10\ \mathrm{m^3/s/cm}$, $U_2 = 75\ \mathrm{m^3/s/cm}$, $U_3 = 125\ \mathrm{m^3/s/cm}$, $U_4 = 100\ \mathrm{m^3/s/cm}$, $U_5 = 60\ \mathrm{m^3/s/cm}$, $U_6 = 20\ \mathrm{m^3/s/cm}$, and $U_7 = 5\ \mathrm{m^3/s/cm}$.

*Assigned Problem 3-2-3*

In the convolution equation method, which of the following hydrographs can be used for convolution if the duration of each net rainfall pulse is 2 h: (a) 2 h unit hydrograph, (b) 1 h unit hydrograph, (c) 0.75 h unit hydrograph, (d) 0.5 h unit hydrograph?

*Assigned Problem 3-2-4*

In the convolution equation method, what should be the time interval between unit hydrograph ordinates if the duration of each net rainfall pulse is 2 h: (a) 2 h, (b) 1 h, (c) 0.75 h, or (d) 0.5 h?

# Problem 3-3: Estimation of the Direct Runoff Hydrograph by the Lagging Method

## *Problem Statement*

Using the lagging method, determine the ordinates of the direct runoff hydrograph (DRH) at the outlet of a watershed due to the following 4 h rainfall event during which the net hourly rainfall depth pulse (P) values were as follows: $P_1 = 0.8$ cm, $P_2 = 2.4$ cm, $P_3 = 4.6$ cm, and $P_4 = 0.7$ cm. The 1 h unit hydrograph ordinates at 1 h intervals are $U_0 = 0\ \mathrm{m^3/s/cm}$, $U_1 = 10\ \mathrm{m^3/s/cm}$, $U_2 = 100\ \mathrm{m^3/s/cm}$, $U_3 = 200\ \mathrm{m^3/s/cm}$, $U_4 = 150\ \mathrm{m^3/s/cm}$, $U_5 = 100\ \mathrm{m^3/s/cm}$, $U_6 = 50\ \mathrm{m^3/s/cm}$, and $U_7 = 0\ \mathrm{m^3/s/cm}$.

## *Background*

This method uses the same equations as in the convolution method, except that the method by which the DRH ordinates are calculated is different. The lagging method calculates the DRH ordinates for each rainfall depth pulse and then adds the DROs *after proper lagging*. The DRH for each pulse is lagged by the duration of the pulse (which is 1 h in this problem). Essentially, when using the convolution equation (say Eq. 2 of Problem 3-2), a single equation is used to calculate $Q_2$. However, in the lagging method, the terms $P_1U_2$ and $P_1U_2$ are calculated separately and then added. The lagging method assumes that all rainfall pulses are of constant or uniform intensity. Therefore, the rainfall intensity during $P_1$ is 0.8 cm/h.

As in the convolution equation method, the lagging method also requires that the ratio of the duration of a pulse, $\Delta t$, to the duration $t$ of the unit hydrograph (1 h in this problem) must be equal to an integer. This rule helps in selecting hydrographs with the proper duration. Moreover, the ratio of the duration of a pulse, $\Delta t$, to the duration $\Delta t'$ between the unit hydrograph ordinates must also be equal to an integer (in the convolution equation method, this ratio must be equal to 1).

## *Assumptions*

(1) All rainfall pulses are of constant or uniform intensity. Therefore, the rainfall intensity during $P_1$ is 0.8 cm/h; (2) the duration of all pulses are equal.

## *Equations*

1. $[Q_1]_i = P_1U_i$ for $i = 1$ to $X$
2. $[Q_2]_i = P_2U_i$ for $i = 1$ to $X$
3. $[Q_3]_i = P_3U_i$ for $i = 1$ to $X$
4. $[Q_4]_i = P_4U_i$ for $i = 1$ to $X$
5. $[Q_{\mathrm{DRO}}]_i = [Q_1]_i + [Q_2]_{i+} [Q_3]_i + [Q_4]_i$ for $i = 1$ to $N$

where $[Q_1]_i$, $[Q_2]_i$, $[Q_3]_i$, and $[Q_4]_i$ are the DRH ordinates due to $P_1$, $P_2$, $P_3$, and $P_4$, respectively; $X$ is the number of nonzero unit hydrograph ordinates; and $N$ is the number of nonzero DRH ordinates.

### *Solution Procedure*

The solution procedure is fairly straightforward as long as you are careful to *lag* the DRH ordinates by $\Delta t$, where $\Delta t$ is the time interval between the pulses. In this problem $\Delta t = 1$ h; therefore, each pulse is lagged by 1 h, whereas the DRH due to $P_1$ starts at $t = 0$, the DRH due to $P_2$ starts at $t = 1$ h, and so on.

### *Solution*

As shown in Spreadsheet 3-3, the DRO hydrograph has nine nonzero ordinates, which are $Q_1 = 8\ \mathrm{m^3/s}$, $Q_2 = 104\ \mathrm{m^3/s}$, $Q_3 = 446\ \mathrm{m^3/s}$, $Q_4 = 1{,}067\ \mathrm{m^3/s}$, $Q_5 = 1{,}430\ \mathrm{m^3/s}$, $Q_6 = 1{,}110\ \mathrm{m^3/s}$, $Q_7 = 635\ \mathrm{m^3/s}$, $Q_8 = 300\ \mathrm{m^3/s}$, and $Q_9 = 35\ \mathrm{m^3/s}$. As expected, these ordinates are identical to the ordinates calculated by the convolution equation in Problem 3-2.

### *Assigned Problem 3-3-1*

Using the lagging method, determine the ordinates of the DRH at the outlet of a watershed due to the following 5 h rainfall event during which the net hourly rainfall depth pulse ($P$) values were as follows: $P_1 = 0.8$ cm, $P_2 = 2.4$ cm, $P_3 = 0$ cm, $P_4 = 4.6$ cm, and $P_5 = 0.7$ cm. The 1 h unit hydrograph ordinates at 1 h intervals are $U_0 = 0\ \mathrm{m^3/s/cm}$, $U_1 = 10\ \mathrm{m^3/s/cm}$, $U_2 = 100\ \mathrm{m^3/s/cm}$, $U_3 = 200\ \mathrm{m^3/s/cm}$, $U_4 = 150\ \mathrm{m^3/s/cm}$, $U_5 = 100\ \mathrm{m^3/s/cm}$, $U_6 = 50\ \mathrm{m^3/s/cm}$, and $U_7 = 0\ \mathrm{m^3/s/cm}$.

### *Assigned Problem 3-3-2*

In the lagging method, which of the following hydrographs can be used for convolution if the duration of each net rainfall pulse is 2 h: (a) 2 h unit hydrograph, (b) 1 h unit hydrograph, (c) 0.75 h unit hydrograph, or (d) 0.5 h unit hydrograph?

### *Assigned Problem 3-3-3*

In the lagging method, what can the time intervals between unit hydrograph ordinates be if the duration of each net rainfall pulse is 2 h: (a) 2 h, (b) 1 h, (c) 0.75 h, or (d) 0.5 h?

# Problem 3-4: Estimation of a *P-h* Unit Hydrograph Given an *R-h* Unit Hydrograph

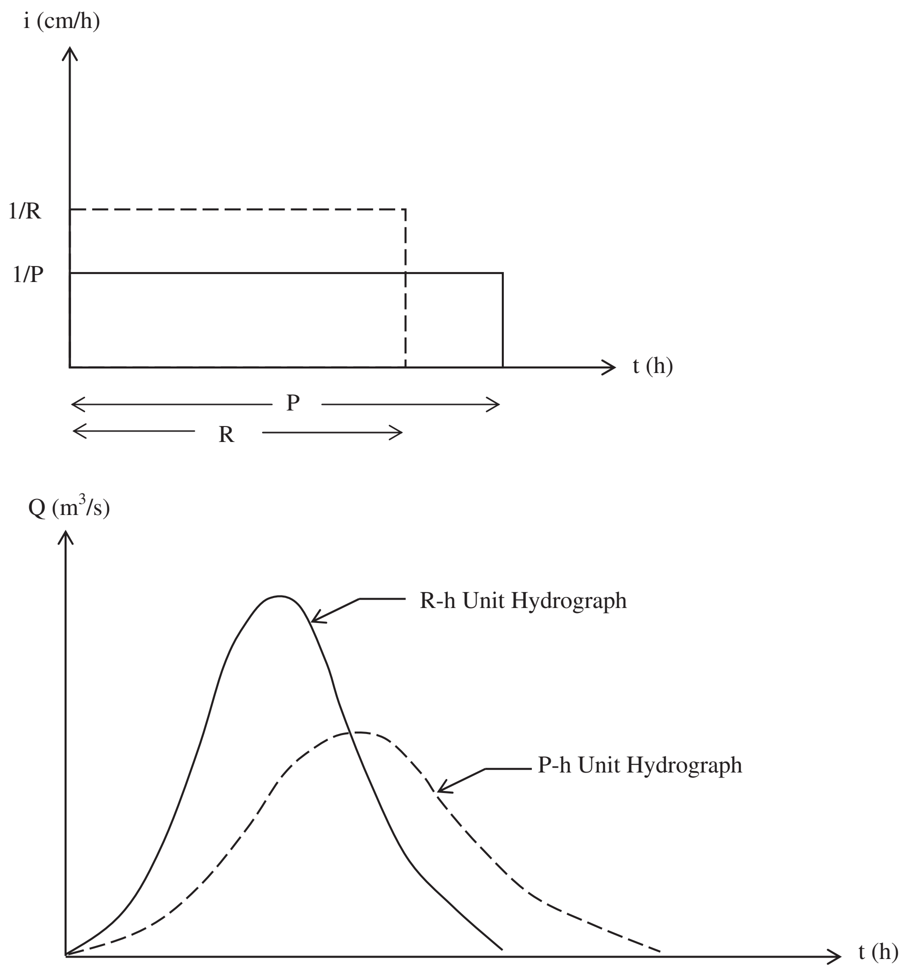
## *Problem Statement*

Determine the ordinates of a 3 h (*P-h*) unit hydrograph given the coordinates of a 2 h unit (*R-h*) hydrograph shown in Spreadsheet 3-4.

## *Background*

Theoretically, the *S*-curve is the sum of infinite *R-h* unit hydrographs, all sequentially lagged by *R-h*. In practice, the *S*-curve ordinates are obtained by adding a sufficient number of *R-h* unit hydrographs so that the *S*-curve reaches a plateau, i.e., the last few ordinates of the *S*-curve do not change in time. The *S*-curve is then lagged by *P-h* to obtain a lagged *S*-curve. The difference, *D*, between the ordinates of the *S*-curve and the lagged *S*-curve are multiplied by $P/R$ to obtain the *P-h* unit hydrograph. If *P* is greater than *R*, as in this problem, you should expect the *P-h* unit hydrograph to have a lower peak flowrate and a higher time base as shown in Fig. 3-6. The opposite would occur if *P* were less

***Fig. 3-6*** **Description of the *P-h* and *R-h* hyetograph and the corresponding unit hydrographs**



than $R$. The lowering of the peak flowrate is referred to as attenuation, while the increase in time base is called "dispersion."

Sometimes, if the unit hydrograph ordinates or $t$-$h$ value are not accurate, the $S$-curve will begin to oscillate around the plateau. In this case, the $S$-curve should be "smoothed" by adjusting the $S$-curve ordinates to achieve a plateau value before determining the lagged $S$-curve, the difference $D$, and the $P$-$h$ unit hydrograph.

### *Equations*

1. $(\mathrm{UH}_{R1})_t = (\mathrm{UH}_{R0})$
2. $(\mathrm{UH}_{R2})_t = (\mathrm{UH}_{R0})$
3. $(\mathrm{UH}_{R3})_t = (\mathrm{UH}_{R0})$
4. $(S)_t = (\mathrm{UH}_{R0})_t + (\mathrm{UH}_{R1})_t + (\mathrm{UH}_{R2})_t + (\mathrm{UH}_{R3})_t$
5. $(S_l)_t = (S)_{t-P}$
6. $(D)_t = (S_l)_t - (S)_t$
7. $(\mathrm{UH}_P)_t = [(D)_t](R/P)$

where $(\mathrm{UH}_{R0})$ is the $R$-$h$ unit hydrograph; $(\mathrm{UH}_{R1})_t$, $(\mathrm{UH}_{R2})_t$, and $(\mathrm{UH}_{R3})_t$ are the unit hydrographs lagged by $R$, $2R$, and $3Rh$, respectively; $(S)_t$ and $(S_l)_t$ are the $S$-curve and lagged $S$-curves; $(D)_t$ is the difference between the $S$-curve and lagged $S$-curve ordinates; and $(\mathrm{UH}_P)_t$ is the $P$-$h$ unit hydrograph.

### *Solution Procedure*

The $S$-curve is obtained by summing the ordinates of the $R$-$h$ unit hydrograph and three lagged $R$-$h$ unit hydrographs. Summing these four hydrographs was sufficient to obtain the $S$-curve, because the $S$-curve ordinates reached a plateau at 350 $\mathrm{m^3/s/cm}$.

### *Solution*

The solutions are shown in Spreadsheet 3-4. The results show the expected attenuation and dispersion; the peak flowrate of the $P$-$h$ unit hydrograph has decreased to 200 $\mathrm{m^3/s/cm}$ (from 250 $\mathrm{m^3/s/cm}$), whereas the time base has increased to 7 h from 6 h.

### *What-If Analysis*

Conduct a what-if analysis to show that

1. Attenuation and dispersion both increase as $P$ increases to $P = h$ and $P = 5\,h$ (instead of $P = 3\,h$).
2. An oscillating $S$-curve is obtained if $R = 3$ h instead of 2 h.

### *Assigned Problem 3-4-1*

Determine the ordinates, peak flowrate and the time base of the 1 h unit hydrograph from the given 2 h unit hydrograph.

# Problem 3-5: Estimation of the Downstream Hydrograph by Routing the Inflow Hydrograph Using the Muskingum Method

## *Problem Statement*

Determine the outflow (downstream) hydrograph, peak flowrate, and time to peak downstream of a channel resulting from the inflow (upstream) hydrograph channel shown in Spreadsheet 3-5. Also, determine the wedge storage ($S_w$), prism storage ($S_p$), and total storage ($S$) at various times. The storage time constant is $K = 2$ days, and the weighting factor is $X = 0.2$. Use a time increment $\Delta t = 24$ h, where $\Delta t$ is the time interval between inflow and outflow ordinates.

## *Background*

The equations used in this problem are described in Section 3. If the initial condition is not given, as in this problem, usual practice is to assume that the initial outflow is equal to inflow at $t = 0$.

## *Equations*

1. $C_1 = (\Delta t - 2KX)/[2K(1-X) + \Delta t]$
2. $C_2 = (\Delta t + 2KX)/[2K(1-X) + \Delta t]$
3. $C_3 = [2K(1-X) - \Delta t]/[2K(1-X) + \Delta t]$
4. $\sum C = C_1 + C_2 + C_3$
5. $Q_{i+1} = C_1 I_{i+1} + C_2 I_i + C_3 Q_i$
6. $S_w = KX(I - Q)$
7. $S_p = KQ$
8. $S = KX(I - Q) + KQ$

## *Solution Procedure*

The solution procedure is straightforward, and the outflow hydrograph and storage calculations are shown in Spreadsheet 3-5.

## *Solution*

As expected, the peak flowrate decreases from 1,699 $m^3/s$ to 1,460 $m^3/s$, whereas the time to peak increases from 192 h to 240 h (Spreadsheet 3-5). Note that the wedge storage becomes negative when $I$ is less than $Q$. Also note that both the prism storage and the total storage increase and then decrease as the water level rises. Although it was not required to calculate the water level in this problem, you can surmise that the highest peak flowrate occurred when the water level was at its peak. The water level during channel routing can be obtained from a stage-discharge graph, a plot of water level versus discharge if such a plot is available.

The inflow and outflow hydrographs are plotted on Chart 1 of Spreadsheet 3-5 to demonstrate the attenuation and dispersion that occurs as water travels through a channel reach.

### *What-If Analysis*

Conduct a what-if analysis by plotting a graph of $K$ versus peak flowrate, $Q_p$. Before conducting the analysis, try to guess the shape of the graph and explain your reasoning.

### *Assigned Problem 3-5-1*

Ignoring the data on alternate days (January 13, January 15, etc.) and using a $\Delta t = 48$ h, determine the flowrate at the downstream section (all other parameters are the same as in Problem 3-5). This shows that the selected value of $\Delta t$ can affect the results. In general, a smaller value of $\Delta t$ will provide more accurate results.

# Problem 3-6: Estimation of the Muskingum $K$ and $X$

### *Problem Statement*

Determine the Muskingum storage time constant $K$ and the weighting factor for the inflow and outflow hydrographs shown in Spreadsheet 3-6.

### *Background*

The Muskingum $K$ and $X$ values for a channel reach can be determined if both the inflow and outflow hydrographs are known for the channel reach. Several methods can be used to determine $K$ and $X$, but the method described here combines the Muskingum and continuity equations to obtain an equation in the form of $K = N/D$, Eq. (3-12), where $N$ and $D$ stand for numerator and denominator, respectively, and are respectively defined by Eqs. (3-13) and (3-14). It can be seen from Eqs. (3-13) and (3-14) that the value of $N$ is dependent on inflow and outflow, whereas the value of $D$ is dependent on inflow, outflow, and $X$. Values for $D$ are computed for various values of $X$, and several graphs are plotted between $N$ and $D$. The graph that plots closest to a straight line is used to determine both $K$ and $X$. Eqs. 2 to 4 (following) are basically Eq. (3-14) written for different values of $X$.

### *Assumptions*

The main assumption in this method is that a graph of $N$ versus $D$ should plot as a straight line and that the slope of that straight line will be equal to $K$.

### *Equations:*

1. $N = [(I_t + I_{t+1}) - (Q_t + Q_{t+1})](0.5\Delta t)$
2. $D_1 = (X)(I_{t+1} - I_t) + (1 - X)(Q_{t+1} - Q_t)$ (for $X = 0.1$)
3. $D_2 = (X)(I_{t+1} - I_t) + (1 - X)(Q_{t+1} - Q_t)$ (for $X = 0.2$)
4. $D_3 = (X)(I_{t+1} - I_t) + (1 - X)(Q_{t+1} - Q_t)$ (for $X = 0.3$)
5. $K = N/D_2$

where $D_1$, $D_2$, and $D_3$ are the values of $D$ at $X = 0.1$, 0.2, and 0.3, respectively.

### *Solution Procedure*

Because the value of $X$ is usually between 0 and 0.3, $D$ is calculated for $X = 0.1$, 0.2, and 0.3. $X$ values other than these three values should be tried if none of these three values produces a straight line. The three graphs, $N$ versus $D_1$, $D_2$, and $D_3$ are shown as Charts 1, 2, and 3, in Spreadsheet 3-6, respectively.

