A General System Structure and Accounting Framework for Socioeconomic Metabolism

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Supporting information is available on the *JIE* Web site

Summary

A wide spectrum of accounting frameworks and models is available to describe socioeconomic metabolism (SEM). Despite the common system of study, a large variety of terms and representations of that system are used by different models. This makes it difficult for practitioners to compare and choose a model or model combination that is fit for purpose. To facilitate model comparison, we analyze the system structure of material flow analysis (MFA); life cycle assessment (LCA); supply and use tables (SUTs); Leontief, Ghosh, and waste input-output analysis; integrated assessment models; and computable general equilibrium models. We show that the typical system structure of MFA and LCA is a directed graph. For the other models and some MFA and LCA studies, the system structure is a bipartite directed graph. We demonstrate that bipartite directed graphs and SUTs are equivalent representations of SEM. We show that the system structures of the models above are special cases of a general system structure, which models SEM as a *bipartite graph*. The general system structure includes industries, markets, the final use phase, products, waste, production factors, resources, and emissions. From the general system structure, we derive an accounting framework in the form of a generalized SUT. The general system structure facilitates the development of clear and unambiguous terminology across models. It helps to identify rules for the correct accounting of waste flows and stock changes. It facilitates model comparison and can serve as a blueprint for a model-independent database of SEM.

Introduction

A Spectrum of Model Families to Describe Socioeconomic Metabolism

Society faces the challenge of reconciling human development with physical constraints that arise as a result of the limited size of the natural environment (UN 2013). To tackle this challenge, society requires scientific knowledge of how human and economic development depends on, and interacts with, the natural environment. In other words, we need to understand and manage *socioeconomic metabolism* (SEM) (Ayres and Simonis 1994; Fischer-Kowalski and Hüttler 1999; Fischer-Kowalski 1997; Baccini and Brunner 1991; Fischer-Kowalski and Weisz 1999; Pauliuk and Müller 2014). Over the last decades, different model families that describe SEM on different scales and with different degrees of physical and economic stringency have evolved. These include material or substance flow analysis (MFA/SFA) (Baccini and Bader 1996), material flow accounting (Eurostat 2001; Fischer-Kowalski et al. 2011), process-based life cycle assessment (LCA) (EU JRC 2010), input-output (I-O) analysis (IOA) (Miller and Blair 2009), energy system and integrated assessment models (IAMs) (Loulou

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et al. 2005), and computable general equilibrium (CGE) models (Burfisher 2011).

To overcome the limitations of individual models and to tackle new research questions, many combinations of accounting frameworks and models are available. Examples include economically extended MFA (Kytzia et al. 2004), the waste input-output (WIO) model (Nakamura and Kondo 2002; Nakamura et al. 2007), multilayer supply and use tables (SUTs) (Schmidt et al. 2010, 2012), and "hybrid" approaches that combine detailed process data with economy-wide system descriptions. The latter include hybrid SUTs (Suh and Lippiatt 2012), hybrid LCA (de Haes et al. 2004; Suh 2004b; Strømman 2009), the hybrid I-O approach (Nakamura et al. 2008), and mixed-unit IOA (Hawkins et al. 2007).

The Need for an Explicit System Structure

The models and their combinations listed above cover different physical and economic aspects of SEM, countries or regions, and time spans. They differ in the resolution of technical and natural processes, boundaries between man-made and natural environment, causal relationships between elements in the system, and model drivers. Moreover, the different models are maintained and developed by often separate scientific communities, which has led to a lack of overview and transparency across fields.

To manage this diversity, a sound comparison and systematization of the different approaches is needed to help practitioners to understand and select the appropriate model or combination of models that is "fit for purpose" (Keirstead 2014). A broad systematic overview of the different accounting frameworks and models is lacking, however. This lack of overview has repeatedly led to reinvention, relabeling of known concepts, and the development of contradictory or even erroneous accounting routines or models. Examples are provided in sections 3 to 5 of the supporting information available on the Journal's website. We assert that this lack of overview partly stems from insufficient understanding of a fundamental property of the different accounting frameworks and models: their *system structure*.

Scope and Research Topics

In this article, we apply a visual and intuitive graph approach to display the system structure of the different models and accounting frameworks. Further, we develop and apply a general system structure that comprises all existing ones.

