



# *A Simple Accounting-based Valuation Model for the Debt Tax Shield*

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## *Abstract*

*This paper describes a simple way to integrate the debt tax shield into an accounting-based valuation model. The market value of equity is determined by forecasting residual operating income, which is calculated by charging operating income for the operating assets at a required return that accounts for the tax benefit that comes from borrowing to raise cash for the operations. The model assumes that the firm maintains a deterministic financial leverage ratio, which tends to converge quickly to typical steady-state levels over time. From a practical point of view, this characteristic is of particular help, because it allows a continuing value calculation at the end of a short forecast period.*

*Keywords: financial statement analysis, equity valuation, financial leverage, corporate income tax, debt tax shield, residual income valuation, cost of capital, Feltham-Ohlson framework*

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## **1 Introduction**

During the past decade, accounting-based valuation has increasingly been advocated as a practical alternative to discounted cash flow analysis (Fairfield and Yohn 2001; Penman 2006, and Ohlson and Gao 2006). It is well recognized that the discounted cash flow model is theoretically equivalent to the residual income concept. Yet, because it focuses on accrual accounting, the latter is of particular help in analyzing financial statements and, therefore, presumably more useful for the practical task of evaluating firms (Penman and Sougiannis 1998; Penman 2001). Based on the Modigliani and Miller notion that financing activities are not value-relevant, Feltham and Ohlson (1995) provided a model in which the analysis of operating activities is distinguished from the analysis of financing activities, and they showed that it is the operating activities that generate value and need to be analyzed. Financing activities generate zero residual income in the future, because a firm's net debt is typically measured close to market value on the balance sheet. Drawing on this work, Nissim and Penman (2001) identified financial ratios that are useful for practical valuation. In particular, a forecast of operating residual income involves a

forecast of operating profitability and growth in operating assets. In addition, they showed that there is a tendency for many of the relevant ratios to revert to typical values over time, which is particularly useful since practical analysis requires a continuing value calculation at the end of the forecast period.

However, the value irrelevancy of debt financing presupposes the absence of possible tax effects. In general, interest on debt is deductible against income in assessing corporate tax. In theory, the equity value of firms that borrow to raise cash for operations is higher compared to firms that are entirely equity-financed. Recent empirical research has provided strong evidence that financing activities are indeed value-relevant. Kemsley and Nissim (2002) estimated the debt tax shield at roughly 40 % of debt balances, or 10 % of the total firm value, and other studies appear to support this result (Graham 2000). Existing concepts of evaluating the debt tax shield are based on discounted cash flow analysis (Ballwieser 2007; Drukarczyk and Schüler 2007) and require information that an external investor presumably does not have access to, in particular about the firm's financing policy (Kruschwitz and Löffler 2006).

For example, the well-known weighted average cost of capital (WACC) technique requires the management to maintain a deterministic debt-to-value ratio that is based on market information (Miles and Ezzell 1980; Löffler 2004). This financing strategy requires a continuous adjustment of debt to the more volatile equity prices. Graham and Harvey (2001) provided evidence that actual debt ratios vary considerably with time and yet firms do not rebalance their debt levels with changes in equity prices. One might expect, therefore, that current ratios will not be a good indicator of the long-term ratios. In fact, firms do adjust their debt levels to previously fixed leverage ratios, but those ratios are usually measured in terms of financial accounting information (Arbeitskreis “Finanzierung” 2009). For the discounted cash flow framework, a few models exist that deal with financing based on book values (Kruschwitz and Löffler 2006; Mai 2008; Scholze 2008).

On the basis of the notion that practical analysis requires short forecast horizons, how can the value-generating capacity of the financing activities be accounted for in a valuation? This paper shows a simple way to do that. As in the Feltham-Ohlson framework, the market value of equity is determined by forecasting residual operating income only. However, residual operating income is calculated by charging operating income for the operating assets at a required return for the operations that is smaller than the existing model. Hence, the value added to operating assets is higher, *ceteris paribus*. The difference measures the tax benefit that comes from borrowing to raise cash for the operations. This paper proposes a financing policy that requires the maintenance of a deterministic financial leverage ratio that is based on financial accounting information in order to calculate this tax benefit. This approach is especially suited for external investors who obtain their information via financial statement analysis. It simplifies the forecast considerably for at least two reasons: first, because the financial leverage ratio tends to converge quickly to typical steady-state levels over time, when measured in book values – as opposed to the market-based measures that are required by the WACC technique. Therefore, as long as the forecast of financial leverage is consistent with the empirical data, the firm behaves as if the managers of the firm consciously follow a strategy as they are assumed to do in this model.

Second, a forecast of financial leverage, based as it is on accounting data, fits nicely into the Feltham-Ohlson framework, since in addition to a forecast of the firm’s financial leverage in the steady state, no further information is needed to calculate a continuing value.

The remainder of the paper is organized as follows: section 2 describes the basic model; section 3 provides the main results and discusses practical issues; and section 4 provides concluding remarks.