*Solution*

The second plot in Chart 2 is closest to a straight line, indicating that $X = 0.2$. The value of $K$ is then determined by Eq. 3 using the values of the $D_2$ column. The average value of $K$ (Spreadsheet 3-6) is the final answer. Alternatively, because $K$ is equal to the slope of the straight line plot between $N$ and $D$, $K$ can also be determined by simply calculating the slope of the line in Chart 2. Answers: $X = 0.2$, $K = 10$ min.

*Assigned Problem 3-6-1*

Determine the value of $K$ if $X = 0.25$.

# Problem 3-7: Determine the Outflow Hydrograph from a Reservoir Using Hydrologic Reservoir (Level Pool) Routing and Calculate the Peak Outflow and the Maximum Water Level in the Reservoir

## *Problem Statement*

The coordinates of an inflow hydrograph to a reservoir are shown in Spreadsheet 3-7. Determine the outflow hydrograph from 0 to 2 days, at intervals of 0.1 days, from a reservoir with the following storage-outflow and stage-storage relationships:

$$S/(2\Delta t) = Q + 10 \tag{3-19}$$

$$H = S/20 \tag{3-20}$$

In Eq. (3-19), $\Delta t$ is the time step in days, $S$ is in $m^3$/s-days, and the number 10 has units of $m^3$/s. In Eq. (3-20), $H$ is in m, $S$ is in $m^3$/s-days, and the number 20 has units of ha. The initial storage in the reservoir is 2 $m^3$/s-days (or 17.28 ha-m).

## *Background*

As discussed in Section 3, the three main equations in a typical reservoir routing problem are the continuity equation and two empirical equations that usually are provided by the engineer who designs the reservoir. The two empirical equations are relationships between storage and outflow, such as Eq. (3-19), and stage and storage, such as Eq. (3-20). Because the continuity equation provides a value of $2S/\Delta t + Q$ instead of $S$, equations like Eq. (3-19) should be rearranged so that $Q$ can be calculated as a function of $2S/\Delta t + Q$. Simple algebraic manipulations are used to accomplish this objective, and Eq. (3-19) is rearranged to

$$Q = [(2S/\Delta t + Q) - 40)]/5 \quad \text{for } Q \geq 0 \tag{3-21}$$

Eq. (3-19) also should be rearranged so that $S$ can be calculated if $Q$ is known. Thus, for convenience in calculations, Eq. (3-19) is rearranged to

$$S = 2\Delta t(Q + 10) \tag{3-22}$$

Substitution of $\Delta t = 0.1$ days and $Q = 0$ in Eq. (3-22) indicates that a minimum storage ($S$) of 2 $m^3$/s-days is needed before there is any outflow from the reservoir. This value is then converted to ha-m (column 10). The value of $Q$ (column 7) is then calculated by rearranging Eq. (3-19) to

$$Q = S/(2\Delta t) - 10 \tag{3-23}$$

A negative $Q$ value would be calculated if the initial value of $S$ were less than 2 $m^3$/s-days. The value of $Q$ is adjusted (column 8) to zero if a negative value is computed (because a negative $Q$ value is not physically possible). The value of $2S/\Delta t + Q$ in row 1 is calculated by simply adding the terms $2S/\Delta t$ and $Q$. The value of $2S/\Delta t - Q$ can be calculated by simply subtracting the value of $Q$ from $2S/\Delta t$. As an alternative, you may use the equation

$$2S/\Delta t - Q = (2\,S/\Delta t + Q) - 2Q \tag{3-24}$$

### Assumption

The reservoir level is the same everywhere at any given time (hence the name "level pool" routing).

### Equations

1. $(I_{\text{sum}})_{j+1} = I_j + I_{j+1}$
2. $H_j = S_j/20$
3. $X_j = 2S_j/\Delta t - Q_j$
4. $Y_j = 2S_j/\Delta t + Q_j$
5. $X_j = Y_j - 2Q_j$
6. $Y_{j+1} = (I_{\text{sum}})_j + X_j$
7. $Q_j = S_j/(2\Delta t) - 10$
8. $Q_j = (Y_j - 40)/5$
9. $Q_j = 0$ (if $Q$ is negative)
10. $S_j = 2\Delta t(Q_j + 10)$

where subscripts $j$ and $j+1$ denote the times before and after the routing period, $\Delta t$, $I_{\text{sum}}$ is the sum of the inflows before and after the routing period, and $X$ and $Y$ are parameters defined by Eqs. 3 and 4, respectively. It should be noted that Eqs. (3-20) to (3-24) have been respectively converted to Eqs. 2, 8, 10, 7, and 5 after adding subscripts.

### Solution Procedure

The solution procedure is shown in Spreadsheet 3-7. This spreadsheet is slightly different from other spreadsheets, because (a) equations for row 1 are different from those used for row 2 onward and (b) the starting point of the spreadsheet in row 1 is column 9. Note that the starting column would be different if the initial water level ($H$) or initial flowrate ($Q$) were given instead of the initial storage ($S$).

### Solution

The flowrates of the routed hydrograph and the corresponding water levels in the reservoir are respectively shown in columns 8 and 11 of Spreadsheet 3-7. The peak outflow of 29 $m^3$/s occurs at $t = 0.8$ days. The highest water level during this storm event also occurs at 0.8 days and is equal to 3.37 m. Note that, as expected, the reservoir has attenuated the flowrate from a peak inflow of $36\,\mathrm{m}^3/\mathrm{s}$ (at $t = 0.6$ days) to a peak outflow of $29\,\mathrm{m}^3/\mathrm{s}$. Also, as expected, the peak outflow occurs later than the peak inflow (at $t = 0.8$ days as compared to $t = 0.6$ days). The inflow and outflow hydrographs are plotted in Chart 1 of Spreadsheet 3-7 to demonstrate the attenuation and dispersion that occurs as water travels through a reservoir.

### *What-If Analysis*

Conduct a what-if analysis by plotting a graph of $Q$ versus $S$ and $H$ versus $S$. Is the change in $Q$ and $H$ as expected?

### *Assigned Problem 3-7-1*

Calculate the initial storage, peak outflow, and water level if the initial water level is 1.5 m.

### *Assigned Problem 3-7-2*

Calculate the peak outflow and water level if the reservoir was (a) initially empty and (b) $H = 1.5$ m. Will the starting column still be column 9?

### *Assigned Problem 3-7-3*

Calculate the peak outflow and water level if the initial $Q = 10\,\mathrm{m}^3/\mathrm{s}$. Will the starting column still be column 9? Why does the water level in the reservoir initially decrease and then increase?

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# 4

# *Stormwater Management*

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# Background

A stormwater management system is required in many states of the United States whenever any watershed is developed. This section covers the processes needed to design a stormwater management system for land development. A stormwater management system for land development is used to control both on-site and downstream flooding and to improve the water quality of the on-site runoff prior to its discharge into a receiving water body. The methods used to store and/or to treat stormwater runoff are sometimes referred to as best management practices (BMPs). The U.S. Environmental Protection Agency defines a BMP as a "technique, measure or structural control that is used for a given set of conditions to manage the quantity and improve the quality of stormwater runoff in the most cost-effective manner." BMPs can be either structural or nonstructural (such as street cleaning). The purpose of this chapter is to discuss the design of some structural BMPs.

# Components of a Stormwater Management System for Land Development

A stormwater management system for land development usually has the following components: (a) a storage area to store and/or treat on-site runoff, (b) storm sewers to carry the on-site runoff to the storage area, and (c) an outlet structure to discharge the water from the storage area to a receiving water body in a controlled manner.

### *Storage Areas*

Many different types of storage areas are used for controlling and treating on-site runoff. These include, but are not limited to, dry retention basins, dry detention basins, dry retention–detention basins, wet detention basins, infiltration trenches, roof top storage, and subsurface storage. The basins are also sometimes referred as "ponds." This book covers the design of dry retention–detention basins and wet detention basins. A key difference between a dry and a wet basin is that the bottom of a dry basin, whether it is a dry retention basin (Fig. 4-1), dry detention basin (Fig. 4-2), or dry retention–detention basin (Fig. 4-3), is above the water table, and the basin is expected to dry up between storm events. The bottom of a *wet* detention basin is below the water table (Fig. 4-4), and as a result water is always visible in a wet detention basin. If properly designed, a wet basin can be aesthetically pleasing as it serves as an artificial "lake."

### *Dry Retention Basins*

A dry retention basin, shown in Fig. 4-1, does not have an engineered outlet structure, such as a weir, pipe, or spillway, and the only way that water is removed from a retention basin is via evaporation or infiltration. A retention basin, therefore, theoretically provides 100% pollutant removal because no water—contaminated or clean—leaves the basin to a receiving water body.

***Fig. 4-1*** **Description of a dry retention basin**

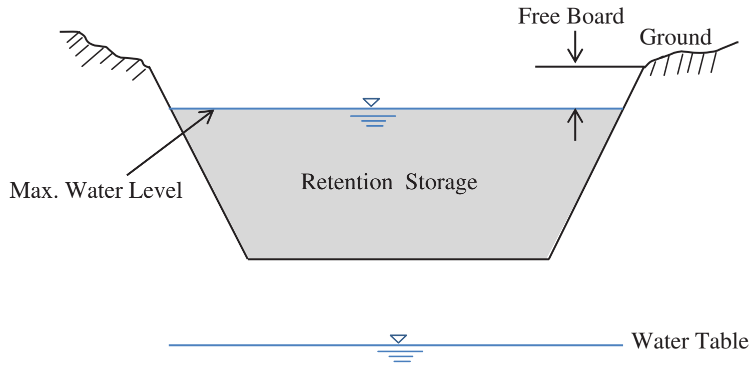



***Fig. 4-2*** **Description of a dry detention basin**

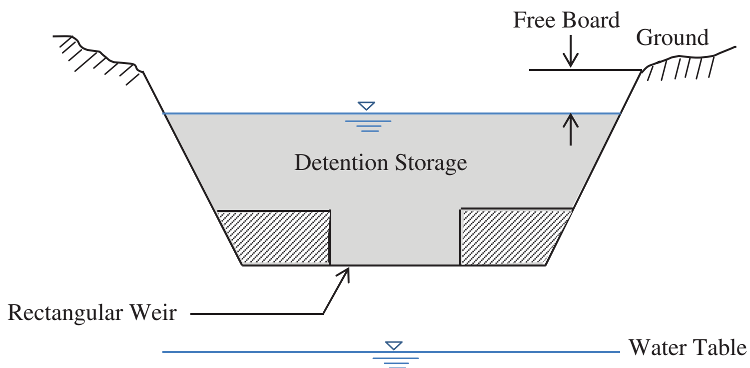
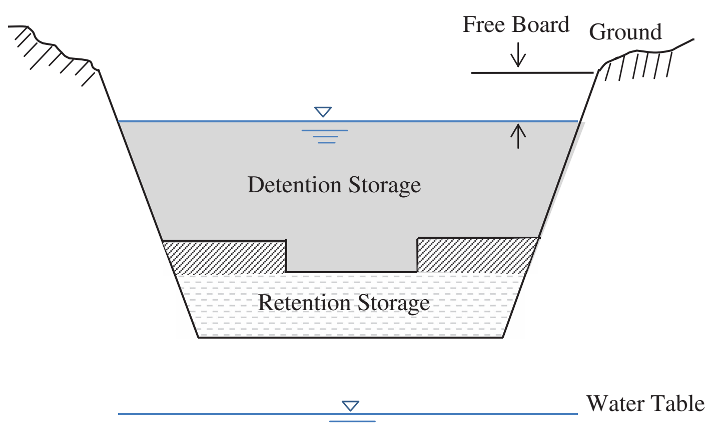


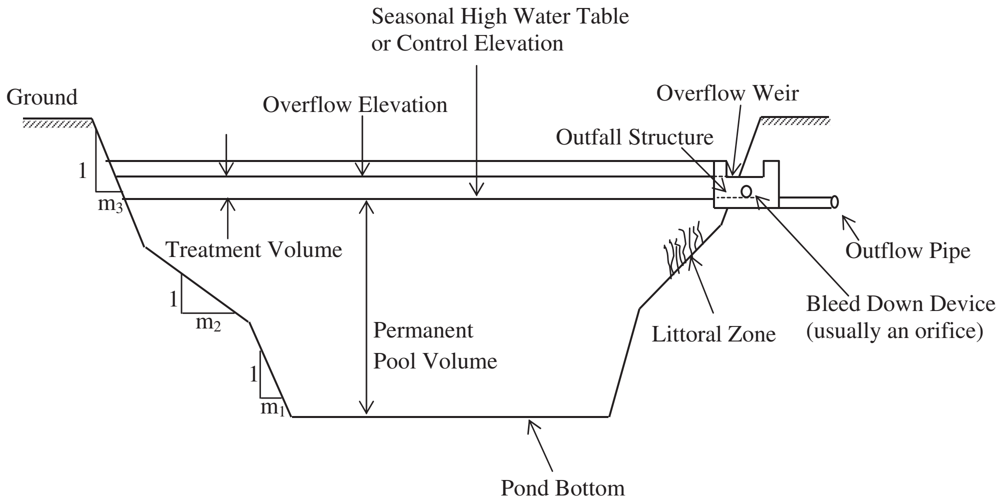

***Fig. 4-3*** **Description of dry retention–detention basin**




*Dry Retention–Detention Basins*

In a dry retention–detention basin, shown in Fig. 4-3, an outlet structure is placed at some elevation above the basin bottom. The water stored below the outlet structure elevation is "retained," as it can leave the basin only via infiltration or evaporation. The water stored above the outlet structure elevation is said to be

***Fig. 4-4*** **Description of a wet detention basin**




"detained" (Fig. 4-3), as it is stored only temporarily in the basin. Usually the runoff due to the first few centimeters of precipitation, also known as the "first flush," is contaminated as it washes the watershed of contaminated dust particles, oil lying on the pavement due to oil leaks from vehicles, and other types of contamination. A dry retention—detention basin provides improved water quality by retaining this first flush. It also provides flood relief by discharging water into a receiving water body in a controlled manner via the outlet structure.

### *Dry Detention Basins*

In a dry detention pond the outlet structure is placed right at the pond bottom (Fig. 4-2). As a result, there is no retention of the first flush but the basin does provide flood relief.

### *Wet Detention Basins*

As shown in Fig. 4-4, a wet-detention pond consists of two outlet structures, a small circular opening, usually referred to as an "orifice" or a "bleed down device" and a larger weir usually referred to as the overflow weir. A small triangular weir could also be used in place of the circular orifice. The smaller outlet structure ideally should be placed at the season high water table (SHWT) mark so that it cannot drain groundwater. The elevation at which the bleed down device is placed is called the control elevation (CE). Ideally, the CE should coincide with the SHWT elevation. The pond bottom is sometimes several meters below the CE, and the volume contained in the basin below the CE is referred to as the "permanent pool volume" (PPV).

The purpose of the PPV is to provide a nice aesthetic effect and also to clean the "dirty" water entering the pond via particle settlement or sedimentation, algal growth, and biological oxidation of organic materials. The bleed down device is designed to discharge the water slowly from the pond to allow the dirty water to stay in the pond for a long enough period so that it can be treated prior to being discharged. The elevation at which the overflow weir is placed is termed the "overflow elevation" (OE). The overflow weir is provided to alleviate flooding conditions within the basin, as it allows the basin water to discharge into a receiving body of water. The volume of water contained between the weir and the bleed down device is referred to as "treatment volume" (TV) as shown in Fig. 4-4. The water in the pond also can be treated biologically by providing a gently sloping "littoral" zone, usually on the periphery of the pond. A littoral zone consists of aquatic plants with the capability of assimilating nutrients from the pond water. The littoral zone should not extend more than 1 m below the CE. As discussed in Problem 4-20, the slopes $m_1$, $m_2$, and $m_3$, shown in Fig. 4-4, must be within an assigned range based on the applicable code.

### *Storm Sewers*

Storm sewers are pipes that are especially designed to convey stormwater or on-site runoff to the basins using gravity flow.

## Design of a Stormwater Management System for Land Development

The design of a stormwater management system requires the following steps:

1. Development of a design hyetograph
2. Estimation of the excess or net rainfall hyetograph or runoff depths
3. Estimation of the time of concentration ($t_c$)
4. Determination of peak flowrate or the direct runoff hydrograph (DRH)
5. Design of conveyance system
6. Design of storage area
7. Design of outlet structure
8. Estimation of the flood elevation by routing
9. Final check to determine if design meets all necessary codes.

Although other methods are discussed in some problems, this book mainly focuses on the design of a stormwater management system using the Natural Resources Conservation Service (NRCS) methods. These methods are also known as the Soil Conservation Service (SCS) methods. The NRCS methods are fairly comprehensive, and a detailed discussion of these methods is provided in SCS (1986). These methods are also described in several hydrology textbooks.

### *Development of a Design Hyetograph*

Several methods are available to determine a design hyetograph. This book covers the following three methods: (i) triangular hyetograph method (Problem 4-2); (ii) NRCS method (Problem 4-3); and (iii) alternating block method (Problem 4-4).

### *Estimation of the Excess or Net Rainfall Hyetograph or Runoff Depths*

Several methods are available to determine excess or net rainfall hyetographs, such as the NRCS method and the Green-Ampt method. This book discusses the NRCS method (Problem 4-5).

### *Estimation of the Time of Concentration* ($t_c$)

Many methods are discussed in the literature for estimating the time of concentration. The two methods covered in this book are the NRCS method (Problem 4-6) and the kinematic wave method (Problem 4-7).

### *Determination of Peak Flowrate or the Direct Runoff Hydrograph (DRH)*

Peak flowrates and/or direct runoff hydrographs are required to design various components of the stormwater management system. The advantage of determining a DRH is that it provides the hydrograph including the peak flowrate. However, there are methods that are used to determine just the peak flowrate. Two of the methods that can be used to predict just the peak flowrate are the NRCS graphical method (Problem 4-8) and the rational method (Problems 4-11 and 4-12). A vast number of methods are available for estimating the DRH. One of the methods, the convolution method, was discussed in Problems 3-2 and 3-3. However, the convolution method may not be possible in land development projects, as the watershed being developed usually is ungagged, and a unit hydrograph is not available. Among the many approaches proposed to determine the DRH for ungagged watersheds, two commonly used methods for determining DRH hydrographs in land development are the NRCS tabular method and the Santa Barbara urban hydrograph (SBUH) method. These methods are discussed in Problems 4-9 and 4-10, respectively.

### *Design of a Conveyance System*

In most land development projects, the runoff is conveyed via storm sewers or by shallow grassy channels, also known as swales. The design of storm sewers is discussed in Problem 4-13, whereas the design of swales will be identical to the methods described in Problems 2-16 and 2-17.