In a related article (Pauliuk et al. n.d.), we show that any system description of SEM can be *structured as a directed graph*, which is an established concept in mathematics (Diestel 2012). A graph model of SEM consists of edges or arrows that represent flows of objects (products, waste, and so on) and vertices or nodes that represent the processes in the system. Using this graph approach, we cover the following five points:

- We show that, without loss of generality, system approaches of SEM form *bipartite (directed) graphs*, which is a special type of directed graph. We show that bipartite graphs and SUTs are equivalent representations of SEM.
- We provide an overview of the system structure of MFA, LCA, I-O, WIO, IAM, and CGE models and show that the system structure of each model is a *bipartite graph* or can be readily reshaped into one.
- We propose a *general system structure* of SEM in the form of a bipartite directed graph. We show that it includes the structures of the different accounting frameworks and modeling families as special cases.
- We derive a general, *multilayer accounting framework* for SEM and show that it is equivalent to the general system structure.
- We explain how the general system structure and the accounting framework can help to establish and use clear, unambiguous nomenclature and facilitate the comparison and combination of different model families.

Bipartite Graphs and Supply and Use Tables

Figure 1a shows a directed graph with five transformation nodes or industries $t_1 \ldots t_5$, one node e representing the environment outside the system boundary, and six different objects (resources, commodities, and so on) $o_1 \ldots o_6$. The black arrows represent the flows within the system, and the gray arrows represent the input from and the output to the environment. The three-dimensional (3D) array (figure 1a, right) is an alternative representation of this system and consists of a stack of six matrices for the flows of $o_1 \ldots o_6$ from and to the processes $t_1 \ldots t_5$ and e. Both representations are equivalent: The array can be constructed from the graph by arranging the flows in matrix form, and the graph can be constructed from the array by considering the row index as the origin node and the column index as the destination node of each flow in the table. The 3D array that describes the system in figure 1a can be called "traceable inventory" because each flow can be traced back to the industrial node where it was generated (Majeau-Bettez et al. 2014b).

In reality, the interindustry flows of a certain commodity shown in figure 1a are not independent of one another. Commodities are scarce and the products consumed by one node cannot be used by another. Industries and end users can often choose between different suppliers and negotiate prices. There is hence a need to include processes that allocate scarce commodities and resources to consumers. These processes are called distribution nodes, market activities (ecoinvent Center 2014a), or commodity nodes (Loulou et al. 2005).

The presence of distribution nodes for all commodities leads to a system where there is no direct exchange between industries because they place their output and buy their requirements on the markets. There is one market for each commodity or resource. Markets do not transform commodities, which is why there is no direct exchange between markets for different