## 2 The Model

### 2.1 Basic Market Valuation

The model considers a neoclassical model with a sequence of dates  $t = 0, 1, \dots$ . The evolution of the economy is uncertain. A corporate income tax is imposed at the firm level, but not at the shareholder level. The difference between operating cash flow after tax and the amount of investment is referred to as the firm’s free cash flow, which is denoted by  $\widetilde{CF}_t^u$ . In the remainder of the paper, variables that are uncertain in the future are marked formally by a tilde added to its symbol. In an all-equity setup, this amount offsets precisely the dividend paid to the owners. The cost of capital for the operations is defined as a conditional expected rate of return and is denoted by  $r_u \equiv R_u - 1$ . The determination of the cost of capital is not addressed here. It is assumed that the free cash flows satisfy the relation:

$$(1) \quad E \left[ \widetilde{CF}_{t+1}^u - \widetilde{CF}_t^u \mid \mathcal{F}_t \right] = \gamma_t \widetilde{CF}_t^u,$$

where  $\gamma_t$  is deterministic and  $E[\cdot \mid \mathcal{F}_t]$  is referred to the investor’s expectations, conditional on the information available at time  $t$ . This assumption is equivalent to saying that the future cash flows form a weak auto-regressive process of degree one (Laitenberger and Löffler 2006). Allowing for time-dependent  $\gamma_t$  imposes virtually no restrictions on the investor’s unconditional expectations, compared to the well-known auto-regressive process with time-independent  $\gamma$ . Given this assumption, it can be shown that  $r_u$  is the appropriate rate for discounting the time  $t$  expected free cash flow today (Kruschwitz and Löffler 2006: 34 et seq.).

A debt-financed (“levered”) firm differs from an equity-financed (“unlevered”) firm by allowing for borrowing or lending. Because interest on debt is deductible against income in the assessment of corporate tax, the levered market value of opera-

tions must be higher than the unlevered value of operations. Let  $\tau$  be the corporate income tax rate and let  $\tilde{i}_t$  be the interest paid at time  $t$ . Levered free cash flow is higher than unlevered free cash flow, where the difference is equal to the product of the tax rate and interest on debt – the so-called “tax benefit”:

$$(2) \quad \widetilde{CF}_t^l = \widetilde{CF}_t^u + \underbrace{\tau \tilde{i}_t}_{\text{tax benefit}}$$

Given no arbitrage opportunities in the capital market, it can be shown that the value of the levered firm,  $V_0^l$ , is determined by the following equation (Kruschwitz and Löffler 2006: 51):

$$(3) \quad V_0^l = V_0^u + \underbrace{R_f^{-1} E_Q [\tilde{\tau i}_1] + R_f^{-2} E_Q [\tilde{\tau i}_2] + \dots}_{\text{present value of tax benefits}},$$

where  $V_0^u$  denotes the value of the unlevered firm. The result rests on the fundamental theorem of asset pricing (Back and Pliska 1991). This theorem maintains that the investor’s subjective expectations  $E$  can be replaced by so-called risk-neutral expectations  $E_Q$ , which can be discounted by the riskless interest rate  $r_f$ . As long as the amount of debt at some particular date is unknown, future tax benefits are uncertain. Thus, in order to put equation (3) into practical use, information is needed about the firm’s particular financing policy.

## 2.2 Accounting Relations

At any time  $t$  the balance sheet reports assets and liabilities employed in operations, and assets and liabilities involved in financing activities. Let  $\tilde{B}_t$  be the book value of equity, then following Feltham and Ohlson (1995), the balance sheet can be restated to distinguish operating and financing activities:

$$(4) \quad \tilde{B}_t = \widetilde{OA}_t - \widetilde{FO}_t,$$

where  $\widetilde{OA}_t$  denotes operating assets, net of operating liabilities, at date  $t$  and  $\widetilde{FO}_t$  denotes financial obligations, net of financial assets, at date  $t$  (Penman 2010). This notion is closely related to a recent joint project of the FASB and the IASB on how financial information will be presented in the future. The first working principle is that financial statements should be presented such that business activities are separated from financing activities in

order to provide information that is more decision useful than that provided in the financial statement formats used today (IASB 2008). Financial obligation is referred to as the book value of debt. The income statement can be reformulated to distinguish income from operating activities and interest paid to debtholders:

$$(5) \quad \widetilde{NI}_t = \widetilde{OI}_t - (1 - \tau)\tilde{i}_t,$$

where  $\widetilde{NI}_t$  denotes earnings, after taxes, for period  $t$ , and  $\widetilde{OI}_t$  denotes operating income, after taxes, in period  $t$ . Given clean surplus accounting, such that

$$(6) \quad \widetilde{OA}_t = \widetilde{OA}_{t-1} + \widetilde{OI}_t - \widetilde{CF}_t^u,$$

the unlevered value of operations is determined by forecasting residual income:

$$(7) \quad V_0^u = OA_0 + \sum_{t=1}^{\infty} R_u^{-t} E [\widetilde{RI}_t],$$

where  $\widetilde{RI}_t \equiv \widetilde{OI}_t - r_u \widetilde{OA}_{t-1}$  is the operating residual income in period  $t$ . Interest expense is recorded such that the book value of debt is measured on the balance sheet at market value. In fact, *all* existing discounted cash flow models, such as the WACC technique, require the following property (Scholze 2009):

$$(8) \quad \tilde{i}_t \equiv r_f \widetilde{FO}_{t-1},$$

where the riskless rate denotes the cost of debt, which reflects given any financial leverage choice, the required return that suppliers of debt apply to the cash flows of the firm. Many financial assets and financial obligations are measured at their fair value under IAS 39. Some financial assets are at least close to market value as a workable approximation, and their fair values are disclosed in footnotes under IFRS 7.