### *Design of Storage Areas*

The design of single and two-stage dry retention–detention basins are described in Problems 4-16 and 4-17. Problems 4-14 and 4-15 demonstrate how useful trendline equations can be obtained in the design of dry retention–detention basins. The design of a wet detention basin is discussed in Problems 4-18 to 4-20.

# Problem 4-1: Estimation of IDF Graphs Using Trendlines

### *Problem Statement*

Determine a trendline equation for the 25-year intensity-duration-frequency (IDF) graph for Zone 7, Florida. Also determine the corresponding $R^2$ for the trendline equation and the percentage error introduced by using the trendline equation.

### *Background*

IDF graphs, if available, are very handy for estimating rainfall intensity if the duration and frequency of the rainfall event are known. The rainfall intensity and duration data for a 25-year storm in Zone 7, Florida, is provided in Spreadsheet 4-1. The initial intensity data were obtained in in./h and were converted to cm/h. Among other applications, IDF graphs are used when engineers estimate peak flowrates using the rational method.

### *Assumptions*

A second-order polynomial has been used to determine the values of rainfall intensity, *s*, as a function of log *t*.

### *Equations*

1. $i_{tr} = 2.681(\log t)^2 - 22.251(\log t) + 40.75$
2. $\mathrm{PE} = (i - i_{tr})/i \times 100\%$

where $i$ is the rainfall intensity (cm/h) obtained from the IDF graph, $i_{tr}$ is the rainfall intensity (cm/h) predicted by the trendline, $t$ is time (minutes), and PE is the percent error in the value estimated by the trendline.

### *Solution Procedure*

A graph was plotted between $i$ and log $t$ (Chart 1 of Spreadsheet 4-1) and was fitted by a second-order trendline equation. The corresponding $R^2$ value was obtained using Excel. The trendline equation was converted to Eq. 1 shown above. Using Eq. 1 will be much more convenient to estimate rainfall intensity than repeatedly reading the IDF graph. The trendline equation should only be used for $t$ between 0 and 2 h as that was the range of data for which it was developed. Extrapolating trendline equations to outside their range can lead to large errors.

### *Solution*

The equation obtained from the trendline analysis is Eq. 1. The maximum percentage error is 1.71% as shown in Spreadsheet 4-1. The trendline equation can be considered accurate because the highest percentage error is less than 2%.

### *Assigned Problem 4-1-1*

Create a spreadsheet to determine the trendline equations for the 5-year ($T = 5$ years) storm for Zone 7, Florida. The intensity-duration data are shown in Spreadsheet 4-1a. Also determine the percentage error introduced by using the trendline equation.

### *Assigned Problem 4-1-2*

Rework Problem 4-1 using a third-order polynomial as a trendline equation and determine if the percentage error reduces compared with the second-order equation.

# Problem 4-2: Estimation of a Design Hyetograph Using the Triangular Hyetograph Method

## *Problem Statement*

Determine the peak rainfall intensity, $i_p$, and time at which peak intensity occurs, $t_a$, for a hyetograph resulting from a 60-min, 25-year storm in Zone 7, Florida. The rainfall depth is 11.43 cm and the storm advancement coefficient, $r$, is 0.40.

## *Background*

The triangular hyetograph method is one of the simplest methods. This method assumes that the hyetograph has a triangular shape with a duration, $t_d$, and peak intensity, $i_p$. The storm advancement constant, $r$, is the ratio of $t_a$ and $t_d$. These definitions lead to the following two equations:

$$i_p = 2P/t_d \tag{4-1}$$

and

$$t_b = t_d(1-r) \tag{4-2}$$

where $P$ is the rainfall depth. Values of $r$ for large cities can be found in Wenzel (1982). The rainfall depth, $P$, for this problem was obtained at http://www.nws.noaa.gov/oh/hdsc/PF_documents/TechnicalPaper_No40.pdf. This website can be used to determine $P$ for at any location in the United States for several assigned values of the return period, $T$, and storm duration, $t_d$.

## *Assumptions*

It is assumed that the hyetograph has a triangular shape and that the value of $r$ is independent of rainfall depth or duration.

## *Equations*

1. $i_p = 2P/t_d$
2. $t_b = t_d(1-r)$
3. $t_a = (t_d - t_b)$

where $P$ is the rainfall depth, and $t_b$ is the duration between the time at which $i_p$ occurs to the end of the rainfall event.

## *Solution Procedure*

The problem solution is straightforward and is shown in Spreadsheet 4-2.

## *Solution*

The maximum intensity of 22.86 cm/h occurs after a period of 24 min.

### *What-If Analysis*

Determine the effect of the storm advancement coefficient $r$ by plotting a graph of $i_p$ versus $r$ for a 60-min storm event if $P = 12$ cm.

### *Assigned Problem 4-2-1*

Determine the triangular hyetograph for a 15-min, 100-year storm in Cleveland, Ohio. The value of $r$ for Cleveland is 0.375.

### *Assigned Problem 4-2-2*

Determine the triangular hyetograph for a 24-h, 100-year storm in Cleveland, Ohio. The value of $r$ for Cleveland is 0.375.

### *Assigned Problem 4-2-3*

Determine the triangular hyetograph for a 24-h, 100-year storm in Chicago, Illinois. The value of $r$ for Chicago is 0.375.

# Problem 4-3: Estimation of a Design Hyetograph Using the NRCS Method

## *Problem Statement*

Determine the precipitation hyetographs resulting owing to 24-h, 25-year storm events at Los Angeles, California; Seattle, Washington; Lincoln, Nebraska; and Miami, Florida, using the Natural Resources Conservation Service (NRCS) method.

## *Background*

The NRCS method for estimating hyetographs is based on four synthetic regional dimensionless rainfall time distributions known as Type I, Type 1A, Type II, and Type III distributions. The rainfall distributions are the ratios of the cumulative precipitation $P$ at a given time and the cumulative precipitation $P_{24}$, which occurs over a 24-h period. The values of $P/P_{24}$ for all four types of precipitation distribution are shown in Spreadsheet 4-3. Type I and IA represent the Pacific maritime climate, Type III represents the Gulf of Mexico and Atlantic Coastal areas, and Type II represents the rest of the country. The exact areas covered by these distributions can be found at http://www.lmnoeng.com/RainfallMaps/RainfallMaps.htm for the remaining part of the United States. Specifically, the cities of Los Angeles, Seattle, Lincoln, and Miami come under Type I, Type 1A, Type II, and Type III distributions respectively. The $P/P_{24}$ values for the four regions can be found at www.hydrocad.net/rftables.htm#SCS%20Rainfalls. The 25-year, 24-h rainfall depths for the four locations are shown in Spreadsheet 4-3 and can be found at the previous link at www.wrcc.dri.edu/pcpnfreq.html for the western states and at www.lmnoeng.com/RainfallMaps/RainfallMaps.htm for the remainder of the country. It is worth noting that the Type II and Type III precipitation distributions yield the highest peak intensities (Lincoln and Miami).

## *Equations*

1. $i_I = [(P/P_{24})_{t+\Delta t} - (P/P_{24})_{t+\Delta t}]_{\mathrm{I}}(P_{24})/\Delta t$
2. $i_{\mathrm{IA}} = [(P/P_{24})_{t+\Delta t} - (P/P_{24})_{t+\Delta t}]_{\mathrm{IA}}(P_{24})/\Delta t$
3. $i_{\mathrm{II}} = [(P/P_{24})_{t+\Delta t} - (P/P_{24})_{t+\Delta t}]_{\mathrm{II}}(P_{24})/\Delta t$
4. $i_{\mathrm{III}} = [(P/P_{24})_{t+\Delta t} - (P/P_{24})_{t+\Delta t}]_{\mathrm{III}}(P_{24})/\Delta t$

where $i_{\mathrm{I}}$, $i_{\mathrm{IA}}$, $i_{\mathrm{II}}$, and $i_{\mathrm{III}}$ are the rainfall intensities for the Type I, Type 1A, Type II, and Type III distributions respectively; $t$ is time; and $\Delta t$ is the incremental duration.

### *Solution Procedure*

The solution is shown in Spreadsheet 4-3.

### *Assigned Problem 4-3-1*

Create a spreadsheet to determine the precipitation hyetographs resulting due to 24-h, 25-year storm events in Sacramento, California; Portland, Oregon; Kansas City, Missouri; and New York City, New York using the Natural Resources Conservation Service (NRCS) method.

# Problem 4-4: Estimation of a Design Hyetograph Using the Alternating Block Method

## *Problem Statement*

Determine the hyetograph ordinates resulting from a 2-h, 25-year storm event in Zone 7, Florida, using the alternating block method. Use a time increment of 5 min.

## *Background*

The alternating block method for estimating hyetographs utilizes IDF curves and is designed to estimate the worst condition hyetograph for the given $T$-year period. The NRCS design hyetographs, discussed in Problem 4-3, were created using the alternating block method.

## *Assumption*

The main assumption of the alternating block method is that the maximum rainfall depth occurs at the center of the storm.

## *Equations*

1. $i = 2.6824(\log t)^2 - 22.251(\log t) + 40.75$
2. $P_t = i_t t$
3. $\Delta P_{t+\Delta t} = P_{t+\Delta t} - P_t$

where $i_t$ is the 25-year rainfall intensity at time $t$, $P_t$, and $P_{t+\Delta t}$ are the cumulative rainfall depths at times $t$ and $t + \Delta t$, $\Delta P$ is the incremental rainfall depth, and $t$ is time.

## *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-4. The value of $t$ is between 0 and 2 h, because the objective is to determine a 2-h hyetograph. The trendline equation determined in Problem 4-1 (Eq. 1 of Problem 4-1) was used to represent the IDF curve. The $\Delta P$ values were calculated using Eqs. 2 and 3. Because the main assumption of the alternating block method is that the maximum rainfall depth occurs at the center of the storm, the highest incremental rainfall depth (2.21 cm) is placed at $t = 60$ min in the last column ($\Delta P_{\text{alt}}$) of Spreadsheet 4-4. The next highest value (1.32 cm) is placed at $t = 65$ min and the following highest number (1.04 cm) at $t = 55$ min. Each value of $\Delta P$ is alternately placed below and above the central number (or block), which is why the method derives its name as the “alternating block method.”

*Solution*

The design hyetograph ordinates are provided in Spreadsheet 4-4.

*Assigned Problem 4-4-1*

Create a spreadsheet to determine the hyetograph resulting from a 2-h, 5-year storm events in Zone 7, Florida, using the alternating block method. (Hint: You will have to use the IDF equation obtained from Problem 4-1.)

*Assigned Problem 4-4-2*

Determine the hyetograph resulting from a 3-h, 25-year storm event in Zone 7, Florida, using the alternating block method. Use a time increment of 5 min.

# Problem 4-5: Estimation of Runoff Depth Using the NRCS Method

## *Problem Statement*

Determine the values of the incremental runoff depth ($\Delta Q$) resulting from a 24-h, 25-year storm event at a commercial watershed in Miami, Florida, using the Natural Resources Conservation Service (NRCS) method. The pervious curve number ($CN_p$) for the land use is 90. The fraction of initial abstraction $f = 0.2$. The 24-h rainfall depth, $P_{24}$, is 28 cm.

## *Background*

The second step in designing a stormwater management system, after determining the design hyetograph (which is the first step), is determining the amount of rainfall that converts into runoff, which is also termed "net precipitation" or "excess precipitation." The NRCS has developed a method for estimating the runoff depth known as the curve number (CN) method. The CN, a dimensionless number, depends on the type of soil (sand, clay, etc.), type of land use (residential, commercial, etc.) and the existing antecedent moisture condition, which is based on the degree of soil saturation prior to the design precipitation event. The CN range is typically from 40 for highly pervious areas to 98 for impervious areas. The fraction of initial abstraction is usually assumed to be 0.2, although it could have a different value. Equations 1 and 4 following were developed by the NRCS.

## *Equations*

1. $S = 2{,}540/\mathrm{CN} - 25.4$
2. $I_a = 0.2S$
3. $\Delta P_t = i_t \Delta t$
4. $P_{t+\Delta t} = \Delta P_{t+\Delta t} + P_t$
5. $Q_t = (P_t - I_a)^2/(P_t - I_a + S)$ for $P_t \geq I_a$ and zero otherwise
6. $\Delta Q_{t+\Delta t} = Q_{t+\Delta t} - Q_t$
7. $\Delta A = \Delta P - \Delta Q$
8. $A_{t+\Delta t} = \Delta A_{t+\Delta t} + A_t$
9. $A_{\mathrm{net}} = A - I_a$ for $A_{\mathrm{net}} \geq I_a$ and zero otherwise

where $i_t$ is the 25-year rainfall intensity for Miami, Florida (obtained from Problem 4-3); $S$ is the potential maximum retention after runoff begins in cm; $I_a$ is the initial abstraction; $\Delta P$, $\Delta Q$, and $\Delta A$ are the respective incremental rainfall, runoff, and abstraction depths during a time interval $\Delta t$; $P$, $Q$, and $A_t$ are the respective cumulative rainfall, runoff, and total abstraction depths; $A_{\mathrm{net}}$ is the net abstraction, which is the difference between the total and initial abstraction depth; and subscripts $t$ and $t + \Delta t$ are used to define values before and after the period $\Delta t$.

### Solution Procedure

The solution is shown in Spreadsheet 4-5. The values of $i$ were imported from Spreadsheet 4-3. It is usually a good idea to combine the spreadsheets of Problems 4-3 and 4-5 into a single spreadsheet. The calculations using Eqs. 5 and 9 require an "IF" statement, as two possible values are possible. Note that runoff cannot occur if $P_t$ is less than or equal to $I_a$. A substantial error can occur if one neglects to put this "IF" statement in the cell equation of Eq. 5, because the term $P_t - I_a$ is squared in the equation and will yield positive numbers even if $P_t - I_a$ is negative.

### Solution

The design precipitation of 27.94 cm results in a runoff depth of 24.82 cm. The precipitation abstracted by the ground is therefore 3.12 cm, and the percentage of precipitation that converts to runoff is 88.83%. The relatively low amount of abstraction is due to the high CN value of the land use. The highest amount of runoff (5.517 cm) occurs in the period between 12.00 and 12.5 h. The total abstraction is 3.123 cm, and the net abstraction is 2.558 cm. Notice that the net abstraction (2.558 cm) is approximately 90% of the maximum possible abstraction, $S$ (2.822 cm).

### What-If Analysis

Determine the effect of $\mathrm{CN}_p$ by plotting (a) a graph between $A_{\mathrm{net}}$ at $t = 24$ h and $\mathrm{CN}_p$ and (b) a graph between the percentage of $S$ that is used for abstraction over a 24-h period versus $\mathrm{CN}_p$. The range of $\mathrm{CN}_p$ should be between 40 and 90. What conclusions can you make from these graphs?

### Assigned Problem 4-5-1

Determine the incremental runoff depths resulting from a 24-h, 25-year storm event at an undeveloped watershed in Los Angeles, California, using the Natural Resources Conservation Service (NRCS) method. The land use CN is 61.

### Assigned Problem 4-5-2

Determine the incremental runoff depths resulting from a 24-h, 25-year storm event at an undeveloped watershed in Lincoln, Nebraska, using the Natural Resources Conservation Service (NRCS) method. The land use CN is 48.

# Problem 4-6: Estimation of Time of Concentration Using the NRCS Method

## *Problem Statement*

Determine the time of concentration for a watershed in Miami, Florida, using the Natural Resources Conservation Service (NRCS) method. The sheet flow length is 30.5 m and occurs over dense grass at a slope of 0.01. The shallow concentrated flow occurs over 427 m of unpaved surface with a slope of 0.01. The open channel flow occurs in a channel of slope 0.01 that has an area of cross section of 2.5 m, a wetted perimeter of 8.60 m, and a flow length of 2,225 m. The 2-year, 24-h precipitation depth in Miami is 9.14 cm, and the Manning's $n$ for sheet flow and open channel flow are 0.24 (dense grasses) and 0.05, respectively.

## *Background*

Several methods are used to estimate the time of concentration, which is defined as the time it takes from the *hydraulically* farthest point of the watershed to the outlet of the watershed. The two methods discussed in this text are the NRCS method and the kinematic wave method. The NRCS method defines three types of flows known as sheet flow, shallow concentrated flow, and open channel flow. Sheet flow occurs over relatively flat surfaces such as parking lots, pavements, lawns, etc. There are two key features of sheet flow: (a) it is spread out and, therefore, occurs at very shallow depths and (b) studies have shown that the maximum distance over which it can occur is 91.5 m. As opposed to sheet flow, shallow concentrated flow is not spread out and can occur on unpaved areas, parking lots, roadside gutters, drainage pipes, or small roadside canals or ditches. In the NRCS method, open channels are defined as channels that are visible on aerial photographs or appear as blue lines on U.S. Geological Survey (USGS) maps. In other words, flow through small channels would be considered shallow concentrated flow, whereas flow through larger channels would be considered open channel flow. Average flow velocity in an open channel is usually determined for bank-full elevation.

## *Equations*

1. $T_{ts} = 0.029(n_s L_s)^{0.8}/[(P_2)^{0.5}(s)^{0.4}]$
2. $V_p = 6.2(s_p)^{0.5}$
3. $T_{tp} = L_p/(3{,}600 V_p)$
4. $V_u = 4.92(\mathrm{s}_u)^{0.5}$
5. $T_{tu} = L_u/(3{,}600 V_u)$
6. $R = A/P$
7. $V_c = (1/n)[(AR^{2/3})(S)^{0.5}]$
8. $T_{tc} = L_c/(3{,}600 V_c)$
9. $T_c = T_{ts} + T_{tp} + T_{tu} + T_{tc}$

where $T_{ts}$, $T_{tp}$, $T_{tu}$, and $T_{tc}$ are the respective travel times (hours) of sheet flow, shallow concentrated paved flow, shallow concentrated unpaved flow, and channel flow; $L_s$, $L_p$, $L_u$, and $L_c$ are the respective lengths (m) of sheet flow, shallow concentrated paved flow, shallow concentrated unpaved flow, and channel flow; $V_p$, $V_u$, and $V_c$ are the respective velocities (m/s) of shallow concentrated paved flow, shallow concentrated unpaved flow, and channel flow; $s$, $s_p$, $s_u$, and $S$ are the respective slopes of sheet flow, shallow concentrated paved flow, shallow concentrated unpaved flow, and channel flow; $n_s$ and $n_c$ are the respective Manning's coefficients for overland flow and channel flow; $P_2$ is the 2-year, 24-h precipitation depth (cm); $R$, $A$, and $P$ are the respective hydraulic radius, cross-sectional area, and wetted perimeter of the open channel; and $T_c$ is the time of concentration. Eqs. 1, 2, and 4 were developed in English units by the NRCS but have been converted to international units. Typically, the Manning's $n$ for overland flow are usually much higher than the Manning's $n$ for channel flow.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-6. The spreadsheet is designed differently from other spreadsheets and follows the structure of the spreadsheet created by the NRCS with some modifications. The shallow concentrated flow is only over an unpaved area, which is why $T_{tp} = 0$. An "IF" statement is required when solving for Eqs. 3 and 5, because there is a division by zero if $V_p$ or $V_u$ are equal to zero.