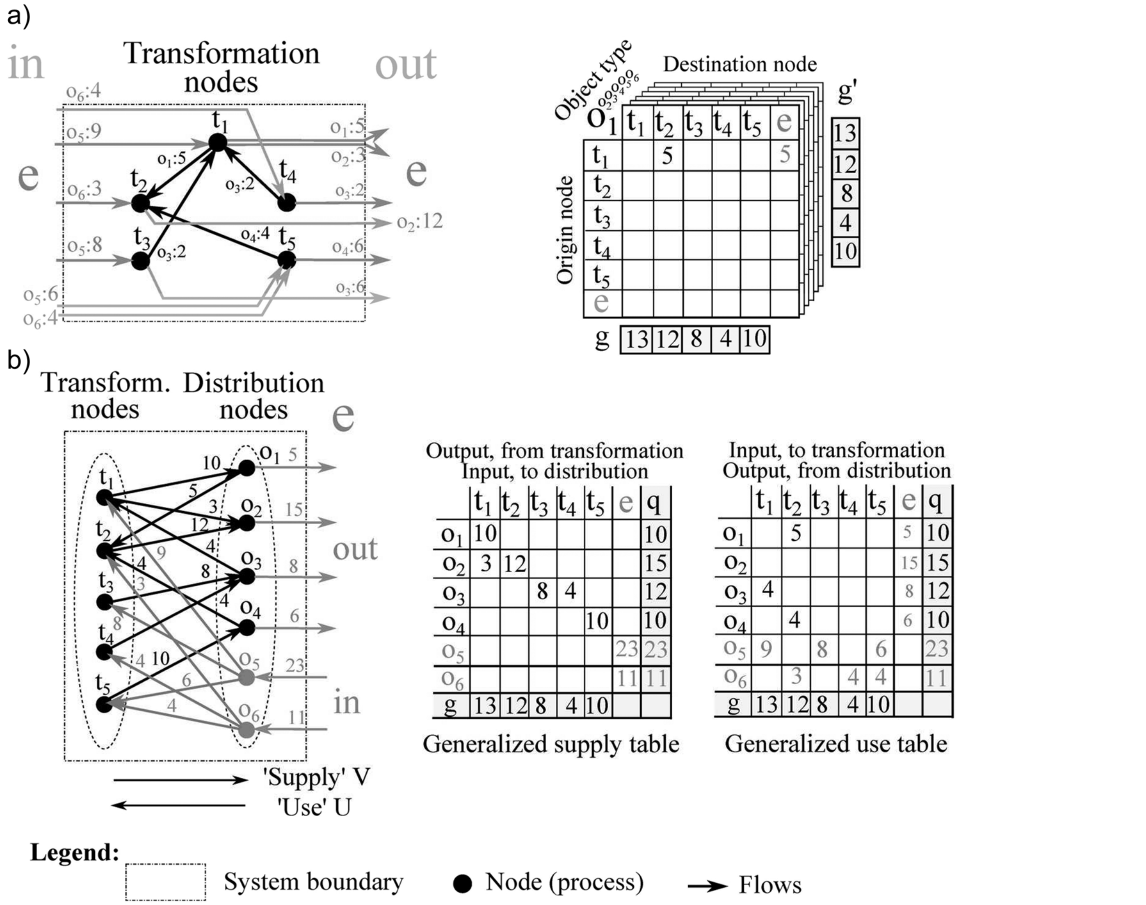

Figure 1 Directed graphs and bipartite directed graphs lead to different accounting frameworks. (a) Directed graph that consists of arrows representing commodity flows between processes (nodes) can be represented as a three-dimensional array (origin × destination × object type). (b) Bipartite graph of transformation nodes (processes or industries) and commodity nodes (markets) can be represented as a supply and use table. In each case, the graph and tabular representations are equivalent, because one can be constructed from the other without loss of information. The vectors ***g*** and ***q*** denote the total throughput of transformation and distribution nodes, respectively. The input-output structure of the transformation nodes is the same in both systems (a) and (b), but the two system descriptions are not equivalent.

commodities. Graphs that have two disjoint sets of nodes and that only have directed edges that connect a node in one set to a node in the other are called *bipartite directed graphs* (Diestel 2012) (figure 1b). The information contained in the bipartite graph in figure 1b can be displayed in tabular form by recording all flows from industries to markets in a transformation output table and all flows from markets to industries in a transformation input table, in which the row numbers represent the distribution nodes and the column numbers the transformation nodes.

Vice versa, the graph in figure 1b can be constructed from these tables by considering each number in the transformation output table as a flow between an industry node represented by the column number and a market node represented by the row number (the same with opposite flow direction for the transformation input table). The tables that represent the bipartite graph are called *SUTs*; they are a common representation of national economies used in the System of National Accounts (SNA). They allow to record co- or joint production (Baumgärtner et al. 2001; UN 2008; Miller and Blair 2009). Bipartite graphs and SUTs are equivalent representations of the system structure of a model describing SEM.

The System Structure of Models that Describe Socioeconomic Metabolism

Material/Substance Flow Analysis and Material Flow Accounting

The system structure of any MFA model is a directed graph, where the nodes (typically drawn as boxes) represent the processes, and the arrows represent the flows (Baccini and Bader 1996; Baccini and Brunner 2012; Fischer-Kowalski et al. 2011). Explicit markets were introduced into MFA by Müller and colleagues (2006), but their use is neither a requirement nor always necessary. MFA systems are therefore not bipartite in general. The systems used in state-of-the-art MFA comprise entire