Assumption (8) implies that debt is not threatened by default. To the extent that an increase in the probability of financial distress is induced by using more debt, the cost of debt will increase with the use of debt. But as a first approximation, one would expect the cost of debt to be roughly constant over wide ranges of debt use.

### 3 Results and Discussion

#### 3.1 A Financing Strategy based on Book Values

Financial leverage is defined as the proportion of operations that is financed through debt, where both variables are measured in terms of book values. Dividing financial obligations by operating assets constitutes a random variable, since numerator and denominator both depict random variables at any date  $t > 0$ . The following financing strategy will now be examined: The firm's management commits to a predetermined schedule for the relative amount of debt to be used in the future by fixing future financial leverage:

**Assumption 1** *Financial leverage remains deterministic for the life of the firm:*

$$(9) \quad \frac{\widetilde{FO}_t}{\widetilde{OA}_t} \equiv \lambda_t \quad \forall t.$$

Assumption 1 requires a ratio that depends on time, not on the state of nature. If, at the end of any period  $t$ , the actual leverage ratio does not equal  $\lambda_t$ , it is assumed that the firm undertakes financial transactions to restore the ratio to its predetermined level. Assumption 1 differs in one important respect from the definitions given by Miles and Ezzell (1980) or Löffler (2004): Their definition requires market values, whereas equation (9) is based on accounting information. For example, according to Miles and Ezzell (1980), the firm maintains a constant market-based leverage ratio,  $L$ , such that  $\widetilde{D}_t = L\widetilde{V}_t^l$ . As already mentioned at the beginning, the preceding assumption is, compared to a market-based measure, quite suitable for describing reality: Firms often adjust their debt levels to previously fixed leverage ratios that are measured in terms of financial accounting information.

Substituting definition (9) into equation (8) leads to  $\widetilde{i}_t = r_f \lambda_{t-1} \widetilde{OA}_{t-1}$ . Period  $t$ 's tax benefit now reflects the risk of the operating activities, at least in principle. Now, consider the following assumption:

**Assumption 2** *The unlevered market to book ratio is deterministic, such that*

$$(10) \quad \widetilde{OA}_t = x_t \widetilde{V}_t^u,$$

where  $x_t \neq 0$  may be any real number.

Basically, assumption 2 is made for technical reasons. Although admittedly heroic, it is indispensable for transforming a theoretical model into a practical valuation tool. Equation (10) implies a ratio between two random variables, namely  $\widetilde{OA}_t$  and  $\widetilde{V}_t^u$ , that depends on time, not on the state of nature. Since the firm is levered, the unlevered market-to-book ratio is not directly observable. In fact,  $x_t$  may be unknown to the investor. Because assumption 2 does not require a constant unlevered market-to-book ratio, it is at least *ex post* consistent with any conceivable sequence of realized values  $\{\widetilde{OA}_t, \widetilde{V}_t^u\}$  over time. The assumption is related to the notion of conservative accounting. Conservatism is defined in two distinct ways in the literature (Beaver and Ryan 2005). First, Feltham and Ohlson (1995) refer to accounting as conservative if on average market values exceed book values, i.e. conservatism is unconditional, or news independent. Examples for unconditional conservatism include historical cost accounting for positive net present value projects and depreciation of assets that is more accelerated than economic depreciation. In contrast, Basu (1997) emphasizes asymmetry in the recognition of anticipated losses as opposed to the non-recognition of anticipated gains, i.e. conservatism can be conditional, or news dependent. Examples of conditional conservatism include lower of cost or market accounting for inventory and impairment accounting for long-lived assets. Assumption 2 is consistent with unconditional conservatism, but inconsistent with conditional conservatism. Typically, the amount of accruals that falls into the second category is small, compared to the amount of the first one. Therefore, the assumption might be a good approximation of reality for many firms. If the assumption holds, the following statement is valid. (All proofs are in the appendix.)

**Proposition 1** *Today's market value of the levered firm can be expressed as follows:*

$$(11) \quad V_0^l = OA_0 + \sum_{t=1}^{\infty} R_u^{-t} E \left[ \widetilde{OI}_t - k_{t-1} \widetilde{OA}_{t-1} \right],$$

where

$$(12) \quad k_t \equiv r_u - \underbrace{\tau r_f \frac{R_u}{R_f}}_{\equiv a} \lambda_t.$$

The term  $\widetilde{RI}_t^{tax} \equiv \widetilde{OI}_t - k_{t-1}\widetilde{OA}_{t-1}$  denotes operating residual income, adjusted for the tax-induced value contribution of the operating activities. Equation (11) differs from the existing residual income model by charging operating income with a cost of capital,  $k_t$ , that is smaller than  $r_u$ , reflecting the benefit to the firm of debt financing.  $k_t$  is similar to the Miles-Ezzell adjustment, except that  $\lambda_t$  is based on book values, not on market values. Intuitively, the operating assets appear to be more profitable when financed by debt, provided that corporate taxes are taken into account. In fact, for  $\tau = 0$ , equation (11) boils down to the well-known formula found, for example, in [Feltham and Ohlson \(1995\)](#) or [Nissim and Penman \(2001\)](#), even if  $\lambda_t \neq 0$ . Instead of using accounting information, the result stated in proposition 1 can be expressed equivalently in terms of future cash flows. See equation (A6) in the appendix.