### *Solution*

The time of concentration is 1.53 h as shown in Spreadsheet 4-6.

### *What-If Analysis*

A what-if analysis shows that

(a) The $T_c$ would increase to 2.02 h, a 32% increase, if the sheet flow occurred over dense underbrush ($n_s = 0.8$). Therefore, it is important to determine the surface description of sheet flow with care.
(b) The $T_c$ decreases to 1.40 h, only an 8% decrease, if $s_u$ increases 500% (from 0.01 to 0.05).

### *Assigned Problem 4-6-1*

Determine $T_c$ if the shallow concentrated flow occurred over a paved surface instead of the unpaved surface (all other parameters remain the same as in Problem 4-6).

# Problem 4-7: Estimation of Time of Concentration Using the Kinematic Wave Method

## *Problem Statement*

Determine the time of concentration, $T_c$, for a watershed in Zone 7 of Florida, during a 25-year storm using the kinematic wave method. The flow length is 30.5 m and occurs over a slope of 0.01. The Manning's $n$ for overland flow is 0.24 (all dimensions are identical to those used for sheet flow in Problem 4-6).

## *Background*

The kinematic wave method was developed primarily for overland or sheet flow such as flow over airports. There is an important difference between the NRCS and kinematic wave methods. The NRCS sheet flow equation is generally used for 24-h storm events, whereas the kinematic wave method assumes that the storm duration *is equal* to the time of concentration. Therefore, in general, the duration of the storm event is much smaller than 24 h in the kinematic wave method. Because precipitation intensity is indirectly proportional to the precipitation duration, the storm intensity determined by the kinematic wave method is usually higher and the predicted $T_c$ is smaller compared with the NRCS method.

## *Equations*

1. $T_d = T_c$
2. $i = 2.681(\log T_d)^2 - 22.251(\log T_d) + 40.75$
3. $T_c = 2.875(nL)^{0.6}/[(i)^{0.4}(s)^{0.3}]$

where $T_d$ is the duration of the storm event (minutes), $T_c$ is the time of concentration (minutes), $i$ is the rainfall intensity (cm/h), $n$ is the Manning's roughness coefficient for overland flow, $L$ is the flow length, and $s$ is the watershed slope. Eq. 2 is Eq. 1 of Problem 4-1. Eq. 3 was developed in English units but has been converted to international units.

## *Solution Procedure*

The solution procedure is iterative and is shown in Spreadsheet 4-7. An iterative solution is required (Spreadsheet 4-7) as there are more than one unknown in all three equations. The first value of $T_d = 30$ min is assumed. After that, the values of $T_d$ are calculated using Eq 1. The iterative procedure continues until $T_d = T_c$.

## *Solution*

$T_c = 11.34$ min (0.19 h) as shown in Spreadsheet 4-7. As expected, the $T_c$ value predicted by the kinematic wave method (0.19 h) is smaller than the travel time, $T_{ts}$, predicted by the NRCS method (0.3 h).

### *What-If Analysis*

A what-if analysis shows that

(a) The $T_c$ would increase to 26.72 min (0.45 h, a 136% increase) if the Manning's $n$ were 0.8 instead of 0.24.
(b) The $T_c$ decreases to 6.5 min, a 43% decrease, if $s_u$ increases 500% (from 0.01 to 0.05).

### *Assigned Problem 4-7-1*

Solve Problem 4-7 using the circulation cell feature of the spreadsheet.

### *Assigned Problem 4-7-2*

Create a spreadsheet to determine $T_c$ for a watershed in Zone 7 of Florida, during a 5-year storm using the kinematic wave method. All parameters are the same as in Problem 4-7.

### *Hint*

Use the solution of Assigned Problem 4-1-1.

# Problem 4-8: Estimation of Peak Flowrate Using the NRCS Graphical Method

### *Problem Statement*

Determine the peak flowrate, $q_p$, occurring at the outlet of a 100 ha watershed located in Type II NRCS zone due to a 15-cm precipitation event. The watershed curve number (CN) is 75, and the time of concentration $T_c$ is 1.53 h. Assume that the pond and swamp factor is one and that the fractional initial abstraction is 0.2. The regression coefficients for estimating unit peak discharge, $q_u$, are $C_0 = 2.553$, $C_1 = -0.615$, and $C_2 = -0.164$. These coefficients can be obtained from Table F-1 of SCS (1986).

### *Background*

The NRCS graphical method is used to estimate the peak discharge at the outlets of homogeneous rural and urban watersheds that can be described by a single CN and $T_c$. The method does not provide the entire hydrograph. The graphical method is a good method to verify the results obtained from other methods such as the Santa Barbara urban hydrograph (SBUH) method and the NRCS tabular method, which are used to determine the DRH. This method should be limited to $T_c$ values ranging from 0.1 to 10 h and $I_a/P$ values ranging from 0.1 to 0.5.

### *Assumption*

The fraction of $S$ that becomes initial abstraction, $f$, is assumed to be 0.2.

### *Equations*

1. $S = 1{,}000/\text{CN} - 10$
2. $I_a = fS$
3. $\text{Ratio} = I_a/P$
4. $X = \log(q_u) = C_o + C_1(\log T_c) + C_2(\log T_c)^2$
5. $q_u = 10^X$
6. $Q = (P - I_a)^2/(P - I_a + S)$ for $P \geq I_a$
7. $q_p = q_u A Q F_p$

where $S$ is the potential maximum retention after runoff begins in cm; $I_a$ is the initial abstraction; $f$ is the fraction of $S$ that becomes initial abstraction (usually assumed to be 0.2); ratio is simply the ratio of $I_a$ and $P$; $q_u$ is the unit peak discharge; $F_p$ is the pond and swamp adjustment factor; $A$ is the watershed area; and $C_o$, $C_1$, and $C_2$ are regression coefficients needed to estimate $\log q_u$. The symbol $(\text{Ratio})_{ro}$ shown in Spreadsheet 4-8 represents the rounded-off value of the ratio $I_a/P$.

## *Solution Procedure*

The solution procedure is described in Spreadsheet 4-8. The ratio $I_a/P$ is rounded off to the nearest $I_a/P$ value published in Table F1 of SCS (1986). In this case, that value is 0.1. Because the regression coefficients were only developed for English units, the input data for the drainage area and the rainfall depth, in international units, are converted to English units (or U.S. customary units) to determine $q_u$, and the final result is converted back to international units. All of the equations, with the exception of Eqs. 2 and 5, were developed for English units. The units of $q_u$ in the English system are csm/in. where csm stands for cubic ft per s per mi$^2$, i.e., $1\,\text{csm} = 1\,\text{ft}^3/(\text{s-mi}^2)$. Eq. 6 requires an "IF" statement as discussed in Problem 4-5.

## *Solution*

The peak flowrate was calculated to be $9.5\,\text{m}^3/\text{s}$.

## *What-If Analysis*

Conduct a what-if analysis to determine the effect of precipitation depth, $P$, on the peak flowrate $q_p$. Plot a graph of $P$ versus $q_p$. Also determine the effect of the curve number (CN) on the peak flowrate $q_p$. Plot a graph of CN versus $q_p$.

## *Assigned Problem 4-8-1*

Determine the peak flowrates at the outlet of the watershed of Problem 4-8 (all parameters remain the same) if the watershed was in Type I, Type IA, and Type III NRCS zones.

## *Hint*

The values of $C_0$, $C_1$, and $C_2$ will have to be read from Table F1 of SCS (1986).

# Problem 4-9: Estimation of Hydrograph Using the NRCS Tabular Method

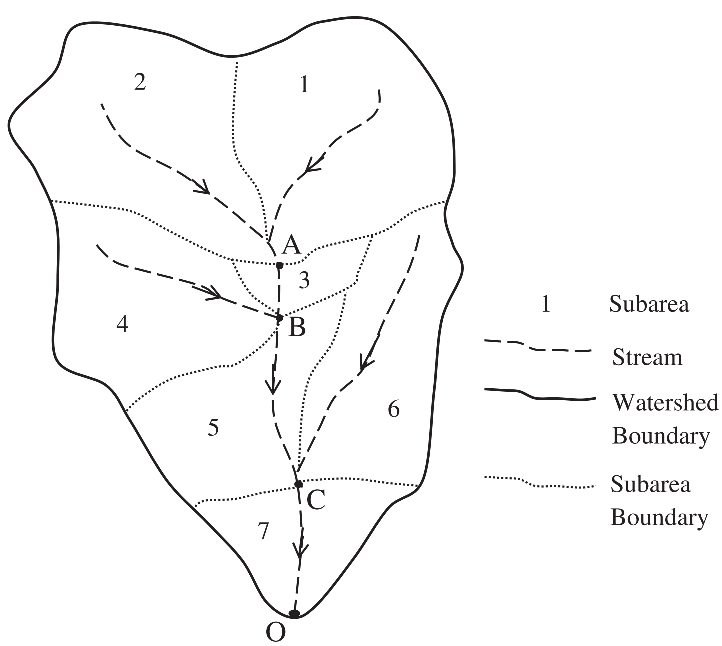
## *Problem Statement*

Determine the peak flowrate occurring at the outlet, *O*, of a 429 ha watershed, shown in Fig. 4-5, due to a 15.24 cm precipitation event. The watershed is located in Type II NRCS zone. The watershed has seven subareas (Fig. 4-5) and the various characteristics of these subareas (area, time of concentration, travel time, and curve number) are shown in Spreadsheet 4-9. The coefficients for estimating unit peak discharge, $q_u$, can be obtained from Tables A-2 to A-5 (Exhibit 5-2 of TR-55, SCS 1986) and are also provided in Spreadsheet 4-9.

## *Background*

The NRCS tabular method is an improvement over the graphical method for two reasons: (a) it can be used for nonhomogeneous watersheds with multiple subareas, and (b) it provides the entire hydrograph instead of just the peak flowrate. Each subarea should be homogeneous, i.e., it must be defined by one value of $t_c$ and CN, and should have no more than one main channel within the subarea. The watershed must have a single main channel, which is Channel AO for this watershed (Fig. 4-5). The time of concentration for each subarea, in this method, is defined as the time it takes for the water to traverse from the hydraulically farthest point to the *outlet of the subarea*. Thus, $T_c = 1.50$ h for Subarea 1 implies that the time it takes for the water to traverse from the

***Fig. 4-5*** **Watershed description for Problem 4-9 (from SCS 1986)**



hydraulically farthest point of Subarea 1 to $A$ is 1.5 h because $A$ is the outlet of Subarea 1. This method introduces the concept of travel time, $T_t$, which is the time that water takes to traverse the main channel. If the main channel passes through subareas, such as Subareas 3, 5, and 7 in this case, then one needs to know (or compute) the travel times $T_{ti}$ for each segment.

The respective times it takes for the water to travel through the main channel in Subareas 1, 5, and 7 are shown in Spreadsheet 4-9. The travel time of 0.5 h for Subarea 3 implies that the time it takes for the water to traverse segment AB of the main channel is 0.5 h. One must read the values of $q_u$ from Exhibits 5-1, 5-1A, 5-2, and 5-3 of SCS (1986), depending on the location of the watershed. In this problem, the watershed is located in Type II NRCS zone, and consequently the $q_u$ values were obtained from Exhibit 5-2. The $q_u$ values are based on values of $I_a/P$, $T_c$, and $T_t$, which is why these three parameters have to be determined prior to reading the values of $q_u$. As in Problem 4-8, the ratio $I_a/P$ is rounded off to the nearest $I_a/P$ values (0.1, 0.3, and 0.5) provided in Exhibit 5-1 through Exhibit 5-3 of SCS (1986).

### *Assumption*

The fraction of $S$ that becomes initial abstraction, $f$, is assumed to be 0.2.

### *Equations*

1. $T_{ts} = \sum T_{ti}$ for $i = 1$ to $n$
2. $S = 1{,}000/\mathrm{CN} - 10$
3. $I_a = 0.2S$
4. $\mathrm{Ratio} = I_a/P$
5. $Q = (P - I_a)^2/(P - I_a + S)$ for $P \geq I_a$
6. $q_p = (q_u A Q F_p)/(35.29)$

where $T_{ti}$ is the travel time in the main channel through Subarea $i$, and $T_{ts}$ is the cumulative travel time through the main channel between the outlet of the subarea to the watershed outlet. All other symbols have the same definitions as in Problem 4-8.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-9. Owing to space constraints in the spreadsheet, the solution only shows the hydrograph between 12.7 and 15.5 h. This time span was selected because peak flowrates usually occur during this period in NRCS precipitation distributions. As in Problem 4-8, the values of $A$ and $P$ have to be converted to English units because $q_u$ values given in Exhibits 5-1 through 5-3 of SCS (1986) are only provided in English units. Values of the peak flowrate, $q_p$, are converted to SI units using Eq 6.

*Solution*

The flowrate at the outlet changes from 17.9 $m^3/s$ to 11.7 $m^3/s$ in the period between 12.7 and 15.5 h with a peak flowrate of 24.8 $m^3/s$ occurring at 13.6 h.

*Assigned Problem 4-9-1*

Determine the hydrograph (for the period between 12.7 and 15.5 hr) and the peak flowrate at the outlet of Subarea 1 (point A in Fig. 4-5) due to the precipitation on Subarea 1 *alone*, for the conditions given in Problem 4-9.

*Hint*

Only Subarea 1 is contributing outflow at A.

*Assigned Problem 4-9-2*

Determine the hydrograph (for the period between 12.7 and 15.5 h) and the peak flowrate at the outlet of Subareas 1 and 2 (point A in Fig. 4-5) due to the precipitation on Subareas 1 and 2, for the conditions given in Problem 4-9.

*Hint*

Both Subareas 1 and 2 are contributing outflow at A.

*Assigned Problem 4-9-3*

Determine the hydrograph (for the period between 12.7 and 15.5 h) and the peak flowrate at the outlet of Subarea 3 (point B of Fig. 4-5) for the conditions given in Problem 4-9.

*Assigned Problem 4-9-4*

Determine the peak flowrate $q_p$ at O (Fig. 4-5) if Subarea 4 of Problem 4-9 is developed and, as a result, the CN of Subarea 4 increases to 90 while its $T_c$ reduces to 0.5 hr. All other parameters remain the same as in Problem 4-9.

*Assigned Problem 4-9-5*

Determine the peak flowrate $q_p$ at O (Fig. 4-5) if the main channel, through Subarea 5 of Problem 4-9, is straightened and, as a result, the $T_t$ through Subarea 5 reduces from 1 h to 0.5 h. All other parameters remain the same as in Problem 4-9.

# Problem 4-10: Estimation of Hydrograph Using the SBUH Method for a Homogeneous Watershed

## *Problem Statement*

Determine the DRH occurring at the outlet of a 100 ha watershed located in Type II NRCS zone owing to a 15-cm precipitation event. The $P/P_{24}$ values at various times are obtained from Problem 4-3 (Lincoln, Nebraska) and are shown in Spreadsheet 4-10. The watershed curve number (CN) is 75, and the time of concentration, $T_c$, is 1.53 h. Use $\Delta t = 2\,\text{h}$. There is no directly connected impervious area.

## *Background*

This problem is identical to Problem 4-8. The SBUH method computes hydrographs by first computing an instantaneous hydrograph and then routing the instantaneous hydrograph through the watershed with the assumption that the watershed behaves as an imaginary linear reservoir with a routing constant equal to the watershed time of concentration. The instantaneous unit hydrograph is computed with the assumption that all runoff, generated during a relatively small time period, $\Delta t$, is removed from the watershed during $\Delta t$. If $\Delta t$ is really small, then the runoff generated must be removed "instantaneously." Clearly, the accuracy of the method should increase if the selected $\Delta t$ is relatively small. Any impervious area within the watershed is considered to be connected, or directly connected, if the runoff from that area flows directly into the drainage system. It is also considered connected if runoff from this area runs over a pervious area as shallow concentrated flow. However, it is considered to be unconnected if runoff from this area runs over a pervious area as sheet flow. Sheet flow and shallow concentrated flows are described in Problem 4-6.

In general, it is assumed that the SBUH method is more accurate than the graphical method, which is considered to be the more conservative of the two methods, i.e., the graphical method predicts slightly higher values of peak flow than the SBUH method. The advantage of using the SBUH method over the graphical method is that it provides a complete hydrograph instead of just peak flowrate, which makes it possible to use the SBUH method for estimating flowrates from nonhomogeneous watersheds with multiple subareas. The advantage of using the SBUH method over the tabular method is that it is simply easier to use.

## *Assumption*

The fraction of $S$ that becomes initial abstraction, $f$, is assumed to be 0.2.

## *Equations*

1. $S = 2{,}540/\text{CN} - 25.4$
2. $I_a = fS$

3. $P = (P/P_{24})(P_{24})$
4. $(\Delta P)_{t+\Delta t} = P_{t+\Delta t} - P_t$
5. $Q = (P - I_a)^2/(P - I_a + S)$ for $P \geq I_a$
6. $I_{t+\Delta t} = (Q_{t+\Delta t} - Q_t)(A) \times 10{,}000/[(\Delta t)(3{,}600)]$
7. $(Q_r)_{t+\Delta t} = (Q_r)_t + [\Delta t/(\Delta t + 2T_c)](I_{t+\Delta t} + I_t - 2Q_t]$

where $Q$ is the cumulative runoff depth, $I$ is the instantaneous flow, $Q_r$ is the routed flow or the actual flowrate leaving the outlet of the watershed, and subscripts $t$ and $t + \Delta t$ are used to define values before and after the period $\Delta t$.

### *Solution Procedure*

The solution is shown in Spreadsheet 4-10.

### *Solution*

The computed peak flowrate is 7.98 $m^3/s$ and occurs at 12.5 h. As expected, the predicted peak flowrate is slightly smaller than the peak flowrate predicted by the graphical method (9.5 $m^3/s$).

### *What-If Analysis*

Conduct a what-if analysis to determine the effect of

(a) $f$ on the peak flowrate by plotting a graph of $q_p$ versus $f$. The range of $f$ should be from 0.05 to 0.5. Do you expect concrete to have the same $f$ as sandy soil? Is the trend shown by the graph similar to what you expected? Explain briefly.
(b) DCIA on the peak flowrate by plotting a graph of $q_p$ versus DCIA. The range of DCIA should be from 0 to 100%. Is the trend shown by the graph similar to what you expected? Explain briefly.
(c) $T_c$ on the peak flowrate by plotting a graph of $q_p$ versus $T_c$. The range of $f$ should be from 0.05 to 0.5. Do you expect concrete to have the same $f$ as sandy soil? Is the trend shown by the graph similar to what you expected? Explain briefly.