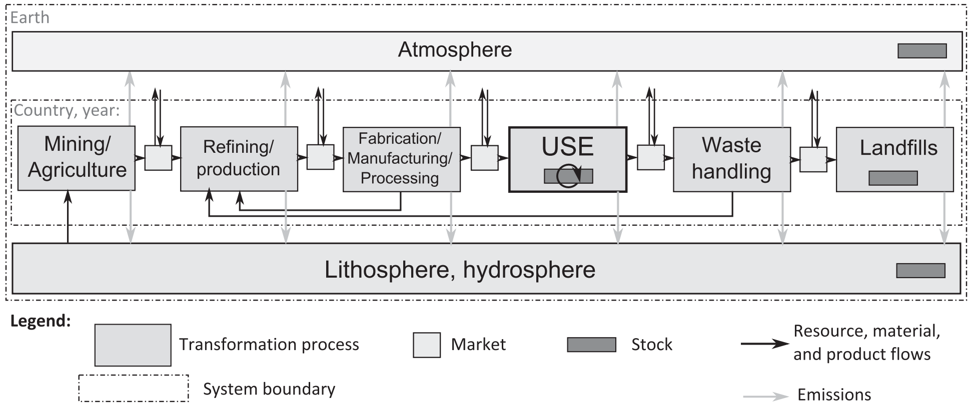

Figure 2 Typical system definition of material flow analysis (Mao et al. 2008; Brunner and Rechberger 2004; Müller et al. 2006; Pauliuk 2013). Large boxes represent industries or transformation processes; small boxes represent markets. Flows entering and leaving the markets in a vertical direction represent trade flows between different regions.

material cycles, including production, use, disposal, recycling, and trade at all stages (figure 2) (Mao et al. 2008; Brunner and Rechberger 2004; Müller et al. 2006; Pauliuk 2013; Liu et al. 2012).

The standardized system definition of material flow accounting (Eurostat 2001; Fischer-Kowalski et al. 2011) is an aggregated version of the scheme in figure 2, where the entire region studied is modeled as a single transformation activity.

System of National Accounts, System of Environmental-Economic Accounting, and Multilayer SUTs

The SNA (UN 2008) provides international standards for the accounting of the monetary aspects of SEM in the form of SUTs, and the System of Environmental-Economic Accounting (SEEA) (UN 2012) describes a compatible accounting framework for physical aspects of society's metabolism and natural assets. Both frameworks record industry output in supply tables $\boldsymbol{V}$ and industry input in use tables $\boldsymbol{U}$. Industrial use of the production factors, labor and capital service, is recorded as value added $\boldsymbol{v}$, and output to end users is called final demand $\boldsymbol{y}$. The SUT provided by the SNA can be recast into an equivalent bipartite graph. Figure 3a shows a simplified representation of this graph using the notation used in MFA. Here, the left box represents the different industrial nodes and the right one the different market nodes. The arrows symbolically represent all flows between nodes. Both industries and markets are balanced.

Waste Generation and Multilayer Supply and Use Tables

Multilayer SUTs allow statisticians to include resource use, waste generation, or emissions into the SUT and determine the process balance on each layer. A well-developed example for multilayer SUTs is the FORWAST and Compiling and Refining Environmental and Economic Accounts (CREEA) system, which not only includes the economic flows in the classical SUT, but also the use of natural resources $\boldsymbol{R}$, emissions to the environment $\boldsymbol{B}$, waste generation $\mathbf{W}_V$ and use $\mathbf{W}_U$, imports $\mathbf{N}_c(n)$ and exports $\mathbf{N}_c(e)$, and stock changes in both industries ($\boldsymbol{\Delta S}$) and markets ($\boldsymbol{S}^+$) (Schmidt et al. 2010, 2012). All flows are recorded in an SUT and therefore the system structure is a bipartite graph (figure 3b). The flows in the SUT can be recorded in multiple units to represent the monetary, mass, energy, or carbon layers of SEM.

Leontief, Ghosh, and Waste Input-Output Models

To create an I-O model from the SUTs $\boldsymbol{V}$ and $\boldsymbol{U}$, a 1:1 correspondence between industries and products needs to be established by introducing additional modeling assumptions, so-called *constructs* (Majeau-Bettez et al. 2014b; Miller and Blair 2009; Lenzen and Rueda-Cantuche 2012). Irrespective of the construct applied, the resulting I-O model can always be recast as a symmetric SUT, where the supply table is the diagonalized output vector $\hat{x}$ and the use table is the interindustry flow matrix $\mathbf{Z}$. Even though a construct may alter the number of transformation nodes or commodity nodes relative to the original SUT, the system structure remains a bipartite graph (figure 3c). In section 1 of the supporting information on the Web, we show that markets are an implicit element of Leontief I-O models: Their balancing equation is identical with the Leontief primary model equation. Nonproduct flows, such as waste, resource use, and emissions, can be included as satellite accounts, which leads to environmentally extended I-O (EEIO) models (Leontief 1970, 1972; Miller and Blair 2009).