Compared to the Feltham-Ohlson framework, a forecast of operating residual income now involves a forecast of three value drivers: (i) operating profitability; (ii) growth in operating assets, as in the existing model; and moreover (iii) tax-induced financing profitability, as can be seen by expressing the residual income measure in ratio form:

$$(13) \quad \widetilde{RI}_t^{tax} = [\widetilde{\Omega}_t - (r_u - \alpha\lambda_{t-1})] \widetilde{OA}_{t-1},$$

where

$$(14) \quad \widetilde{\Omega}_t \equiv \frac{\widetilde{OI}_t}{\widetilde{OA}_{t-1}}$$

is the return on operating assets as a measure of operating profitability, and financial leverage  $\lambda_t$  as a measure of financing profitability. Apparently, growth in operating assets influences both operating and financing profitability:

1. Firms increase residual income with their operating activities by either increasing  $\widetilde{\Omega}$  above  $r_u$ , or by growth in operating assets that will earn at this return.
2. Alternatively, firms increase residual income with their financing activities by increasing  $\lambda$ , or by growth in operating assets for a given financial leverage.

### 3.2 Long-Term Forecasting: Steady State Valuation of the Debt Tax Shield

The infinite-horizon forecasting required by equation (11) is considered impractical and, in practice,

forecasts are made for a finite number of years. A “continuing value” is added at the forecast horizon to operationalize the truncation ([Penman 1998](#)). For ease of notation, the current date 0 is considered as the forecast horizon. In order to develop a steady-state version, equation (11) is restated as:

$$(15) \quad V_0^l = OA_0 + \sum_{t=1}^{\infty} R_u^{-t} E \left[ (\widetilde{\Omega}_t - r_u) \widetilde{OA}_{t-1} \right] + \dots \\ \dots + \underbrace{\sum_{t=1}^{\infty} R_u^{-t} E \left[ \alpha \lambda_{t-1} \widetilde{OA}_{t-1} \right]}_{\equiv P_0^{tax}},$$

There is an extensive literature about the calculation of the continuing value for the operating activities, the first part of equation (15) ([Penman 2010: 486 et seq.](#)); however, only the value of the debt tax shield, denoted by  $P_0^{tax}$ , is considered in the following. In particular, the question is, what forecasts of future tax benefits can be obtained from current financial statements? The amount of debt at a certain date is determined by financial leverage times operating assets: First, similar to [Feltham and Ohlson \(1995\)](#) and others, operating assets are expected to grow at a constant rate  $0 \leq g < r_u$ :

$$(16) \quad E \left[ \widetilde{OA}_{t+1} \mid \mathcal{F}_t \right] = (1 + g)\widetilde{OA}_t.$$

Second, it is assumed that the financial leverage ratios  $\lambda_t, t = 0, 1, \dots$ , are generated by the following deterministic process:

$$(17) \quad (\lambda_t - \lambda^*) = \omega (\lambda_{t-1} - \lambda^*),$$

where  $\omega \in [0, 1]$  and  $\lambda^*$  refers to the steady-state level in the long run. The preceding assumption is supported by empirical evidence: [Nissim and Penman \(2001\)](#) document the typical evolution of financial ratios over time. Of particular interest was the question of whether those ratios tend to converge to typical values over time. Among other things, [Nissim and Penman \(2001\)](#) examined how the debt-to-equity ratio typically behaved for NYSE and AMEX firms over five-year periods between 1964 and 1999. The authors placed firms into one of 10 groups on the basis of their current (year 0) debt to equity ratio, with the firms with the highest 10 % in the top group and firms with the lowest 10 % in the bottom group. The median measure for each group was then tracked over the subsequent

5 years. What can be observed is that financial leverage is fairly constant; in addition, portfolios with extreme (high or low) measures tend to become more like the average measure as time goes on (Nissim and Penman 2001). The preceding process captures the mean reversion feature observed empirically by Nissim and Penman (2001). This characteristic can be seen more clearly by expressing (17) in the following form:

$$(18) \quad \lambda_t = \omega\lambda_{t-1} + (1 - \omega)\lambda^*.$$

It is readily seen that the leverage process is a convex combination of a mean reversion process and a forecast of constant leverage. When  $\omega = 0$ ,  $\lambda_t = \lambda^*$ , the process exhibits extreme mean reversion. When  $\omega = 1$ , future leverage ratios remain at the current level,  $\lambda_t = \lambda_{t-1}$ . Except for the extreme case  $\omega = 1$ , financial leverage ratios are time varying. In the long run, financial leverage ratios converge to a constant long-run average  $\lambda^*$ , that is,  $\lim_{t \rightarrow \infty} \lambda_t = \lambda^*$ . The smaller the value chosen for  $\omega$ , the sooner a steady state will be reached.