### *Assigned Problem 4-10-1*

Use the SBUH method to determine the hydrograph from the watershed described in Problem 4-9.

### *Hint*

Unlike the tabular method, there is no travel time concept in the SBUH method. Therefore, to make the two methods equivalent, the time of concentration used in the SBUH method should be the *sum* of the time of concentration and the cumulative travel time of the tabular method. For example, the time of concentration for Subarea 1 in the SBUH method should be $1.5 + 2.0 = 3.5$ h.

# Problem 4-11: Estimation of Peak Flowrate Using the Rational Method for a Homogeneous Watershed

## *Problem Statement*

An open channel running from Point 1 to 3 connects the outlets of four subareas as shown in Fig. 4-6. Points 1 and 2 are the respective outlets of Subarea 1 and 2, whereas Point 3 is the outlet of Subareas 3 and 4 and also the entire watershed. Under design conditions, water travels from Point 1 to 2 in 3 min and from 2 to 3 in 2 min. All four subareas are in Zone 7 of Florida. Subarea 1 is a 1.2 ha suburban residential watershed ($C_1 = 0.4$), Subarea 2 is a commercial area ($A_2 = 1.5$ ha, $C_2 = 0.8$), Subarea 3 is an undeveloped area ($A_3 = 1.5$ ha, $C_3 = 0.2$), and Subarea 4 is an industrial park ($A_4 = 1.3$ ha, $C_2 = 0.7$). The times of concentrations of the four subareas, $T_{c1}$, $T_{c2}$, $T_{c3}$, and $T_{c4}$, are 12, 8, 20, and 7 min, respectively. Determine the peak flowrate at the outlet of Subarea 1 (Point 1 of Fig. 4-6) for a 25-year storm event using the rational method.

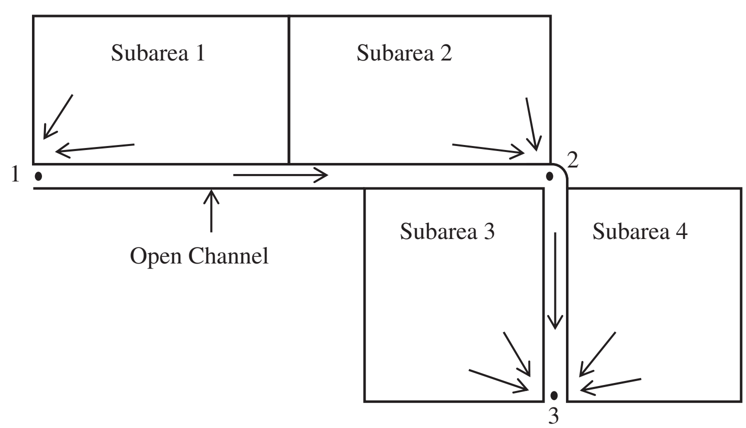
## *Background*

The rational method was first introduced in the United States in 1889 and is still widely used because of its simplicity. Similar to the graphical method, the rational method computes just the peak flowrate at the watershed outlet and not the entire hydrograph. The rational method is based on a very simple equation:

$$Q = CiA \tag{4-3}$$

where $Q$ is the peak flowrate occurring at the outlet of the watershed, $C$ is the runoff coefficient, $i$ is the rainfall intensity and $A$ is the watershed area. The rational method equation, Eq. (4-3), was initially developed for the English system where $Q$, $i$, and $A$ had units of $\mathrm{ft}^3/\mathrm{s}$, in./h, and acres, respectively.

***Fig. 4-6*** **Watershed description of Problem 4-11**



Because 1 cfs is approximately equal to 1 acre-in./h (1 acre-in./h = 1.008 cfs), the runoff coefficient $C$ was considered to be dimensionless.

The rational method runoff coefficient, $C$, is a function of the average abstraction rate of the watershed, which in turn is dependent on the watershed land use and soil type. Similar to the kinetic wave method, the rational method typically assumes that the duration of the storm event, $D$, must be equal to the time of concentration ($T_c$) of the watershed. Under these conditions, the entire watershed area will contribute runoff to the outlet. However, some regulatory agencies may require the design engineer to determine peak flowrates for storm durations less than $T_c$ and to select the highest estimated peak flowrate for design purposes (see Assigned Problem 4-11-1). When the storm duration is less than $T_c$, the entire watershed does not contribute runoff to the outlet, and the engineer needs to have a mechanism, such as time-area curves, to determine the percentage of area contributing runoff to the watershed (see Assigned Problem 4-11-1). In these cases, the watershed area contributing runoff to the watershed outlet will be less than the total watershed area, and the rational method equation can be modified to

$$Q = CiA_c \tag{4-4}$$

where $A_c$ is the contributing watershed area. Eq. 4-4 has been converted to a metric equivalent for the purposes of this book, and the metric equation takes the form of

$$Q = 0.028CiA_c \tag{4-5}$$

where $Q$ is in m$^3$/s, $i$ is in cm/h, and $A$ is in ha. The number 0.028 ensures that the runoff coefficients, determined for the English system, also can be used in Eq. (4-5). The rainfall intensity of the storm event is calculated by the trendline equation obtained found in Problem 4-1.

### *Assumption*

The storm duration is equal to the time of concentration of the watershed.

### *Equations:*

1. $D = T_c$
2. $i = 2.681(\log D)^2 - 22.251(\log D) + 40.75$
3. $Q = 0.028CiA_c$

where $i$ is the rainfall intensity (cm/h), $D$ is the duration of the storm event (h), $Q$ is the peak flowrate (m$^3$/s) at the outlet of the watershed, $C$ is the runoff coefficient, and $A_c$ is the area (ha) contributing to the watershed after time $D$.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-11.

### *Solution*

The computed peak flowrate is $0.267\,\mathrm{m^3/s}$.

### *What-If Analysis*

Conduct a what-if analysis to determine

(a) The effect of $C$ on $Q$ by plotting a graph of $Q$ versus $C$. The range of $C$ is from 0.2 to 1.0.
(b) The effect of $T_c$ on $Q$ by plotting a graph of $Q$ versus $T_c$. The range of $T_c$ is from 0.25 h to 2 h.

### *Assigned Problem 4-11-1*

Develop spreadsheets to determine the peak flowrate at the outlet of Subarea 1 of a 1.2 ha suburban, residential watershed ($C = 0.4$), in Zone 7 of Florida, for a 25-year storm event using the rational method for storm events of the following duration: (a) 5 min, (b) 10 min. The time–area curves indicate that the percentages of watershed area contributing to the watershed after 5 and 10 min are 20% and 70%, respectively.

# Problem 4-12: Estimation of Peak Flowrate Using the Rational Method for a Nonhomogeneous Watershed

## *Problem Statement*

An open channel running from Point 1 to 3 connects the outlets of four subareas as shown in Fig. 4-6. Points 1 and 2 are the respective outlets of Subarea 1 and 2, whereas Point 3 is the outlet of Subareas 3 and 4. Under design conditions, water travels from Point 1 to 2 in 3 min and from 2 to 3 in 2 min. All four subareas are in Zone 7 of Florida. Subarea 1 is a 1.2 ha suburban residential watershed ($C_1 = 0.4$), Subarea 2 is a commercial area ($A_2 = 1.5$ ha, $C_2 = 0.8$), Subarea 3 is an undeveloped area ($A_3 = 1.5$ ha, $C_3 = 0.2$), and Subarea 4 is an industrial park ($A_4 = 1.3$ ha, $C_2 = 0.7$). The times of concentration of the four subareas, $T_{c1}$, $T_{c2}$, $T_{c3}$, and $T_{c4}$, are 12, 8, 20, and 7 min, respectively. Determine the peak flowrate at the outlet of Subarea 2 (Point 2 of Fig. 4-6) for a 25-year storm event using the rational method.

## *Background*

Watersheds, or subareas, are considered nonhomogeneous if they have different times of concentration or $C$ values. Because Subareas 1 and 2 are contributing to the outlet at 2, and because they have different times of concentration and $C$ values, the entire watershed consisting of both Subareas 1 and 2 should be described as a nonhomogeneous watershed. For nonhomogeneous watersheds, the rational method equation is modified to

$$Q = 0.028 C_w i \sum A_{ci} \qquad (4\text{-}6)$$

where $i$ is the number of subareas contributing runoff to the outlet, $A_{ci}$ is the contributing area of the $i$th subarea, and $C_w$ is the weighted runoff coefficient, which is defined as

$$C_w = \left[\sum (C_i A_{ci})\right] / \sum A_{ci} \qquad (4\text{-}7)$$

The time of concentration of a nonhomogeneous watershed is determined by finding the time it takes for the water to travel from the hydraulically farthest point of each subarea to the outlet and then selecting the largest of these values. In this problem, the time it takes for the water to travel from the hydraulically farthest point of Subarea 2 to the outlet is 8 min. Also, the time it takes for the water to travel from the hydraulically farthest point of Subarea 1 to Point 2 is 12 min plus 3 min or 15 min. Therefore, the time of concentration for the entire watershed will be 15 min.

## *Equations*

1. $\sum A_{ci} = A_1 + A_2$ for $i = 2$
2. $C_w = [\sum (C_i A_{ci})] / \sum A_{ci}$ for $i = 2$

3. $T_{t1} = T_{c1} + t_2$
4. $T_{t2} = T_{c2}$
5. $T_{\max}$ = greater of $T_{t1}$ and $T_{t2}$
6. $D = T_{\max}$
7. $i = 2.681(\log D)^2 - 22.251(\log D) + 40.75$
8. $Q = 0.028 C_w i A_c$

where $n$ = number of subareas contributing to the outlet, $A_c$ is the total area (ha) contributing to the outlet (ha), $T_{t1}$ and $T_{t2}$ are the respective times it takes from the hydraulically farthest points of Subareas 1 and 2 to get to the outlet, $T_{\max}$ is the time it takes for water to travel from the farthest hydraulic point to the outlet, and $Q$ is the peak flowrate at the outlet. Here, the outlet is defined as the location where the peak flowrate is desired.

### *Solution Procedure*

The solution is shown in Spreadsheet 4-12. It takes 15 and 8 min for the water to travel from the hydraulically farthest points of Subareas 1 and 2 to the outlet, respectively. Therefore, $T_{\max}$, and consequently $D$ is equal to 15 min, which is the greater of the two travel times. This is a fairly simple problem, because there are only two subareas, and therefore only two travel times have to be calculated. However, several travel times would have to be calculated in watersheds, with several subareas.

### *Solution*

The computed peak flowrate is $0.853\,\text{m}^3/\text{s}$.

### *What-If Analysis*

Conduct a what-if analysis to determine the effect of $T_{c1}$ on $Q$ by plotting a graph of $Q$ versus $T_{c1}$. The range of $T_{c1}$ is from 3 to 15 min.

### *Assigned Problem 4-12-1*

Determine the peak flowrate at Point 3 of the watershed shown in Fig. 4-6 during a 25-year design storm.

### *Assigned Problem 4-12-2*

Prove that Eqs. 2 and 8 can be combined to yield

$$Q = 0.028\left(\sum C_i A_i\right) i.$$

# Problem 4-13: Design of a Storm Sewer System Using the Rational Method

## *Problem Statement*

A watershed consisting of three subareas, in Zone 7 of Florida, is draining into a lake (Fig. 4-7). Points A, B, and C are the respective outlets of the Subareas 1 to 3, and Pipes AB, BC, and CD carry the runoff from the three subareas into the lake. Subarea 1 is a residential area ($A_1 = 0.3$ ha, $C_1 = 0.6$, $T_{c1} = 4$ min), Subarea 2 is a parking lot ($A_2 = 0.2$ ha, $C_2 = 0.8$, $T_{c2} = 5$ min), and Subarea 3 is a grassy area ($A_3 = 0.4$ ha, $C_3 = 0.1$, $T_{c3} = 9$ min). The Manning's $n$ value for the selected pipes is 0.015. The inlet elevations at A, B, C, and D are 10 m, 9.12 m, 8.63 m, and 8.12 m, respectively. The pipe lengths are given in Spreadsheet 4-13. Determine the diameters of the storm sewer pipes AB, BC, and CD for a 25-year design storm.

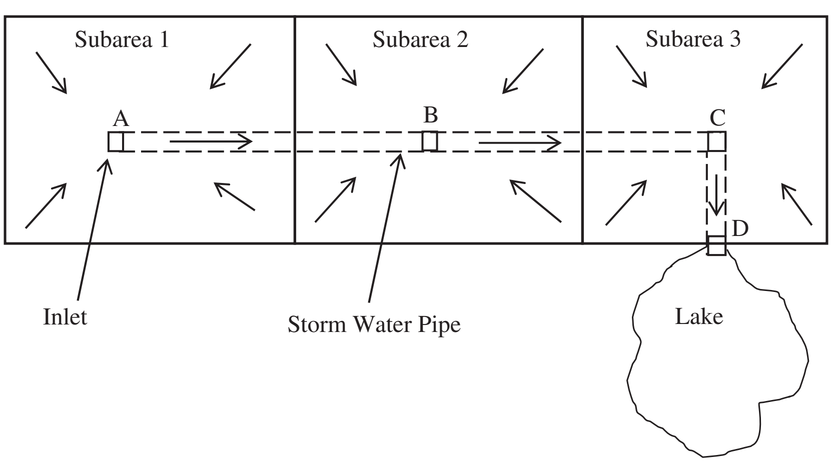
## *Background*

Eqs. 2 and 8 of Problem 4-12 can be combined to yield $Q = [\sum(C_i A_{ci})]i$. Using this equation is sometimes more convenient than using Eqs. 2 and 8. In the rational method, pipe diameters are found in sequence with the diameter of the most upstream pipe (AB) found first. There are two points worth noting: (1) Each pipe drains a different area; pipe AB drains Subarea 1, pipe BC drains Subareas 1 and 2, and pipe CD drains all three subareas; and (2) the pipe diameters can be determined by several different methods; however, the Manning's $n$ equation (Eq. 10) is used in this problem.

## *Assumptions*

(1) It is assumed that all pipes will be flowing full during storm conditions (Eq. 10 was derived from the Manning's $n$ equation, with the assumption that the pipe is flowing full). (2) The location of the EGL is not known initially; therefore, $S$ is

***Fig. 4-7* Watershed description of Problem 4-13**



calculated with the assumption that the water level, during storm conditions, will be at the top of the inlet. (3) It is assumed that pipe diameters are available in increments of 20 cm; therefore, the selected pipe size, $d_s$, is the next higher size available. Although not done in this problem, engineers should calculate the actual slope of the EGL, once the pipe diameter is calculated, and use that value instead of Eq. 1 to recompute $d$ and $d_s$.

### *Equations*

1. $S = (E_u - E_d)/L$
2. $A_c = \sum A_{cj}$
3. $T_{t1} = T_{c1} + \sum t_{tk}$
4. $T_{t2} = T_{c2} + \sum t_{tk}$
5. $T_{t3} = T_{c3} + \sum t_{tk}$
6. $T_{\max} = \text{largest } T_{ti}$
7. $D = T_{\max}$
8. $i = 2.681(\log D)^2 - 22.251(\log D) + 40.75$
9. $Q = 0.028(\sum C_i A_i)i$
10. $d = (3.21Qn/S^{0.5})^{3/8}$
11. $V = 4Q/\pi d_s^2$
12. $T_t = L/V$

where $S$ is the slope of the energy grade line (EGL); $E_u$ and $E_d$ are the upstream and downstream inlet elevations of a pipe; $L$ is the length of the pipe; $A_c$ is the total area (ha) contributing to the outlet; $j$ is the number of subareas draining into a pipe; $A_{cj}$ is the area of the $j$th subarea; $T_{c1}$, $T_{c2}$, and $T_{c3}$ are the times of concentrations of the three subareas (also known as inlet times); $t_{tk}$ is the travel time through the $k$th pipe, where $k$ is the number of pipes that water has to travel from a subarea to reach the outlet; $T_{t1}$, $T_{t2}$, and $T_{t3}$ are the respective times it takes from the hydraulically farthest points of Subareas 1, 2, and 3 to get to the outlet; $T_{\max}$ is the time it takes for water to travel from the farthest hydraulic point to the outlet; $D$ is the rainfall duration; $i$ is the rainfall intensity; $Q$ is the peak flowrate at the outlet; $d$ is the pipe diameter; $d_s$ is the selected pipe diameter; and $V$ is the flow velocity in the pipe. Once again, the outlet is defined as the location where the peak flowrate is desired.

### *Solution Procedure*

The solution is shown in Spreadsheet 4-13. Pipe AB is sized initially, and its diameter is found to be 0.375 m. Because the pipes are available in increments of 0.2 m, the selected pipe size is 0.4 m. The travel time through Pipe AB, $t_{t1}$, is 1.47 min. The travel times from the hydraulically farthest points of Subareas 1 and 2 to Point B were computed to be 5.47 and 5 min, respectively. The larger of the two times was selected to determine the diameter for Pipe BC. The travel time through Pipe BC was found to be 1.92 min. The entire process was repeated for Pipe CD.

*Solution*

The selected diameters for pipes AB, BC, and CD are 0.4, 0.6, and 0.8 m, respectively.

*What-If Analysis*

Conduct a what-if analysis to determine the effect of $L$ on $Q$ by plotting a graph of $Q$ versus $L$. The range of $L$ is from 30 m to 100 m.

*Assigned Problem 4-13-1*

Determine the size of pipe CD if the grassy area, Subarea 3, is developed into a parking lot ($C = 0.8$) and its inlet time decreases to 5 min from 9 min.

*Assigned Problem 4-13-2*

Assume that instead of an open channel, there are two pipes, Pipes 1-2 and 2-3, draining Subareas 1 and 2 of the watershed shown in Fig. 4-6. The inlet elevations of Points 1, 2, and 3 are 10 m, 9.75 m, and 9.25 m, respectively. Both pipes are 75 m long and Manning's $n$ of each pipe is 0.015. Determine the pipe sizes by the rational method if pipe diameters are available in 0.2 m increments.

# Problem 4-14: Estimation of TR-55 Storage Volume Graphs Using Trendlines

## *Problem Statement*

Using the $q_{\text{pre}}/q_{\text{post}}$ and $S_d/V_r$ data for Type I/IA and Type II/III SCS rainfall distributions given in Spreadsheet 4-14, determine third-order trendline equations by plotting the necessary graphs. Find the corresponding $R^2$ values and estimate the percentage errors as a result of using the trendline equations. The data were obtained from SCS (1986).