The WIO model (Nakamura and Kondo 2002) can be built from an SUT (Lenzen and Reynolds 2014). Its system structure is therefore a bipartite graph (figure 3d). The WIO model comprises industries and markets for main products, markets for different types of waste, and waste treatment activities. The waste treatment part of the WIO model is a mirror-inverted I-O model with waste flowing into treatment activities. A detailed description of the system structure of the WIO model, including the definition of all system variables, is contained in section 2 of the supporting information on the Web.

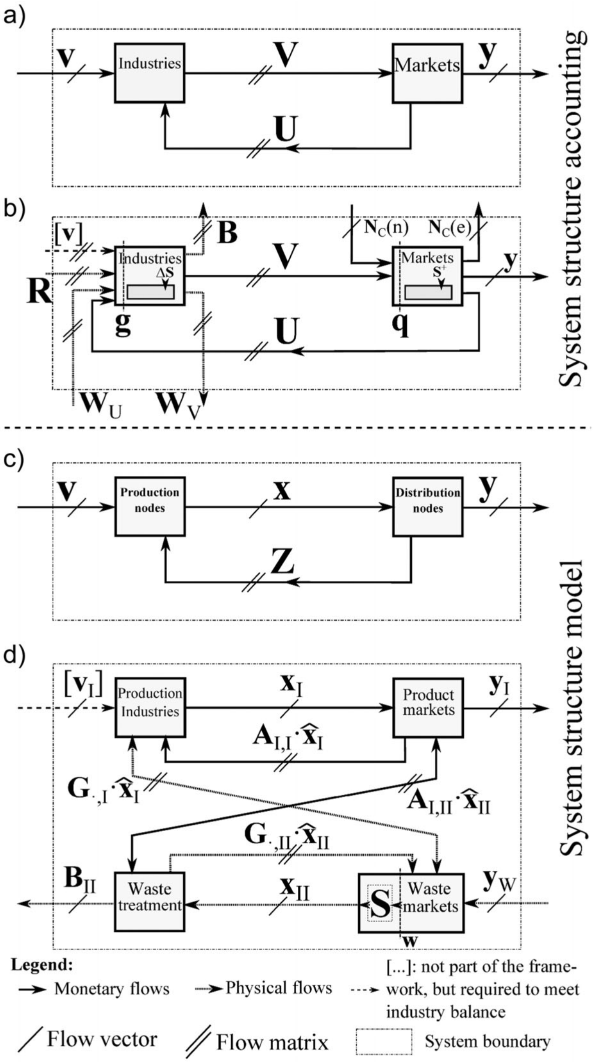

Figure 3 System structures of (a) the supply and use table (SUT) of the System of National Accounts. (b) Multilayer SUTs, exemplified by the EU FORWAST and Compiling and Refining Environmental and Economic Accounts (CREEA) projects (Schmidt et al. 2010, 2012). Here, solid arrows also represent physical flows. (c) Leontief and Ghosh input-output (I-O) models. (d) Waste input-output model (Nakamura and Kondo 2002). Parts (a) and (b) show the system structure of accounting frameworks, and parts (c) and (d) the system structure of I-O models. All variables are explained in the text or in the Supporting Information on the Web. In all cases, satellite accounts that do not enter the balancing equations were omitted. In some frameworks, value added and final demand can also be recorded as matrices.