From a practical point of view, the time pattern of  $\lambda_t$  is determined by two elements: (1) The current level  $\lambda_0$  relative to its typical level  $\lambda^*$  for a comparison set of firms. (2) The rate of reversion  $\omega$  to a long-run level. Element 1 is established by the analysis of current and past financial statements: historical industry patterns are a good starting point if the future is likely to be similar to the past. Element 2 is the subject of forecasting. How long will a nontypical financial leverage ratio take to fade to the typical long-run level? Or, put it differently: how long will a nontypical leverage persist? The analysis of Nissim and Penman (2001) suggests that 3 up to 5 years might be a reasonable forecast. For a given forecast  $t$ , the appropriate “fading rate”  $\omega(t)$  can be derived endogenously by picking a number  $\varepsilon$ , such that  $\omega^t |\lambda_0 - \lambda^*| < \varepsilon$ , and solving for  $\omega(t)$ :

$$(19) \quad \omega(t) < \exp \left\{ \frac{1}{t} \ln \left( \frac{\varepsilon}{|\lambda_0 - \lambda^*|} \right) \right\},$$

where  $\varepsilon$  serves as a means to make the difference between the firm’s individual leverage and the long-run industry level as small as desired.

Although both forecasts are linear,  $E[\lambda_t \widetilde{OA}_t | \mathcal{F}_t]$  does not specify a linear expectation. In fact, by defining the forward difference operator  $\Delta u_t \equiv u_{t+1} - u_t$  the conditional expectation of the change

in future debt levels can be expressed as follows:

$$(20) \quad E[\Delta \widetilde{FO}_t | \mathcal{F}_t] = E[\lambda_t \Delta \widetilde{OA}_t | \mathcal{F}_t] + \dots \\ \dots + \Delta \lambda_t \widetilde{OA}_t + E[\Delta \lambda_t \Delta \widetilde{OA}_t | \mathcal{F}_t].$$

The last term is nonzero but small for reasonable growth rates  $g$ . Moreover, because of its mean reverting characteristic, it vanishes within a short time, since equation (18) implies

$$(21) \quad \Delta \lambda_t = \omega^t (\omega - 1) (\lambda_0 - \lambda^*) \leq 0.$$

Therefore, neglecting the last term in equation (20) will not do much harm. Substituting equations (16) and (18) into equation (20) and rearranging leads to the following expectation:

$$(22) \quad E[\widetilde{FO}_{t+1} | \mathcal{F}_t] \approx (\omega + g)\widetilde{FO}_t + (1 - \omega)\lambda^* \widetilde{OA}_t.$$

Figure 1 illustrates the measurement error that results from applying equation (22), the continuous line, instead of equation (20), the broken line, for different values of  $g$ . Further parameter values are:  $\omega = 0.7$ ,  $\lambda^* = 40\%$ ,  $OA_0 = 100$  and  $\lambda_0 = 80\%$ . It can be observed that the measurement error tends to increase with  $g$  but always converges to zero.

Equations (16) and (22) form a linear information dynamics of  $\widetilde{FO}_t$  and  $\widetilde{OA}_t$ . Since  $\widetilde{FO}_t$  is itself linear in  $\widetilde{OA}_t$ , there exist a constant  $\theta$ , such that the market value of future tax benefits at date  $t$  can be expressed as  $\widetilde{P}_t^{tax} = \theta \cdot \widetilde{OA}_t$ . The specific formula is expressed in the following proposition:

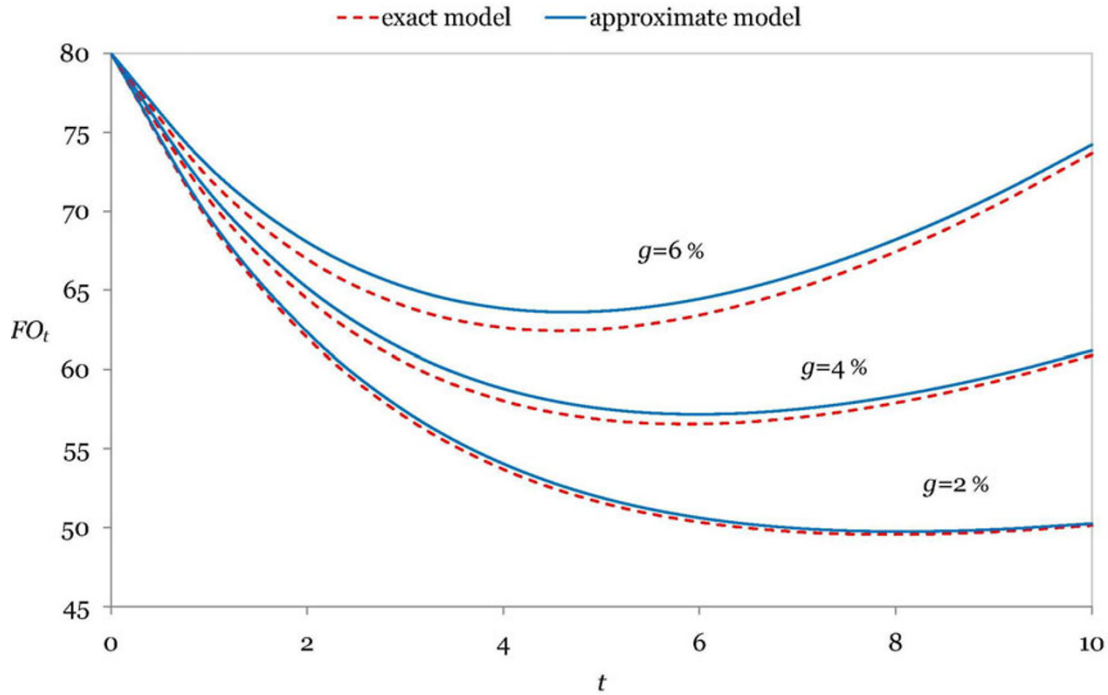
**Proposition 2** *The following is valid for the continuing value of the debt tax shield:*

$$(23) \quad P_0^{tax} = OA_0 \cdot \alpha \frac{(r_u - g)\lambda_0 + (1 - \omega)\lambda^*}{(1 - \omega)(r_u - g) + (r_u - g)^2}.$$