## *Background*

Retention–detention basins are designed to meet the requirement that the peak flowrate after development (post-development flowrate) must not exceed the flowrate that existed prior to development (predevelopment flowrate) under design conditions. In other words, the post-development peak flowrate discharge from the retention–detention basin must be attenuated to the predevelopment peak flowrate during the design storm event. This requirement is sometimes referred to as "pre versus post criteria." In addition, the basins should be large enough to store sufficient runoff to prevent flooding. The volume of a detention pond storage, $V_s$, necessary to provide the desired attenuation and storage can be determined by appropriate routing procedures. However, the NRCS (Fig. 6-1, SCS 1986) has provided a quick method for estimating the necessary volume of a detention pond by developing two graphs that represent Type I and IA, and Type II and Type III rainfall distributions respectively. As demonstrated in Problem 4-16, it is convenient to use these graphs if an appropriate trendline equation is available.

## *Equations*

1. $(S_d/V_r)_{\text{I,IA}} = -0.7323(Q_o/Q_i)^3 + 1.9470(Q_o/Q_i)^2 - 1.7456(Q_o/Q_i) + 0.6564$
2. $(E)_{\text{I,IA}} = (100)[(S_d/V_r)_{\text{I,IA}} - (S_d/V_r)_{tr}]_{\text{I,IA}}/(S_d/V_r)_{\text{I,IA}}$
3. $(S_d/V_r)_{\text{II,III}} = -0.7197(Q_o/Q_i)^3 + 1.5103(Q_o/Q_i)^2 - 1.3692(Q_o/Q_i) + 0.6732$
4. $(E)_{\text{II,III}} = (100)\{(S_d/V_r)_{\text{II,III}} - [(S_d/V_r)_{tr}]_{\text{I,IA}}\}/(S_d/V_r)_{\text{II,III}}1$

where $V_r$ is the runoff volume after development, $Q_i$ and $Q_o$ are the respective peak flowrates before and after development, $(S_d/V_r)_{tr}$ is the ratio predicted by the trendline, and $E$ is the percent error in the value estimated by the trendline. $Q_i$ and $Q_o$ are also referred to the pre- and post-development peak flowrates.

## *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-14. The data shown in the first two columns of Spreadsheet 4-14 were obtained from SCS (1986).

*Solution*

The trendline equations, obtained for rainfall distributions I/IA and II/III, are shown as Eqs. 1 and 3. The trendline equations and the corresponding $R^2$ values are also shown in Charts 1 and 2 of Spreadsheet 4-14. The errors in using the trendline equations are less than 2% for Type I and IA, and less than 1% for Type II and III distributions.

*Assigned Problem 4-14-1*

Determine the trendline equations using first-, second-, fourth-, and fifth-order polynomial equations and compare the errors obtained with the third-order polynomial equation.

# Problem 4-15: Estimation of Trendline Equations for Stage–Storage Data

## *Problem Statement*

Using the stage–storage data given in Spreadsheet 4-15, determine the fourth-order trendline equation by plotting the necessary graph. Find the corresponding $R^2$ values and estimate the percentage errors as a result of using the trendline equation.

## *Background*

Water levels in rivers, lakes, ponds, etc., are sometimes referred to as "stage." The term "storage" is used to define the volume of water in a water body such as a lake, pond, or a river reach. Engineers either need to determine or measure the relationship between storage and stage. Among other uses, the stage–storage relationship is used to design retention–detention ponds. The relationship is also used to estimate water elevation during routing. Although the stage–storage relationship is usually depicted in the form of a table (e.g., Spreadsheet 4-15) or a graph, it is convenient to convert the relationship to an equation using the trendline feature of Excel. Trendline equations are much more convenient in designing retention–detention ponds and to perform routing rather than reading the values of a graph or by interpolating the tabular values.

## *Equations*

1. $E = 6.0761S^4 - 10.0790S^3 + 2.2372S^2 + 3.7327S + 30.5$
2. $E = (S - S_{tr})/i \times 100\%$

where $E$ is the water level elevation or stage (m NGVD), $S_{tr}$ is the storage (ha-m) predicted by the trendline, and $E$ is the percent error introduced by using the trendline equation.

## *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-15.

## *Solution*

The fourth-order trendline equation, shown as Eq. 1, was obtained from Chart 1 of Spreadsheet 4-15. The trendline equation is very accurate, as the highest percentage error computed in Spreadsheet 4-15 is less than 0.02%.

## *Assigned Problem 4-15-1*

The retention–detention basin shown in Spreadsheet 4-15 was expanded to yield the stage–storage data shown in Spreadsheet 4-15a. Determine a fourth-order stage–storage trendline equation for the modified retention–detention basin, and compute the associated errors.

# Problem 4-16: Design of a Single Stage Retention–Detention Pond Using the TR-55 Method

## *Problem Statement*

Design a retention–detention pond with a rectangular weir for a 30 ha watershed located in the NRCS Type II zone. The required retention depth, $R_d$, is 1 cm. The retention–detention pond should be designed to retain the first centimeter of precipitation and to provide sufficient attenuation to meet the "pre- versus post-condition" during a 25-year, 24-h NRCS storm event with a precipitation depth ($P$) of 20 cm. The 25-year pre- and post-development flows, $q_i$ and $q_0$, are 5 and 10 $m^3/s$ respectively, and the post-development curve number, $CN_0$, is 61. The bottom of the pond is at 30.5 m NGVD. The trendline equation for the stage–storage relationship for the pond is given by Eq. 1 of Problem 4-15. Assume that the fraction initial abstraction, $f$, is 0.2 ($I_a = 0.2S$). The weir coefficient is 1.81.

## *Background*

Retention–detention basins equipped with single-stage structures are designed to attenuate the peak flowrate of a single storm event. The design of a retention–detention basin with multiple-stage structures is discussed in Problem 4-17. The specified retention depth is usually obtained from local stormwater management codes related to retention–detention pond design. Usually the first few centimeters of precipitation (the first "flush") is highly contaminated, which is why the codes usually specify the first 1 to 5 cm of precipitation to be retained. The pre- and post-development flowrates could be determined by any of the three methods described in Problems 4-8 to 4-10. The flow over a rectangular weir is given by

$$L_w = q_0/(CH_w^{1.5}) \tag{4-8}$$

where $L_w$ is the length of the weir in meters, $H_w$ is the water level over the weir crest during flood conditions, $q$ is the flowrate over the weir in $m^3/s$, and $C$ is the weir coefficient.

## *Equations*

1. $S_r = AR_d/100$
2. $E_w = 6.0761S_r^4 - 10.0790S_r^3 + 2.2372S_r^2 + 3.7327S_r + 30.5$
3. $S = 2{,}540/\mathrm{CN} - 25.4$
4. $I_a = 0.2S$
5. $Q = (P - I_a)^2/(P - I_a + S)$ for $P \geq I_a$
6. $V_r = (Q)(R_d)/100$
7. $S_d/V_r = -0.7197(q_o/q_i)^3 + 1.5103(q_o/q_i)^2 - 1.3692(q_o/q_i) + 0.6732$
8. $S_d = (S_d/V_r)(V_r)$
9. $S_{rd} = S_r + S_d$

10. $E_{\mathrm{m}} = 6.0761S_{rd}^4 - 10.0790S_{rd}^3 + 2.2372S_{rd}^2 + 3.7327S_{rd} + 30.5$
11. $H_w = E_m - E_w$
12. $L_w = q_0/(CH_w^{1.5})$

where $S_r$, $S_d$, and $S_{rd}$ are the retention, detention, and retention–detention storage provide in the pond; $V_r$ is the runoff volume after development; $q_i$ and $q_o$ are the peak flowrates before and after development; and $E_m$ and $E_w$ are the maximum stage and the weir stage.

### *Solution Procedure*

The calculations are shown in Spreadsheet 4-16a, and the results are summarized in Spreadsheet 4-16b.

### *Solution*

A 3.3 m rectangular weir will be placed at an elevation of 31.6 m NGWD. Because the pond bottom is at 30.5 m NGVD and the flood or peak stage is computed to be 32.5 m NGVD, there will be 2 m of water in the pond during the 25-year flood. The retention–detention pond needs to be enlarged if the 2 m pond depth is unacceptable. Enlargement of the pond will change the stage–storage relationship, and new trendline equations will have to be determined to replace Eqs. 2 and 10, which will no longer be applicable.

### *What-If Analysis*

Conduct a what-if analysis to determine the effect of the watershed area ($A$) on the flood stage ($E_m$) by plotting a graph of $E_m$ versus $A$. The range of $A$ is from 10 ha to 30 ha.

### *Assigned Problem 4-16-1*

Determine the detention volumes, peak elevation, weir elevation, and weir length if no retention was required.

### *Assigned Problem 4-16-2*

The engineer has decided to enlarge the retention–detention pond to provide 4 cm of retention depth (instead of 1 cm). The stage–storage data for the expanded basin are shown in Spreadsheeet 4-15a. Determine the retention and detention volumes, peak elevation, weir elevation, and weir length if the expanded retention–detention pond is used instead of the one described in Problem 4-16; all other parameters remain the same.

### *Hint*

Use the trendline equation determined for Assigned Problem 4-15-1 instead of Eqs. 2 and 10; the other equations will not change.

# Problem 4-17: Design of a Two-Stage Retention–Detention Basin Using the TR-55 Method

## *Problem Statement*

Design a two-stage retention–detention basin with a rectangular weir for a 30 ha watershed located in the NRCS Type II zone. The required retention depth, $R_d$, is 1 cm. The retention–detention pond should be designed to provide sufficient attenuation to meet the "pre- versus post-condition" during both 2-year and 25-year, 24-h NRCS storm events. The 2-year and 25-year precipitation depths ($P_1$ and $P_2$) are 13.25 and 20 cm, whereas the pre- and post-development flows are 1.4 and 2.6 $m^3/s$ and 5 and 10 $m^3/s$, respectively. The post-development curve number, $CN_0$, is 61. The bottom of the pond is at 30.5 m NGVD. The trendline equation for the stage–storage relationship for the pond is given by Eq. 1 of Problem 4-15. Assume that the fraction initial abstraction, $f$, is 0.2 ($I_a = 0.2S$). The weir coefficient is 1.81.

## *Background*

Retention–detention basins, equipped with single-stage structures designed for storm events with relatively large return periods, are unable to attenuate the higher-frequency storm events, i.e., storm events with relatively smaller return periods.

In other words, a weir designed to meet the pre- versus post-condition for a 25-year storm event will not be able to meet the pre- versus post-condition for a 2-year storm. The frequent passage of runoff due to a 2-year storm event, under post-development conditions, can create erosion problems in downstream channels. Therefore, some codes require retention–detention ponds to have multistage structures to attenuate runoffs from both high- and low-frequency events. In this problem, the discussion is confined to the design of two-stage rectangular weirs. *A* typical two-stage structure consisting of rectangular weirs is shown in Spreadsheet 4-17.

## *Equations*

1. $S_r = AR_d/100$
2. $E_{w1} = 6.0761S_r^4 - 10.0790S_r^3 + 2.2372S_r^2 + 3.7327S_r + 30.5$
3. $S = 2{,}540/\mathrm{CN} - 25.4$
4. $I_a = 0.2S$
5. $Q_1 = (P_1 - I_a)^2/(P_1 - I_a + S) \quad \text{for } P_1 \geq I_a$
6. $V_{r1} = (Q_1)(R_d)/100$
7. $S_{d1}/V_{r1} = -0.7197(q_{o1}/q_{i1})^3 + 1.5103(q_{o1}/q_{i1})^2 - 1.3692(q_{o1}/q_{i1}) + 0.6732$
8. $S_{d1} = (S_{d1}/V_{r1})(V_{r1})$
9. $S_{rd1} = S_r + S_{d1}$

10. $E_{m1} = 6.0761S_{rd1}^4 - 10.0790S_{rd1}^3 + 2.2372S_{rd1}^2 + 3.7327S_{rd1} + 30.5$
11. $E_{w2} = E_{m1}$
12. $H_{w1} = E_{m1} - E_{w1}$
13. $L_{w1} = q_{01}/(CH_{w1}^{1.5})$
14. $Q_2 = (P_2 - I_a)^2/(P_2 - I_a + S) \quad$ for $P_2 \geq I_a$
15. $V_{r2} = (Q_2)(R_d)/100$
16. $S_{d2}/V_{r2} = -0.7197(q_{o2}/q_{i2})^3 + 1.5103(q_{o2}/q_{i2})^2 - 1.3692(q_{o2}/q_{i2}) + 0.6732$
17. $S_{d2} = (S_{d2}/V_{r2})(V_{r2})$
18. $S_{rd2} = S_r + S_{d2}$
19. $E_{m2} = 6.0761S_{rd2}^4 - 10.0790S_{rd2}^3 + 2.2372S_{rd2}^2 + 3.7327S_{rd2} + 30.5$
20. $H'_{w1} = E_{m2} - E_{w1}$
21. $H_{w2} = E_{m2} - E_{w2}$
22. $q'_{o1} = CL_{w1}H_{w1}^{1.5}$
23. $q'_{o2} = q_{o2} - q'_{o1}$
24. $L_{w2} = q'_{02}/(CH'^{1.5}_{w2})$

where $S_r$ is the retention volume, $S_{d1}$ and $S_{d2}$ are the detention storages, $S_{rd1}$ and $S_{rd2}$ are the retention–detention storages, $E_{m1}$ and $E_{m2}$ are the maximum stages, $E_{w1}$ and $E_{w2}$ are the weir stages, and $V_{r1}$ and $V_{r2}$ are the runoff volumes after development at stages 1 and 2, respectively. Also, $q_{i1}$ and $q_{o1}$ are the peak pre- and post-development flowrates for the stage 1 flood, whereas $q_{i2}$ and $q_{o2}$ are the peak pre- and post-development flowrates for the stage 2 flood. $H_{w1}$ is the head above the stage 1 weir crest during the stage 1 flood, $H_{w2}$ is the head above the stage 2 weir crest during the stage 2 flood, $H'_{w1}$ is the head above the stage 1 weir crest during the stage 2 flood, and $L_{w1}$ and $L_{w2}$ are the weir lengths of the stage 1 and stage 2 weirs, respectively. Also, $q'_{01}$ and $q'_{02}$ are the flowrates over stage 1 and stage 2 weirs during the stage 2 flood, and $Q_1$ and $Q_2$ are the runoff depths during stage 1 and stage 2 floods, respectively.

### *Solution Procedure*

The calculations for stage 1 and 2 designs are respectively shown in Spreadsheet 4-17a and b, and the results are summarized in Spreadsheet 4-17c. The key to the solution procedure is to first size the stage 1 weir before designing the stage 2 weir, as one needs to calculate the flow that goes over the stage 1 weir ($q'_{01}$) when designing the stage 2 weir. The flowrate going over the stage 2 weir is the difference between peak flowrate during the stage 2 flood and $q'_{01}$ as shown in Eq. 23.

### *Solution*

The peak stage is 32.48 m NGVD, and the weir lengths for stages 1 and 2 are 1.86 m and 6.95 m.

### *Assigned Problem 4-17-1*

Determine the peak elevation if no retention was required. All other parameters are the same as in Problem 4-17.

### *Assigned Problem 4-17-2*

Design a three-stage retention–detention basin, with a rectangular weir, for a 30 ha watershed located in the NRCS Type II zone. The required retention depth, $R_d$, is 1 cm. The retention–detention pond should be designed to provide sufficient attenuation to meet the "pre- versus post-condition" during 2-year, 10-year, and 25-year, 24-h NRCS storm events. The 2-year, 10-year, and 25-year precipitation depths ($P_1, P_2$, and $P_2$) are 13.25 cm, 17 cm, and 20 cm, whereas the pre- and post-development flows are 1.4, 2.3, and 2.6 $m^3/s$, and 5, 8, and 10 $m^3/s$, respectively. The post-development curve number, $CN_0$, is 60. The bottom of the pond is at 30.5 m NGVD. The trendline equation for the stage–storage relationship for the pond is given by Eq. 1 of Problem 4-15. Assume that the fraction initial abstraction, $f$, is 0.2 ($I_a = 0.2S$). The weir coefficient is 1.81.

# Problem 4-18: Estimation of Minimum Required Treatment Volume for a Wet Detention Basin

## *Problem Statement*

The treated water from a wet basin is to be discharged into Class III waters. The minimum required treatment volume needed for water to be discharged into Class III waters should be calculated using the following codes: hold the greater of (a) 2.54 cm of rainfall on the entire site that does not provide treatment or (b) 6.35 cm of the rainfall that falls on the impervious area. The site, located in Melbourne, Florida, has an area ($A$) equal to 8 ha, and the site area is divided into the following land uses: area of parking and internal roads (asphalt) $A_{pr} = 50\%$, building area $A_b = 12\%$, area of lawns in good condition $A_l = 8\%$, wooded area $A_w = 15\%$, area of wetlands $A_{wl} = 5\%$, and area of the wet-detention pond $A_p = 10\%$. Determine the required treatment volume.

## *Background*

The Clean Water Act is the primary federal law in the United States governing water pollution. Passed in 1972, the objective of the Federal Water Pollution Control Act, commonly referred to as the Clean Water Act, is to restore and maintain the chemical, physical, and biological integrity of the nation's waters by preventing point and nonpoint pollution sources, providing assistance to publicly owned treatment works for the improvement of wastewater treatment, and maintaining the integrity of wetlands. The state of Florida in the United States has classified all state waters into six classes (www.dep.state.fl.us/water/wqssp/classes.htm) based on the designated uses of the waters. The lowest water classes require the most protection. Thus, Class 1 waters require more protection than Class 2 waters, and so on. Class 2 waters are defined as waters that are generally coastal waters where shellfish harvesting occurs. Class 3 waters are utilized for fish consumption, recreation, propagation, and maintenance of a healthy, well-balanced population of fish and wildlife.

The codes for estimating the required treatment volume for a wet detention pond are based on the realization that the first few centimeters of runoff will be "dirty" and once the dirty water, often referred to as the "first flush" carries all the accumulated pollutants from the site (parking lot, etc.), the remaining water discharged into the wet basin will be relatively "clean." The codes, therefore, specify that the runoff owing to the first few centimeters of rainfall be "held" within a wet detention pond, treated, and then slowly discharged into the receiving water body. Sites with large impervious areas are expected to provide more and dirtier runoff than relatively impervious sites; therefore, the codes usually require engineers to compute the treatment volume by two methods, as specified in the Problem Statement, and to choose the larger of the two treatment volumes. In general, precipitation falling on the wetlands and the pond itself need not be treated, as these land uses provide intrinsic treatment. Therefore, the combined wetland and wet pond area is removed when computing the site area.

### *Equations*

1. $A_c = (A_{wl} + A_p)(A)/100$
2. $A_d = A - A_c$
3. $A_I = (A_{\text{pr}} + A_b)(A)/100$
4. $\text{TV}_1 = (A_d)(d_1)/100$
5. $\text{TV}_2 = (A_I)(d_2)/100$

where $A_c$ is the combined wetland and pond area and runoff from this area does not need to be treated, $A_d$ is the area that provides no treatment and runoff from this area needs to be treated, and $\text{TV}_1$ and $\text{TV}_2$ are the treatment volumes that need to be treated based on the two criteria stated in the Problem Statement.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-18.