Life Cycle Assessment

Explicit system definitions in the form of directed graphs are part of good practice in LCA (EU JRC 2010), but there is no common system structure for LCA. Markets are often not explicitly modeled in LCA, but their importance for LCA has been stated (Weidema 2003; Weidema et al. 2009) and they were included in the latest version of ecoinvent (ecoinvent Center 2014a, page 'market activity'). Moreover, the application of SUTs and I-O models to LCA (Suh et al. 2010; de Haes et al. 2004; Suh et al. 2004; Strømman et al. 2009) implies the presence of markets, because both SUTs and I-O models have a bipartite directed graph as their system structure. Markets connect the different industrial activities in the supply chain of a product. In ecoinvent 3, markets are activities that "do not transform inputs, but simply transfer the intermediate output of one or more transforming activities to the activities that consume this intermediate exchange as an input" (ecoinvent Center 2014a, page 'market activity'). Hence, the system structure of an LCA study can be displayed as a bipartite graph. Markets are convenient if there is more than one supplier for a certain commodity. Their function becomes trivial if there is only one supplier for a certain commodity ("one-brand axiom"), and thus they are often omitted (Heijungs and Suh 2002).

Integrated Assessment and General Equilibrium Models

IAMs combine models of SEM with climate models (Pindyck 2013). We include IAMs in this analysis to better understand their relation to industrial ecology (IE) models. Some documentations of IAMs explicitly mention the divide between industries and markets ("commodity nodes") in their systems, for example, the MARKAL and TIMES model family (Loulou et al. 2005). This divide leads to a bipartite directed graph as a system structure, which has long been recognized by the IAM community (figure 4). Most IAMs not only use bipartite graphs to model flows, but they also contain dynamic models of in-use stocks of capital and consumer goods as part of their description of SEM (Loulou et al. 2005).

CGE models stem from the neoclassical economic tradition. Their system structure is given in the form of a social accounting matrix, which describes the spending of each activity (column account) and the source of income to each activity (row account) (Burfisher 2011). Depending on the scope of the model, activities include production activities (industries), households, or the government. The social accounting matrix is constructed from SUTs, similar to the construction of I-O models. According to our comments on figure 3c, the system structure of CGE models is therefore a bipartite directed graph.

Figure 5 summarizes our findings on system structure and coverage of different activity groups and extensions by the different model families. It shows that no model or accounting framework comes close to covering the entire SEM on both the monetary and the physical layer. Several models used in IE have a bipartite system structure, but markets are often implicit.

A General System Structure of Socioeconomic Metabolism

The different IE, energy systems, and economic models of SEM share a common system structure, which can be presented as a bipartite directed graph. For models without a bipartite systems structure, markets can be introduced without loss of generality by rerouting each interindustry flow through a

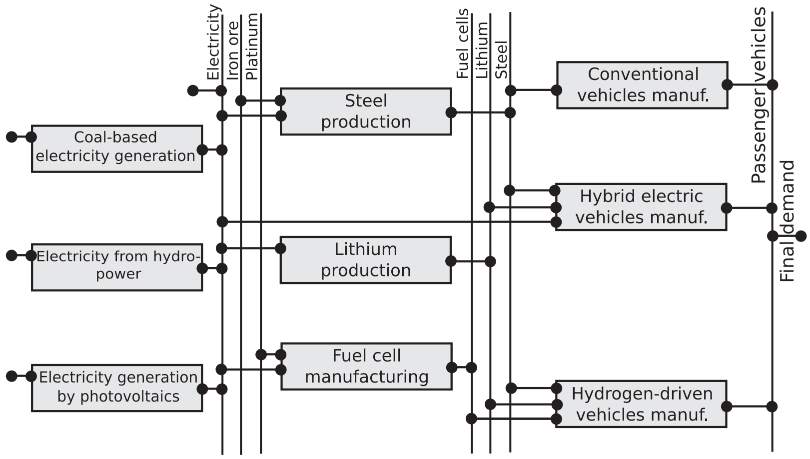

Figure 4 System structure of integrated assessment models (IAMs) in the notation of Loulou and colleagues (2005). A bipartite directed graph, consisting of technologies (process nodes, drawn as boxes) and markets (commodity nodes, drawn as vertical lines) represents the system structure of socioeconomic metabolism in IAMs. The direction of flows is from left to right.