According to proposition 2, the value of future tax benefits can be calculated by applying a multiplier to today’s operating assets. The multiplier is greater than zero whenever the firm is financed by debt, i.e. whenever  $\lambda_0$  or  $\lambda^*$  is positive. Given a certain financial leverage choice, more value is added when operating assets grow at a faster rate.

Growing operating assets at a constant rate in expectation is a simple, yet widely used assumption in the literature – and presumably in practice, as well. But in principle, the result stated in the proposition, can be extended to a broader class of stochastic processes, if the forecasted value of

**Figure 1: Evolution of  $FO_t$  over time by applying the approximate model compared to the exact model**



expected operating assets was assumed to be generated by a linear process of the following form:

$$(24) \quad E \left[ \widetilde{OA}_{t+1} \mid \mathcal{F}_t \right] = \delta_1 \widetilde{X}_1 + \dots + \delta_n \widetilde{X}_n,$$

where  $\delta_1, \dots, \delta_n$  are arbitrary parameters and  $\widetilde{X}_1, \dots, \widetilde{X}_n$  could be any random variables that drive the expectation – including  $\widetilde{OA}_{t+1}$ . Garman and Ohlson (1980) show that there exist constants  $\theta_1, \dots, \theta_n$ , such that

$$(25) \quad \widetilde{P}_t^{tax} = \theta_1 \widetilde{X}_1 + \dots + \theta_n \widetilde{X}_n.$$

Different assumptions about the stochastic process generating the information variables can be found in Fama (1977) or Myers and Turnbull (1977). In particular, consider the following forecast (Feltham and Ohlson 1995):

$$(26) \quad E \left[ \widetilde{OA}_{t+1} \mid \mathcal{F}_t \right] = (1 + g) \widetilde{OA}_t + \widetilde{\Phi}_t,$$

where the scalar variable  $\widetilde{\Phi}_t$  captures all “other information” that represent potential omitted correlated information variables that are valuation relevant. Using equation (26) instead of (16) changes

equation (22) to:

$$(27) \quad E \left[ \widetilde{FO}_{t+1} \mid \mathcal{F}_t \right] \approx (\omega + g) \widetilde{FO}_t + \dots \\ \dots + (1 - \omega) \lambda^* \widetilde{OA}_t + \lambda_t \widetilde{\Phi}_t,$$

i.e.  $\widetilde{\Phi}_t$  “scales” expectations of next-year debt by today’s leverage ratio. The linear information dynamics, consisting of equations (26) and (27), yield the following valuation equation that is presented here without proof:

$$(28) \quad P_0^{tax} = \theta \left( OA_0 + \frac{\Phi_0}{1 + r_u} \right),$$

where  $\theta$  is equal to the multiplier applied to  $OA_0$  in proposition 2. Although interesting for empirical work (Liu and Ohlson 2000; Callen and Segal 2005), the major problem with using the forecast (26) for practical purposes is that  $\widetilde{\Phi}_t$  is typically undefined and unknown *a priori* to external investors. However, equation (28) can be put to practical use if the expectation of next-year operating assets is publicly observable. Analysts’ consensus forecasts would seem to be a reasonable measure. Using the notation  $\widehat{OA}_t^{t+1}$  as the date  $t$  observable expectation for next-year operating assets, then  $\widetilde{\Phi}_t$  can be inferred as a date  $t$  observable

variable by using equation (26):

$$(29) \quad \tilde{\Phi}_t = \widehat{OA}_t^{t+1} - (1 + g)\widehat{OA}_t.$$

In this context one can think of  $(1 + g)\widehat{OA}_t$  as the external investor's first-cut estimate of next-period operating assets, and  $\tilde{\Phi}_t$  capturing all "other information" relevant to forecasting the future.

### 3.3 An Example

For illustration purposes, consider a setting where three simple forecasts will yield a valuation of the levered firm. Suppose, an external investor predicts that

1. the return on operating assets is constant:  
 $E[\tilde{\Omega}_t] = \bar{\Omega}$ ;
2. operating assets grow at a constant rate  $g$ , and
3. Financial leverage remains constant at its current level:  $\omega = 1$ .