### *Solution*

The treatment volumes calculated based on the two requirements are 0.17 ha and 0.31 ha. The desired treatment volume is 0.31 ha, which is the greater of the two numbers. This site had a high percentage of impervious area, which is why the treatment volume calculated based on the impervious area was greater.

### *What-If Analysis*

Conduct a what-if analysis to determine the effect of the wetlands and parking/internal road areas on the required treatment volume by plotting a graph of required treatment volume versus wetland area. The range of wetland area should be from 0% to 20%. Adjust the parking and internal road area according to the selected wetland area such that the sum of the two areas is always 55%. All other parameters are the same as in Problem 4-18.

### *Assigned Problem 4-18-1*

The treated water from a wet basin is to be discharged into Class 2 waters. The minimum required treatment volume needed for water to be discharged into Class 2 waters is 50% more than required for discharging into Class 2 waters. The codes for estimating the Class 3 treatment are the same as stated in Problem 4-18. The site, located in Melbourne, Florida, has an area ($A$) equal to 15 ha, and the site area is divided into the following land uses: area of parking and internal roads (asphalt) $A_{pr} = 35\%$, building area $A_b = 35\%$, area of lawns in good condition $A_l = 5\%$, wooded area $A_w = 2\%$, area of wetlands $A_{wl} = 2\%$, and the area of the wet-detention pond $A_p = 21\%$. Determine the required treatment volume.

# Problem 4-19: Estimation of Minimum Required Permanent Volume for a Wet Detention Basin

## *Problem Statement*

Determine the minimum required permanent pool volume (PPV) for an 8 ha site located in Melbourne, Florida, if the residence time ($T_r$) is 21 days and the average daily precipitation depth ($P_w$) during the wet period is 0.5 cm. The site area is divided into the following land uses: area of parking and internal roads (asphalt) $A_{pr} = 50\%$, building area $A_b = 12\%$, area of lawns in good condition $A_l = 8\%$, wooded area $A_w = 15\%$, area of wetlands $A_{wl} = 5\%$, and the area of the wet-detention pond $A_p = 10\%$.

## *Background*

The storage capacity of the permanent pool volume (PPV) must be large enough to detain untreated runoff long enough so that the treatment processes described in Problem 4-18 can take place. Uptake by algae is probably the most important process for nutrient removal from the wet-detention pond; therefore, the average residence time of the water in the pond must be long enough to ensure algal growth. A residence time of three weeks is considered to be the minimum duration that ensures adequate opportunity for algal growth, although a longer residence time is preferred for better treatment. A three-week residence time implies that the PPV should be large enough to hold the runoff generated from three weeks of precipitation. Ideally, the PPV should be large enough to hold the runoff generated from three weeks of the most intense precipitation period. It should be noted that wetlands should not discharge into the wet basin.

The average wet-season rainfall may be available from local sources; otherwise, it will have to be calculated based on locally available precipitation climatological data. The required residence time can be smaller than three weeks if the runoff is pretreated prior to being discharged into the wet pond or if there is a littoral zone. The required residence time also should depend on the quality of the water of the downstream water body where the runoff from the pond is being discharged. The engineer may decide to increase residence time to provide improved treatment, as one should avoid discharging polluted water into a clean environment. For example, in Florida, the required minimum residence time for Class 3 waters is 21 days, whereas for Class 2 waters it is 31.5 days.

## *Equations*

1. $A_f = 1 - P_w/100$
2. $C_w = (\sum P_i C_i)/100$
3. $P_r = (P_w)(T_r)$
4. $R_r = (P_r)(C_w)$
5. $V_r = (R_r)(A_f)$
6. $\text{PPV} = V_r$

where $A_f$ is the percentage of area discharging into the set basin, $C_w$ is the weighted rational method runoff coefficient, $C$ is the rational method coefficient for any land use, $P_r$ is the total precipitation depth during the required residence time, and $R_r$ and $Vr$ are the respective runoff depth and runoff volume that are generated during the required residence time.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-19.

### *Solution*

The desired treatment volume is 0.55 ha-m.

### *What-If Analyses*

Conduct a what-if analysis to determine the effect of the wetlands and parking/internal road areas on the required permanent pool volume by plotting a graph of required permanent pool volume versus wetland area. The range of wetland area should be from 0% to 20%. Adjust the parking and internal road area according to the selected wetland area such that the sum of the two areas is always 55%. All other parameters are the same as in Problem 4-19.

### *Assigned Problem 4-19-1*

Determine the minimum required permanent pool volume (PPV) for an 8 ha site located in Melbourne, Florida, if the residence time ($T_r$) is 31.5 days and the average daily precipitation depth ($P_w$) during the wet period is 0.5 cm. The site has an area ($A$) equal to 15 ha, and the site area is divided into the following land uses: area of parking and internal roads (asphalt) $A_{pr} = 35\%$, building area $A_b = 35\%$, area of lawns in good condition $A_l = 5\%$, wooded area $A_w = 2\%$, area of wetlands $A_{wl} = 2\%$, and the area of the wet-detention pond $A_p = 21\%$.

# Problem 4-20: Sizing of a Rectangular Wet Detention Basin

## *Problem Statement*

Determine the dimensions of a rectangular wet-detention basin from the basin bottom to an elevation of 20.55 m at increments ($\Delta z$) of 0.15 m. The pond bottom (PB) elevation should be at 18 m NGVD. The wet basin should provide a minimum treatment volume (TV) of 0.31 ha-m, and a minimum permanent pool volume (PPV) of 0.55 ha-m. The season high water table (SHWT) elevation is 19.5 m above NGVD. The minimum pond length to pond width ratio at the control elevation must be two to one. The pond must have a littoral zone from the overflow elevation (OE) to a depth of 0.75 m below the CE. The minimum pond slope ($m_2$) should be 6 : 1 from the bottom of the littoral zone to the overflow elevation, 4 : 1 ($m_3$) above the overflow elevation, and 3 : 1 ($m_1$) below the littoral zone (all ratios are horizontal to vertical). The OE must be no more than 0.5 m above the CE. The maximum allowable pond depth is 3.75 m, and the mean depth should be between 0.6 m and 2.4 m. All parameters are described in Fig. 4-4.

## *Background*

As stated in the background section of Chapter 4, a littoral zone helps in removing nutrients from the wet basin through plant uptake. The slope $m_2$ shown in Fig. 4-4 must be within an assigned range based on the applicable code. A gentle slope is required at the littoral zone to allow plants to establish roots, and in this problem $m_2 = 6$. Similarly, the values of $m_1$ and $m_3$ (Fig. 4-4) are equal to 3 and 4, respectively. A minimum slope of 3 : 1 is usually required below the littoral zone based on soil stability considerations, whereas a gentler slope of 4 : 1 (or higher) is usually required above the overflow elevation for safety considerations. The difference between the OE and CE cannot be too large if a basin is designed with a littoral zone, because the plants in the littoral zone may get completely submerged with water and decay. In this problem, the difference between the OE and CE cannot be more than 0.5 m. The depth of the wet basin, below the CE, is also critical in providing proper treatment. A shallow basin can cause excessive algal bloom, whereas anaerobic conditions can develop in a very deep basin. Excessive algal blooms are not aesthetically pleasing, and anaerobic conditions can cause nutrients and metals to seep into the basin from bottom sediments. Therefore, the basin depth below the CE should not be greater than 4 m, and a mean depth (pond volume divided by the pond area at the CE) between 0.4 m and 2.4 m is also recommended (St. Johns River Water Management District, 1991). Some regulatory agencies may require a minimum length to width ratio of 2 : 1 at the CE to avoid "short circuiting," which is the term used when runoff enters the pond close to the outlet structure and does not stay in the basin long enough to be treated.

### *Equations*

1. $b_{z-1} = b_z - m_{z-1}(\Delta z)$ for $z < \text{CE}$
2. $b_{z+1} = b_z + m_{z+1}(\Delta z)$ for $z > \text{CE}$
3. $l_{z-1} = l_z - m_{z-1}(\Delta z)$ for $z > \text{CE}$
4. $l_{z+1} = l_z + m_{z+1}(\Delta z)$ for $z > \text{CE}$
5. $A_z = (b_z)(l_z)$
6. $(V_c)_z = (V_c)_{z-1} + [(A_{z-1} + A_z)/2](\Delta z)$
7. $E_s = E - E_{\text{CE}}$
8. $V_{cs} = (V_c)_z - (V_c)_{\text{CE}}$
9. $\text{PPV} = (V_c)_{\text{CE}}$
10. $\text{TV} = (V_c)_{\text{OE}} - (V_c)_{\text{CE}}$
11. $d_m = (V_c)_{\text{CE}}/(A)_{\text{CE}}$
12. $d = (E)_{\text{CE}} - (E)_{\text{PB}}$

where $l$ and b are the length and width of the basin, $\Delta z$ is elevation increment, m is the slope (horizontal to vertical), $A$ is the pond area, $(A)_{\text{CE}}$ is the pond area at the CE, $E$ is the water level in the pond, $E_{\text{CE}}$ is the control elevation, $E_{\text{PB}}$ is the elevation of the pond bottom, $E_{\text{s}}$ is the water level in the pond above the CE, $V_c$ is the cumulative storage, $(V_c)_{\text{CE}}$ is the cumulative storage at the CE, $V_{cs}$ is the cumulative storage above the CE, $(V_c)_{\text{OE}}$ is the cumulative storage above the overflow elevation, $d_m$ is the mean depth, and $d$ is the pond depth below the CE. The subscript $z$ is used to signify a given elevation, whereas the subscripts $z+1$ and $z-1$ signify an elevation $\Delta z$ above and below elevation $z$. All elevations are in NGVD.

### *Solution Procedure*

One possible solution is shown in Spreadsheet 4-20. The CE was set at the SHWT elevation, and the OE of the pond was set at 19.95 m NGVD. The OE could be set at a different elevation as long as the difference in elevation between the OE and CE did not exceed 0.5 m. The littoral zone is between 18.75 m and 19.50 m. The length and width of the wet basin were obtained by trial and error by assuming a basin width at the CE. A basin width of 65 m at the CE satisfied all required criteria, although other basin widths that satisfy all required criteria are also possible.

### *Solution*

The designed wet-detention basin is 130 m by 65 m at the CE and provides PPV of 1.05 ha-m and a TV of 0.40 ha-m. The wet-detention basin meets all the slope considerations. The basin depth of 1.50 m and the mean depth of 1.24 m meet the depth criteria. The provided PPV and CE are both higher than the required PPV and CE, and it may be possible to design a smaller basin. Sometimes engineers will design a larger pond than necessary, because the soil removed by digging the pond can be used as fill at the site.

*Assigned Problem 4-20-1*

Design a wet-detention basin with the exact same requirements as the basin in Problem 4-20 with the exception that the length to width ratio at the CE should be 3:1 (instead of 2:1).

*Assigned Problem 4-20-2*

Plot a stage–storage graph between $Z_s$ and $V_{cs}$ calculated in Problem 4-20 and determine an appropriate trendline equation, along with an $R^2$ value for the graph.

*Assigned Problem 4-20-3*

Design a wet-detention basin with the exact same requirements as the basin in Problem 4-20 with the exception that the pond bottom is at 18.15 m NGVD (instead of 18.00 m NGVD).

*Assigned Problem 4-20-4*

Determine the PPV and TV provided by the wet-detention pond if the length and width at the CE are 120 m and 60 m, respectively. Do these values exceed the required PPV and TV?

*Assigned Problem 4-20-5*

Design a wet-detention basin with the exact same requirements as the basin in Problem 4-20 with the exception that the OE is at 19.8 m NGVD instead of 19.95 m NGVD.

# Problem 4-21: Sizing a Circular Orifice for a Wet Detention Pond

## *Problem Statement*

Determine the diameter of a circular orifice required to discharge no more than half the treatment volume over a 48-h period ($t_d$) for the pond designed in Problem 4-20. The TV provided for the wet-detention pond is 0.40 ha-m. Use the stage–storage graph determined for Assigned Problem 4-20-2. The centerline of the orifice will be placed at the CE, which is 19.5 m NGVD. The minimum orifice diameter will be 5 cm.

## *Background*

The requirement to discharge no more than half the TV over a 48-h period is provided so that the TV is discharged slowly and stays in the wet detention pond for a sufficient time so that it can be treated. This type of requirement can be found from the local codes. Because the orifices are small, they are liable to become plugged by floating debris; therefore, a minimum orifice diameter is usually specified by codes. The orifice coefficient, $C_0 = 0.6$. The respective control elevation (CE) and the overflow elevation (OE) are at 19.5 m and 19.95 m above NGVD.

## *Assumption*

The initial water level is at the overflow elevation.

## *Equations*

1. $Q_0 = (100^2)(\mathrm{TV}/2)/(t_d)(3{,}600)$
2. $h_1 = \mathrm{OE} - \mathrm{CE}$
3. $h_2 = -(0.1105)(\mathrm{TV}/2)^2 + 1.1515(\mathrm{TV}/2) + 0.0017$
4. $h = (h_1 + h_2)/2$
5. $A_0 = Q_0/[(C_0)(2gh)^{0.5}]$
6. $d_0 = [(4A_0)/(\pi)]^{0.5}(100)$

where $Q_0$ is the average allowable flowrate through the orifice, $h_1$ is the initial water elevation above the CE prior to discharge, $h_2$ is the water level above the CE when half the treatment volume has been discharged, $h$ is the average water elevation above the CE during the 48-h period when the TV is being discharged, $A_0$ is the area of the orifice, and $d_o$ is the orifice diameter. The orifice coefficient used in Eq. 4 accounts for frictional losses that would occur as water passes through a relatively small orifice.

### *Solution Procedure*

The solution procedure is shown in Spreadsheet 4-21. Eq. 3 is a trendline equation that was obtained as a solution to Assigned Problem 4-20-2. Eq. 5 is a standard equation used to calculate flowrates through circular openings. The orifice coefficient used in Eq. 5 accounts for frictional losses that would occur as water passes through a relatively small orifice.

### *Solution*

The orifice diameter was found to be 9.76 cm, so the selected orifice diameter will be 10 cm.

### *Assigned Problem 4-21-1*

Determine the orifice diameter if half the TV can be discharged over 24 h instead of 48 h. All other parameters are the same as in Problem 4-21.

# Problem 4-22: Sizing a Rectangular Weir for a Wet Detention Pond

*Problem Statement*

Determine the length of a rectangular weir that will be needed for the pond designed in Problem 4-20 with an overflow elevation (OE) at 19.95 m NGVD. Assume that the maximum stage during flood conditions is 20.55 m NGVD. The predevelopment flowrate, $q_0$, is $0.8\,\mathrm{m^3/s}$. The weir coefficient, $C_w = 1.81$.

*Background*

The rectangular weir is placed at the overflow elevation (OE), and its main purpose is to control flooding in the pond by allowing water to discharge at a rate not higher than the predevelopment flowrate. The weir length is designed by making an assumption about the maximum stage that would occur during flood conditions. The validity of this assumption is checked via routing, and the weir length is adjusted if needed.

*Equations*

1. $E_w = \mathrm{OE}$
2. $H_w = E_{\max} - \mathrm{OE}$
3. $L_w = q_0/(CH_w^{1.5})$

where $E_w$ is the elevation at which the weir is placed, $H_w$ is the water elevation above the weir crest in m, $E_{\max}$ is the maximum stage desired under flood conditions, $L_w$ is the weir length. In Eq. 3, $L_w$ and $H_w$ must be in m, and $q_0$ must be in $\mathrm{m^3/s}$.

*Solution Procedure*

The solution procedure is shown in Spreadsheet 4-22.

*Solution*

The length of the rectangular weir was found to be 0.95 m.

# Problem 4-23: Development of Storage–Outflow Relationship for a Wet Detention Basin

## *Problem Statement*

Determine the $Q$ versus $2S/\Delta t + Q$ data for the wet detention pond designed in Problem 4-20. Plot these data and determine a trendline equation for the graph and the corresponding $R^2$ value. The stage–storage data are identical to that found in Spreadsheet 4-20. The control elevation (CE) and the overflow elevation (OE) are at 0.00 m and 0.45 m, respectively. The circular orifice diameter ($d_0$) and the rectangular weir length ($L_w$) are 0.10 m and 0.95 m, respectively. The routing period, $\Delta t = 0.25$ h, and the orifice coefficient and the weir coefficients are 0.6 and 1.81, respectively.

## *Background*

The purpose of finding a trendline equation between $Q$ versus $2S/\Delta t + Q$ was described in Problem 3-7. Routing is the final step in designing a stormwater management system. The purpose of routing is to (a) determine the outflow hydrograph, (b) determine the water levels in the basin under flood conditions, (c) determine the peak water level under flood conditions, (d) ensure that the peak water level under flood conditions is less than the assumed peak water level when designing the weir, and (e) ensure that the post-development flowrate discharged through the control structure does not exceed the predevelopment flowrate. Developing the trendline equation requires knowledge of stage–storage and stage–outflow relationships. The stage–storage data are obtained when the engineer sizes the wet basin as described in Problem 4-20. The stage–outflow relationships can be obtained after the weir and the orifice have been designed as demonstrated in this problem.

If, after routing, the actual maximum water level is found to be less than the assumed maximum stage of 1.05 m, then no changes may be required. However, if the actual maximum water level is above the assumed stage, the engineer may (a) consider redesigning the pond and making it larger, (b) consider increasing the weir size but still keeping the discharge over the weir less than the predevelopment flowrate, or (c) raise the ground elevation near the pond although this may require raising the elevation of the entire site.

## *Equations*

1. $Q_0 = (C_0)(\Pi d_0^2/4)[2g(Z - \mathrm{CE})]^{0.5}$
2. $Q_w = C_w L_w (Z - \mathrm{OE})^{1.5}$
3. $Q = Q_0 + Q_w$
4. $f(S,Q) = 2S/\Delta t + Q$

in which $Q_0$ is the flowrate through the orifice, $Q_w$ is the flowrate over the weir, $Q$ is the total flowrate being discharged from the outlet structure, $Z$ is stage, $f(S,Q)$ is defined in Eq. 4, and $S$ is the storage in the basin.

### *Solution Procedure*

The data between $Q$ versus $2S/\Delta t + Q$ are shown in Spreadsheet 4-23. The graph between $Q$ and $2S/\Delta t + Q$ is shown in Spreadsheet 4-23.

### *Solution*

A sixth-order trendline equation best fit the graph (Chart 1 of Spreadsheet 4-23); the equation is $Q = (1.1092)(10^{-7})[f(S,Q)]^6 - (7.7206)(10^{-6})[f(S,Q)]^5 + (1.9364)(10^{-4})[f(S,Q)]^4 - (2.0119)(10^{-3})[f(S,Q)]^3 + (8.5088)(10^{-3})[f(S,Q)]^2 - (9.2856)(10^{-3})[f(S,Q)] + (6.0346)(10^{-5})$. The corresponding $R^2 = 0.9999$. The order of the trendline equation and the number of decimal places in a trendline equation can affect the answer significantly, especially if higher-order trendline equations are used. Hence, the desired trendline equation is provided up to four decimal places. The number of decimal points should be increased based on the required accuracy.