	Graph type	Final use phase	Product Industries	Waste treatment industries	Product markets	Waste markets	Labor, capital service	Emissions	Resources
MF accounting	**D**	X (all merged into one activity)					---	X	X
MF analysis/SFA	**D/(B)**	X	X	X	X	X	---	X	X
SNA+SEEA	**B**	X	X	X	---[I]	---[I]	X	X	X
Multi-layer SUTs	**B**	---	X	X	---[I]	---[I]	---	X	X
EE-IO	**B**	---	X	X	---[I]	---[I]	X[b]	X	X
WIO	**B**	---	X	X	---[I]	---[I]	---	X	---
LCA	**D/(B)**	X	X	X	---[I,c]	---[I,c]	X	X	X
IAM	**B**	X	X	---	X	---	X	X	X
CGE	**B**	X	X	---[a]	---[I]	---[I,a]	X	X	X

Figure 5 Graph type (D = directed; B = bipartite directed) and rough indication of the coverage of different system elements by the different accounting and model frameworks. This table shows the typical coverage of system elements, such as industries, emissions, and so on, by the different models. We acknowledge that there are special cases where models have more extensive coverage of the system than shown here. Examples include hybrid LCA, economically extended MFA, and secondary steel production in IAMs. I = implicit, not recognized as separate activity; a = aggregated together with main industries or markets; b = included in models with closure for labor or capital service; c = included in ecoinvent 3. Color code: monetary layer: yellow; physical layer: blue; both layers: green. MF = material flow; SFA = substance flow analysis; SNA = System of National Accounts; SEEA = System of Environmental-Economic Accounting; SUTs = supply and use tables; EE-IO = environmentally extended input-output; WIO = waste input-output; LCA = life cycle assessment; IAM = integrated assessment model; CGE = computable general equilibrium.

commodity node. We now propose a general system structure of SEM that contains the three types of transformation activities listed in figure 5: the final use phase; goods and services producing industries ("production industries"); and waste treatment industries. It also contains markets (distribution activities) for each of the five object flow categories in figure 5: natural resources; goods or products; waste; emissions; and the man-made production factors, labor and capital service.

Each of the three transformation activities is connected to all five distribution activities. All flows and stocks that are part of the general system structure are listed and explained in table 1. The choice of a common notation was difficult because the established conventions in different fields are not compatible. We chose a main symbol for each flow, which indicates the type of object and the direction of the flow ($\boldsymbol{R}$: resources, $\boldsymbol{V}$: products from industries, ...), and an index, which indicates the process involved (Y: final use phase, T: waste treatment industries, ...). Exceptions were made for final demand and trade. Industries and final use phase consume natural resources ($\boldsymbol{R}_I, \boldsymbol{R}_T, \boldsymbol{R}_Y$) and emit process waste to nature ($\boldsymbol{B}_I, \boldsymbol{B}_T, \boldsymbol{B}_Y$). The capital goods in the final use phase supply capital service and the human agents supply labor. Both are distributed to the industries on the factor markets ($\boldsymbol{f}, \boldsymbol{F}_I, \boldsymbol{F}_T$) (Samuelson and Nordhaus 2005; Duchin 2009). Industries supply products and generate

Table 1 List of a) sets and b) system variables and examples for the general system structure (figure 6), general accounting framework (figure 7), and balancing equations (1) to (7)

a)

Sets		
Description	*Symbol*	*Example*
Set of all product groups	P	Passenger cars, mobile phones, steel
Set of all waste types	W	Municipal solid waste, blast furnace slag
Set of all production industries	I	Primary steel making
Set of all waste treatment industries	T	Incinerators, secondary steel making
Set of all factors, resources, and emissions, respectively	F, R, B	Labor and capital service (value added), iron ore in ground, CO_2 to air
Set of all final use (Y) and stock change (S) categories	Y, S	Households, government, industries, work in progress
Set of all trade nodes	M	Industries in other countries, etc.

b)