Substituting those forecasts into equation (13) implies that future residual income is predicted to grow at a rate  $g$ , as well as future debt levels. Now, the value of the levered firm can be expressed as follows:

$$(30) \quad V_0^l = \underbrace{\frac{\bar{\Omega} - g}{r_u - g} OA_0}_{=V_0^u} + \frac{1 + r_u}{1 + r_f} \cdot \frac{\tau r_f \lambda_0}{r_u - g} OA_0,$$

where the first part of the right-hand side is the unlevered value of operations (Ohlson and Gao 2006; Rajan, Reichelstein, and Soliman 2007): The multiplier is the unlevered market-to-book ratio that compares the return on operating assets to the cost of capital. For  $\bar{\Omega} > r_u$ , the multiplier is greater than one, and more value is added to book value the higher the return on operating assets is relative to the growth rate. For  $\bar{\Omega} = r_u$ , the multiplier is equal to one, and future growth does not matter. The second part of equation (30) captures the debt tax shield according to proposition 2: Next period's tax benefits are equal to  $\tau r_f \lambda_0 OA_0$  and grow with a rate  $g$ , afterwards. The factor  $\frac{1+r_u}{1+r_f}$  adjusts for the fact that the uncertainty of period  $t$ 's level of tax benefits is resolved just a second after time  $t$  – and has thus to be discounted with the riskless rate from time  $t + 1$  to  $t$ , and with the risk-adjusted rate from time  $t$  to 0. Substituting the following numbers

$\bar{\Omega}$	$OA_0$	$g$	$r_u$	$\tau$	$r_f$	$\lambda_0$
15 %	100	2 %	12 %	45 %	8 %	40 %

into equation (30) yields:

$$(31) \quad V_0^l = \frac{0.15 - 0.02}{0.12 - 0.02} \cdot 100 + \dots \\
\dots + \frac{1.12 \cdot 0.45 \cdot 0.08 \cdot 0.4}{1.08 \cdot (0.12 - 0.02)} \cdot 100 \\
= 1.3 \cdot 100 + 0.1493 \cdot 100 \\
= 130 + 14.93 = 144.93.$$

The debt tax shield of  $P_0^{tax} = 14.93$  reflects about 38 % of debt balances, or roughly 10 % of total firm value. Suppose now that the current leverage is currently at a high level of  $\lambda_0 = 80\%$ , and will thus revert to the long-run industry level of  $\lambda^* = 40\%$  within  $t = 5$  years. Using equation (19) and  $\varepsilon = 0.01$ , this implies for the fading rate  $\omega(5)$ :

$$(32) \quad \omega(5) < \exp \left\{ \frac{1}{5} \ln \left( \frac{0.01}{|0.8 - 0.4|} \right) \right\} \\
< 0.47817.$$

Choosing  $\omega = 0.478$  and plugging the numbers into equation (23) yields a value of the debt tax shield of:

$$(33) \quad P_0^{tax} = 0.1733 \cdot 100 = 17.33,$$

which is obviously higher than the result above, because of the higher leverage for the next five years.

As a last comment, the unlevered value of operations can also be expressed in terms of future free cash flows. Clean surplus accounting implies that

$$(34) \quad E[\widetilde{CF}_t^u] = E[\widetilde{OI}_t - (\widetilde{OA}_t - \widetilde{OA}_{t-1})].$$

Substituting this equation into the expression for  $V_0^u$  in equation (30) yields:

$$(35) \quad V_0^u = \frac{E[\widetilde{CF}_1^u]}{r_u - g}.$$

Since  $E[\widetilde{CF}_1^u] = 0.15 \cdot 100 + (102 - 100) = 13$ , equation (35) amounts to:

$$(36) \quad V_0^u = \frac{13}{0.12 - 0.02} = 130.$$



## 4 Concluding Remarks

Based on the notion that the financing activities adds to the market value of equity when taxes are taken into account, this paper shows a simple way to value the debt tax shield. Penman lists a few criteria for a practical valuation model (Penman 2010: 92 et seq.):

1. *Finite forecast horizons*: A valuation method is preferred for which a finite-horizon forecast does the job.
2. *Validation*: A forecast makes the method credible if it can be validated in the firm's future financial statements.
3. *Parsimony*: A practical model needs to forecast something for which the information gathering and analysis is relatively straightforward. If that information is to be found in the financial statements, all the better.

The model presented here fulfills these criteria. Calculating the continuing value of the debt tax shield involves a short forecast horizon, because financial leverage typically reverts to a steady-state level within a few years. Furthermore, since the Feltham-Ohlson framework already requires a forecast of operating profitability and growth in operating assets, only two additional pieces of information are required for valuing the debt tax shield; namely, the steady-state level of financial leverage, and a time horizon for the convergence to this level. Needless to say, the financial leverage is based on accounting information that can be validated after the fact to confirm that the forecast was good (or poor).

However, as with any practical tool, there are tradeoffs that have to be explored: first, the model requires, in particular, a deterministic unlevered market to book ratio in the future. Second, so far, the discussion of leverage has been related to the notion that taxes that a firm must pay generate value for the shareholders. Unfortunately for investors, interest payments received from debt are taxed as income as well as dividends and capital gains received from equity. Personal taxes reduce cash flows to investors and thus have the potential to offset some of the tax benefits of leverage. Notwithstanding these facts, the impact of personal taxes on firm value has been omitted

from the model, primarily because empirical evidence suggests that the magnitude of the personal tax disadvantage of debt is only limited, if dividends and interest payments are to be taxed at the same rate. Furthermore, the objective of this paper was to enlarge the scope of the residual income model, without complicating its practical applicability. Considering taxes that are raised both at the investor and at the company level leads to a model that is presumably only in special circumstances easy to handle.