### *Assigned Problem 4-23-1*

Solve Problem 4-23, assuming that the weir length is 0.75 m instead of 0.95 m.

# Problem 4-24: Estimation of Discharge from the Control Structure and Stage in the Wet Pond during Flood Conditions

## *Problem Statement*

Determine the discharge from the control structure and the water level elevations in the wet detention pond during flood conditions via routing for the pond designed in Problem 4-20. The inflow hydrograph (post-development flow) during flood conditions entering the wet-pond is shown in Spreadsheet 4-24. Use the $Q$ versus $2S/\Delta t + Q$ trendline equations for Problem 4-23 and the $Z$ versus $S$ ($V_s$) trendline equation found in Assigned Problem 4-20-2. The initial water level is at the control elevation (CE). The routing period, $\Delta t = 0.25$ h.

## *Background*

This is simply a routing problem discussed in Problem 3-7. The main difference is that whereas empirical equations were given in Problem 3-7, the empirical equations used in this example were developed by the design engineer.

## *Equations*

1. $I_s = I_t + I_{t-\Delta t}$
2. $f_2(S,Q)_t = I_s + f_1(S,Q)_{t-\Delta t}$
3. $Q = (1.1092)(10^{-7})[f(S,Q)]^6 - (7.7206)(10^{-6})[f(S,Q)]^5 + (1.9364)(10^{-4})$ $[f(S,Q)]^4 - (2.0119)(10^{-3})[f(S,Q)]^3 + (8.5088)(10^{-3})[f(S,Q)]^2 - (9.2856)$ $(10^{-3})[f(S,Q)] + (6.0346)(10^{-5})$
4. $Q' = Q$ (if $Q$ is positive)

   $Q' = 0$ (if $Q$ is negative)
5. $f_1(S,Q)_t = f_2(S,Q)_{t-\Delta t} - 2Q$
6. $S_t = [f_2(S,Q)_t - Q](3{,}600\Delta t)/2$
7. $(V_{cs})_t = S_t/10{,}000$
8. $Z_s = -0.1105 S_t'^2 + 1.1515 S_t' + 0.0017$

in which subscripts $t$ and $t$–$\Delta t$ respectively denote the values at the end and the beginning of the routing period, $Q$ is the flowrate calculated by the trendline equation (Eq. 3), $Q'$ is the adjusted flowrate, $f_1(S,Q) = 2S/\Delta t - Q$, $f_2(S,Q) = 2S/\Delta t + Q$, $S$ is the storage in the basin in m$^3$, $S'$ is the storage in the basin in ha-m, and $Z_s$ is the water level.

*Solution Procedure*

The solution procedure is identical to the one described for Problem 3-7. It is important to note that the equations for row 1 are different from those used in row 2 and subsequent rows. For row 1, the starting elevation is given as the control elevation. Knowing this, one can obtain from Spreadsheet 4-21 that the stage, $Z_s$, and the cumulative storage, $S'$, are also zero. Furthermore, one determines that the initial $Q$ is also zero from Spreadsheet 4-21. For row 1, the functions $f_1(S,Q)$ and $f_2(S,Q)$ are determined by the following equations:

1. $f_1(S,Q)_{t=0} = 2[(S)_{t=0})]/\Delta t - (Q')_{t=0}$
2. $f_2(S,Q)_{t=0} = 2[(S)_{t=0}]/\Delta t - (Q')_{t=0}$

The trendline equation, Eq. 3, sometimes computes negative values of $Q$ (when $Q$ is small) if the curve fitting using the trendline equation is not completely accurate. In these cases, it becomes necessary to use Eq. 4 and adjust the negative value of $Q$ to zero, as negative values are not possible.

*Solution*

The maximum flowrate from the wet pond is 0.045 $m^3/s$ and occurs after 3.75 h. The peak elevation in the pond is 0.56 m above the CE.

# *Appendix*

***Table A-1.*** **Description of Unknowns and Equations Needed to Solve Problems 1-8 to 1-11**

| *Prob. No* | *Assumptions/ Specifications* | *Unknowns* | *Equation Name* | *Equation* | *Modified Equation* | *Solution Type* |
|---|---|---|---|---|---|---|
| 1-8 | No Minor losses | $h_f, f, V$ or $Q$ | Energy | $Z_1 - Z_2 = h_f$ | no modification | Direct |
| | Fully Rough Turbulent Flow | | Colebrook-White | $1/f^{0.5} = 1.14 + 2\log_{10}(d/k_s)$ | $f = 1/[1.14 + 2\log_{10}(d/k_s)]^2$ | |
| | Use Darcy-Weisbach Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | $V = [2gdh_f/(fL)]^{0.5}$ | |
| | Use Colebrook-White Method | | | | | |
| 1-9 | No Minor losses | $h_f, f, R_e, V$ or $Q$ | Energy | $Z_1 - Z_2 = h_f$ | no modification | Trial and Error |
| | Use Darcy-Weisbach Method | | Swamee and Jain | $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/\mathrm{R}_e^{0.9}]\}^2$ | no modification | |
| | Use Swamee and Jain Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | $V = [2gdh_f/(fL)]^{0.5}$ | |
| | | | Reynolds | $\mathrm{R}_e = Vd\rho/\mu$ | no modification | |
| 1-10 | No Minor losses | $h_f, R, S, Q$ | Energy | $Z_1 - Z_2 = h_f$ | no modification | Direct |
| | Use Hazen-Williams Method | | Hazen-Williams | $Q = 0.85C_H R^{0.63} S^{0.54} A$ | no modification | |
| | | | Hydraulic Radius | $R = d/4$ | no modification | |
| | | | Slope of EGL | $S = h_f/L$ | no modification | |
| 1-11 | Include Minor losses | $h_f, h_{mi}, R_e, f, V$ or $Q$ | Energy | $Z_1 - Z_2 = h_f + h_{mi}$ | no modification | Trial and Error |
| | Use Darcy-Weisbach Method | | Swamee and Jain | $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/\mathrm{R}_e^{0.9}]\}^2$ | no modification | |
| | Use Swamee and Jain Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | no modification | |
| | | | Reynolds | $\mathrm{R}_e = Vd\rho/\mu$ | no modification | |
| | | | Minor Loss | $h_{mi} = K'[V^2/(2g)]$ | no modification | |

Note that in the minor loss equation of Problem 1-11, $K'$ is the sum of minor loss coefficients $K_e$ and $K_{ex}$.

**Table A-2. Description of Unknowns and Equations Needed to Solve Problem 1-12 to 1-14**

| Prob. No | Assumptions/ Specifications | Unknowns | Equation Name | Equation | Modified Equation | Solution Type |
|---|---|---|---|---|---|---|
| 1-12 | Include Minor losses | $h_f$, $h_{mi}$, $h_p$, $\mathrm{R}_e$, $f$, $P$ | Energy | $Z_1 - Z_2 = h_f + h_{mi} - h_p$ | no modification | Trial and Error |
| | Use Darcy-Weisbach Method | | Swamee and Jain | $f = 0.25/\{log_{10}[k_s/(3.7d) + 5.74/\mathrm{R}_e^{0.9}]\}^2$ | no modification | |
| | Use Swamee and Jain Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | $V = [2\,gdh_f/(fL)]^{0.5}$ | |
| | | | Reynolds | $\mathrm{R}_e = Vd\rho/\mu$ | no modification | |
| | | | Minor Loss | $h_{mi} = K'[V^2/(2g)]$ | no modification | |
| | | | Power | $P = \Upsilon h_p Q$ | no modification | |
| 1-13 | No Minor losses | $h_f$, $f$, $\mathrm{R}_e$, $d$ | Energy | $Z_1 - Z_2 = h_f$ | no modification | Trial and Error |
| | Use Darcy-Weisbach Method | | Swamee and Jain | $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/\mathrm{R}_e^{0.9}]\}^2$ | no modification | |
| | Use Swamee and Jain Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | $d = K(LQ^2/h_f)^{0.2}; K = [8f/(g\pi^2)]^{0.2}$ | |
| | | | Reynolds | $\mathrm{R}_e = Vd\rho/\mu$ | no modification | |
| 1-14 | Include Minor losses | $h_f$, $h_{mi}$, $\mathrm{R}_e$, $f$, $d$ | Energy | $Z_1 - Z_2 = h_f + h_{mi}$ | no modification | Trial and Error |
| | Use Darcy-Weisbach Method | | Swamee and Jain | $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/\mathrm{R}_e^{0.9}]\}^2$ | no modification | |
| | Use Swamee and Jain Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | no modification | |
| | | | Reynolds | $\mathrm{R}_e = Vd\rho/\mu$ | no modification | |
| | | | Minor Loss | $h_{mi} = K'[V^2/(2g)]$ | no modification | |

***Table A-3.*** **Description of Unknowns and Equations Needed to Solve Problem 1-15**

| *Prob. No* | *Assumptions/ Specifications* | *Unknowns* | *Equation Name* | *Equation* | *Modified Equation* | *Solution Type* |
|---|---|---|---|---|---|---|
| 1-15 | Include Minor losses | $h_f$, $h_{mi}$, $h_p$, $\mathrm{R}_e$, $f$, $V$ or $Q$ | Energy | $Z_1 - Z_2 = h_f + h_{mi} - h_p$ | no modification | Trial and Error |
| | Use Darcy-Weisbach Method | | Swamee and Jain | $f = 0.25/\{\log_{10}[k_s/(3.7d) + 5.74/\mathrm{R}_e^{0.9}]\}^2$ | no modification | |
| | Use Swamee and Jain Method | | Darcy-Weisbach | $h_f = fLV^2/(2gd)$ | no modification | |
| | | | Reynolds | $\mathrm{R}_e = Vd\rho/\mu$ | no modification | |
| | | | Minor Loss | $h_{mi} = K'[V^2/(2g)]$ | no modification | |
| | | | Pump Performance | $h_p = 20 - 4{,}713Q^2$ | no modification | |

Note that in the minor loss equation of Problem 1-15, $K'$ is the sum of minor loss coefficients $K_e$, $K_{ex}$, $K_b$, and $K_v$.

**_Table A-4_. Description of Unknowns and Equations Needed to Solve Problem 1-16**

| *Prob. No* | *Assumptions/Specifications* | *Unknowns* | *Equation Name* | *Equation* | *Modified Equation* | *Solution Type* |
|---|---|---|---|---|---|---|
| 1-16 | No Minor losses | $h_{f1}$, $h_{f2}$, $h_{f3}$ | Energy Eq. 1 | $Z_1 = E_j + h_{f1}$ | $h_{f1} = Z_1 - E_j$ | Trial and Error |
| | Use Darcy-Weisbach Method | $V_1$ or $Q_1$ | Energy Eq. 2 | $Z_2 = E_j + h_{f2}$ (if $Z_2 > E_j$) | $h_{f2} = Z_2 - E_j$ (if $Z_2 > E_j$) | |
| | $f$ = constant | $V_1$ or $Q_2$ | Energy Eq. 2 | $E_j = Z_2 + h_{f2}$ (if $Z_2 < E_j$) | $h_{f2} = E_j - Z_2$ (if $Z_2 < E_j$) | |
| | | $V_1$ or $Q_3$ | Energy Eq. 3 | $E_j = Z_3 + h_{f3}$ | $h_{f3} = E_j - Z_3$ | |
| | | $E_J$ | Darcy-Weisbach Eq. 1 | $h_{f1} = fL_1V_1^2/(2gd_1)$ | $V_1 = [2gd_1h_{f1}/(fL_1)]^{0.5}$ | |
| | | | Darcy-Weisbach Eq. 2 | $h_{f2} = fL_2V_2^2/(2gd_2)$ | $V_2 = [2gd_2h_{f2}/(fL_2)]^{0.5}$ | |
| | | $V_2$ or $Q_2$ | Darcy-Weisbach Eq. 3 | $h_{f3} = fL_3V_3^2/(2gd_3)$ | $V_3 = [2gd_3h_{f3}/(fL_3)]^{0.5}$ | |
| | | | Continuity Eq. 1 | $Q_1 + Q_2 = Q_3$ (if $Z_2 > E_j$) | Net$Q = Q_1 + Q_2 - Q_3 = 0$ (if $Z_2 > E_j$) | |
| | | | Continuity Eq. 1 | $Q_1 = Q_2 + Q_3$ (if $Z_2 < E_j$) | Net$Q = Q_1 - Q_2 - Q_3 = 0$ (if $Z_2 < E_j$) | |

***Table A-5.* Description of Unknowns and Equations Needed to Solve Problem 1-17**

| *Prob. No* | *Assumptions/Specifications* | *Unknowns* | *Equation Name* | *Equation* | *Modified Equation* | *Solution Type* |
|---|---|---|---|---|---|---|
| 1-17 | No Minor losses | $h_{f1}$, $h_{f2}$, $h_{f3}$, $h_{f4}$, $h_{f5}$ | Energy Eq. 1 | $E_A + h_p = E_{J1} + h_{f1}$ | $E_{J1} = E_A + h_p - h_{f1}$ | Trial and Error |
| | Use Darcy-Weisbach Method | $V_1$ or $Q_1$ | Energy Eq. 2 | $E_{J1} = E_{J2} + h_{f2}$ | $h_{f2} = E_{J1} - E_{J2}$ | |
| | | $V_1$ or $Q_2$ | Energy Eq. 3 | $E_{J1} = E_{J3} + h_{f3}$ | $h_{f3} = E_{J1} - E_{J2}$ | |
| | | $V_1$ or $Q_3$ | Energy Eq. 4 | $E_{J2} = E_B + h_{f4}$ | $h_{f4} = E_{J2} - E_B$ | |
| | | $V_1$ or $Q_4$ | Energy Eq. 5 | $E_{J2} = E_C + h_{f5}$ | $h_{f5} = E_{J2} - E_C$ | |
| | | $V_1$ or $Q_5$ | Darcy-Weisbach Eq. 1 | $h_{f1} = fL_1V_1^2/(2gd_1)$ | no modification | |
| | | $E_{J1}$, $E_{J2}$ | Darcy-Weisbach Eq. 2 | $h_{f2} = fL_2V_2^2/(2gd_2)$ | $V_2 = [2gd_2h_{f2}/(fL_2)]^{0.5}$ | |
| | | $h_p$ | Darcy-Weisbach Eq. 3 | $h_{f3} = fL_3V_3^2/(2gd_3)$ | $V_3 = [2gd_3h_{f3}/(fL_3)]^{0.5}$ | |
| | | | Darcy-Weisbach Eq. 4 | $h_{f4} = fL_4V_4^2/(2gd_4)$ | $V_4 = [2gd_4h_{f4}/(fL_4)]^{0.5}$ | |
| | | | Darcy-Weisbach Eq. 5 | $h_{f5} = fL_5V_5^2/(2gd_5)$ | $V_5 = [2gd_5h_{f5}/(fL_5)]^{0.5}$ | |
| | | | Continuity Eq. 1 | $Q_1 = Q_4 + Q_5$ | no modification | |
| | | | Continuity Eq. 2 | $Q_1 = Q_2 + Q_3$ | no modification | |
| | | | Pump Characteristic | $h_p = 60 - 10Q_1^2$ | no modification | |

Note that only **one** of the two continuity equations listed in Problem 1-16 can be used; an IF statement can also be used.

***Table A-6.*** **Description of Unknowns and Equations Needed to Solve Problem 1-18**

| *Prob. No* | *Assumptions/ Specifications* | *Unknowns* | *Equation Name* | *Equation* | *Modified Equation* | *Solution Type* |
|---|---|---|---|---|---|---|
| 1-18 | No Minor losses | $h_{f1}$, $h_{f2}$, $h_{f3}$, $h_{f4}$, $h_{f5}$ | Energy Eq. 1 | $E_A + h_p = E_{J1} + h_{f1}$ | $E_{J1} = E_A + h_p - h_{f1}$ | Trial and Error |
| | Use Darcy-Weisbach Method | $V_1$ or $Q_1$ | Energy Eq. 2 | $E_{J1} = E_{J2} + h_{f2}$ | $h_{f2} = E_{J1} - E_{J2}$ | |
| | $f$ = constant | $V_1$ or $Q_2$ | Energy Eq. 3 | $E_{J1} = E_{J3} + h_{f3}$ | $h_{f3} = E_{J1} - E_{J2}$ | |
| | | $V_1$ or $Q_3$ | Energy Eq. 4 | $E_{J2} = E_B + h_{f4}$ | no modification | |
| | | $V_1$ or $Q_5$ | Energy Eq. 5 | $E_{J2} = E_C + h_{f5}$ | $h_{f5} = E_{J2} - E_C$ | |
| | | $E_{J1}$, $E_{J2}$ | Darcy-Weisbach Eq. 1 | $h_{f1} = fL_1V_1^2/(2gd_1)$ | no modification | |
| | | $h_p$ | Darcy-Weisbach Eq. 2 | $h_{f2} = fL_2V_2^2/(2gd_2)$ | $V_2 = [2gd_2h_{f2}/(fL_2)]^{0.5}$ | |
| | | $d_4$ | Darcy-Weisbach Eq. 3 | $h_{f3} = fL_3V_3^2/(2gd_3)$ | $V_3 = [2gd_3h_{f3}/(fL_3)]^{0.5}$ | |
| | | | Darcy-Weisbach Eq. 4 | $h_{f4} = fL_4V_4^2/(2gd_4)$ | no modification | |
| | | | Darcy-Weisbach Eq. 5 | $h_{f5} = fL_5V_5^2/(2gd_5)$ | $V_5 = [2gd_5h_{f5}/(fL_5)]^{0.5}$ | |
| | | | Continuity Eq. 1 | $Q_1 = Q_4 + Q_5$ | no modification | |
| | | | Continuity Eq. 2 | $Q_1 = Q_2 + Q_3$ | no modification | |
| | | | Pump Characteristic | $h_p = 60 - 10Q_1^2$ | no modification | |

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Dr. Pandit has more than 100 technical publications, including journal papers, proceedings papers, technical reports, and book chapters. As a ground water and surface water modeler, he has developed two finite-element ground water models, GROSEEP (Groundwater Seepage) and SOLTRA (Solute Transport), and two non-point source models, CALSIM (Continuous Annual Load Simulation Model) and WEANES (Wet Pond Annual Efficiency Simulation). These models have been used in several research and consulting projects.

Dr. Pandit is a registered professional engineer. He has consulted in more than 30 projects related to the modeling of stormwater quality and quantity; the evaluation of structural and nonstructural stormwater best management practices; the evaluation and design of stormwater drainage systems; and streambed sediment loadings.