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## Appendix

### Proof of Proposition 1

Consider the following claim (Laitenberger and Löffler 2006; Kruschwitz and Löffler 2006):

**Lemma** *If the cost of capital is deterministic and free cash flows are weak auto-regressive, then the following is valid for all times  $i > t$ :*

$$(A1) \quad R_u^{t-i} E \left[ \tilde{V}_i^u \mid \mathcal{F}_t \right] = R_f^{t-i} E_Q \left[ \tilde{V}_i^u \mid \mathcal{F}_t \right].$$

Since  $OA_0$  is already known today, equation (3) can be expressed as follows, using assumption 1 and equation (8):

$$(A2) \quad V_0^l = V_0^u + \frac{\tau r_f \lambda_0 OA_0}{R_f} + \dots \\ \dots + \sum_{t=2}^{\infty} \tau r_f \lambda_{t-1} R_f^{-t} E_Q \left[ \tilde{OA}_{t-1} \right].$$

$\tilde{OA}_t$  is, of course, an uncertain quantity from today's perspective. But this uncertainty is resolved just a second after time  $t$ . Therefore, the interest payments to be rendered at time  $t + 1$  are, in contrast, certain. The resulting tax benefits are thus to be discounted with the cost of debt from time  $t + 1$  to time  $t$ . But the discounting from  $t$  to 0 may not be carried out with  $r_f$ , since it is uncertain from today's perspective as to how much tax the firm proportionately financed by debt will save.

By assumption, operating assets are always a deterministic multiple of the unlevered market

value. Using the Lemma's result therefore implies for any date  $t$ :

$$(A3) \quad R_u^{-t} E \left[ \widetilde{OA}_t \right] = R_f^{-t} E_Q \left[ \widetilde{OA}_t \right].$$

$R_f^{-t} E_Q \left[ \widetilde{OA}_t \right]$  can thus be substituted for  $R_u^{-t} E \left[ \widetilde{OA}_t \right]$  to yield the following expression for equation (A2):

$$(A4) \quad V_0^l = V_0^u + \tau r_f R_f^{-1} \sum_{t=1}^{\infty} R_u^{1-t} \lambda_{t-1} E \left[ \widetilde{OA}_{t-1} \right].$$

Expanding with  $R_u/R_u$  and rearranging terms yields:

$$(A5) \quad V_0^l = V_0^u + \sum_{t=1}^{\infty} R_u^{-t} \tau r_f \frac{R_u}{R_f} \lambda_{t-1} E \left[ \widetilde{OA}_{t-1} \right].$$

Substituting equation (7) yields the result stated in the proposition.

Instead of using accounting information, the result stated in proposition 1 can be expressed in terms of future cash flows by substituting the equations (7) and (34) into equation (11). After some rearranging, the following corollary can be stated:

**Corollary** *Today's market value a levered firm can be expressed in terms of future cash flows as follows:*

$$(A6) \quad V_0^l = \sum_{t=1}^{\infty} \frac{E \left[ \widetilde{CF}_t^u \right]}{(1+r_u)^t} + \sum_{t=1}^{\infty} \frac{E \left[ \tau r_f \lambda_{t-1} \widetilde{OA}_{t-1} \right]}{(1+r_f)(1+r_u)^{t-1}}.$$

Equation (A6) requires a forecast of future operating assets in order to value the market value of future tax benefits.

### Proof of Proposition 2

The present value of tax benefits can be expressed as

$$(A7) \quad P_0^{tax} = \sum_{t=1}^{\infty} R_u^{-t} E \left[ \alpha \widetilde{FO}_{t-1} \right].$$

With a capital market free of arbitrage, equation (A7) implies for any date  $t \geq 0$

$$(A8) \quad R_u \widetilde{P}_t^{tax} = \alpha \widetilde{FO}_t + E \left[ \widetilde{P}_{t+1}^{tax} \mid \mathcal{F}_t \right].$$

Conjecture that the value function  $\widetilde{P}_t^{tax}$  is a linear function of  $\widetilde{FO}_t$  and  $\widetilde{OA}_t$  for all  $t$ , with zero intercept,

$$(A9) \quad \widetilde{P}_t^{tax} = \pi_1 \widetilde{FO}_t + \pi_2 \widetilde{OA}_t,$$

and substitute into the above, using (16) and (22):

(A10)

$$\begin{aligned} R_u \left[ \pi_1 \widetilde{FO}_t + \pi_2 \widetilde{OA}_t \right] &= \dots \\ \dots &= \pi_1 \left[ (\omega + g) \widetilde{FO}_t + (1 - \omega) \lambda^* \widetilde{OA}_t \right] + \dots \\ \dots &+ \pi_2 (1 + g) \widetilde{OA}_t + \alpha \widetilde{FO}_t. \end{aligned}$$

Collecting terms associated with each of the two variables yields two equations in two unknowns:

$$(A11) \quad R_u \pi_1 = \pi_1 (\omega + g) + \alpha,$$

(A12)

$$R_u \pi_2 = \pi_1 (1 - \omega) \lambda^* + \pi_2 (1 + g).$$

Solving this system of equations yields

(A13)

$$\pi_1 = \frac{\alpha}{(1 - \omega) + r_u - g},$$

(A14)

$$\pi_2 = \frac{\alpha(1 - \omega) \lambda^*}{[(1 - \omega) + r_u - g] \cdot [r_u - g]}.$$

Substituting the solution into the expression for the value function, and using  $\widetilde{FO}_t = \lambda_t \widetilde{OA}_t$  yields the multiplier stated in the proposition.

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