System variables			
Description	*Symbol*	*Size*	*Example*
Supply of products by production industries (supply table)	$\boldsymbol{V}_I$	$\#P \cdot \#I$	Supply of wheat by agricultural sector
Use of products by production industries (use table)	$\boldsymbol{U}_I$	$\#P \cdot \#I$	Use of electricity by car manufacturing sector
Treatment of waste type by waste treatment industry	$\boldsymbol{T}_T$	$\#W \cdot \#T$	Municipal solid waste sent to landfills
Generation of waste type by waste treatment industry	$\boldsymbol{G}_T$	$\#W \cdot \#T$	Slag generated by incinerators
Generation of waste type by primary industry	$\boldsymbol{G}_I$	$\#W \cdot \#I$	Fabrication scrap generated by car industry
Use of waste type by primary industry	T_I	$\#W \cdot \#I$	Use of old car tires in cement kilns
Use of products by waste treatment industry	$\boldsymbol{U}_T$	$\#P \cdot \#T$	Use of electricity in car shredders
Supply of products by waste treatment industry	$\boldsymbol{V}_T$	$\#P \cdot \#T$	Supply of electricity by incineration plants
Factor use by primary and waste treatment industries, throughput, and total	$\boldsymbol{F}_I$, $\boldsymbol{F}_T$, $\boldsymbol{q}_f$, $\boldsymbol{f}$	$\#F \cdot \#I$, $\#F \cdot \#T$, $\#F \cdot 1$	Labor costs in incineration plant
Use of natural resources by primary and waste treatment industries, use phase, throughput, and total	$\boldsymbol{R}_I$, $\boldsymbol{R}_T$, $\boldsymbol{R}_U$, $\boldsymbol{q}_r$, $\boldsymbol{r}$	$\#R \cdot \#I$, $\#R \cdot \#T$, $\#R \cdot \#Y$, $\#R \cdot 1$	Use of mineral resources, harvested biomass, or air

(*Continued*)

Table 1 Continued

	System variables		
Description	*Symbol*	*Size*	*Example*
Emissions by primary and waste treatment industries, final use phase, throughput, and total	$\boldsymbol{B}_I$, $\boldsymbol{B}_T$, $\boldsymbol{B}_Y$, $\boldsymbol{q}_b$, $\boldsymbol{b}$	$\#B \cdot \#I$, $\#B \cdot \#T$, $\#B \cdot \#Y$, $\#B \cdot 1$	Emissions of CO_2
Net stock additions on product and waste markets (P,W), primary and waste treatment industries (I,T), and the final use phase (Y)	$\boldsymbol{\Delta S}_x$, $x = P,W,I,T,Y$	$\#P \cdot 1$, $\#W \cdot 1$, $\#S \cdot \#I$, $\#S \cdot \#T$, $\#S \cdot \#Y$	Addition to inventories of primary aluminum on markets
Stocks on product and waste markets (P,W), primary and waste treatment industries (I,T), and the use phase (Y)	$\boldsymbol{S}_x$, $x = P,W,I,T,Y$	*Not specified here*	Inventories of primary aluminum on markets
Final demand for products	$\mathbf{Y}_P$	$\#P \cdot \#Y$	Final demand for passenger cars
Supply of postconsumer waste	$\mathbf{Y}_W$	$\#W \cdot \#Y$	EoL cars sent to shredders
Imports of products and waste	$\mathbf{N}_P$, $\mathbf{N}_W$	$\#P \cdot \#M$, $\#W \cdot \#M$	Import of passenger vehicles or obsolete electronic equipment
Exports of products and waste	$\boldsymbol{E}_P$, $\boldsymbol{E}_W$	$\#P \cdot \#M$, $\#W \cdot \#M$	Export of passenger vehicles or obsolete electronic equipment
Industry throughput, primary and waste treatment, use phase throughput	$\boldsymbol{g}_I$, $\boldsymbol{g}_T$, $\boldsymbol{g}_Y$	$1 \cdot \#I$, $1 \cdot \#Y$, $1 \cdot \#Y$	Total throughput of passenger car manufacturing, total throughput through incinerator
Market throughput, products, and waste	$\boldsymbol{q}_P$, $\boldsymbol{q}_W$	$\#P \cdot 1$, $\#W \cdot 1$	Total turnover of passenger cars or municipal solid waste

Note: Matrices are denoted as bold capital letters and vectors as bold lowercase letters. The # operator denotes the cardinality of the set: #*P* is the number of product groups, and so on. CO_2 = carbon dioxide; EoL = end of life.

waste ($\boldsymbol{V}_I$, $\boldsymbol{G}_I$ for production and $\boldsymbol{V}_T$, $\boldsymbol{G}_T$ for waste treatment industries) and consume products and treat waste ($\boldsymbol{U}_I$, $\boldsymbol{T}_I$, for production and $\boldsymbol{U}_T$, $\boldsymbol{T}_T$ for waste treatment industries). Agents and institutions enjoy the services by the products in the final use phase (not quantified here). Products enter and waste/end-of-life (EOL) products leave the final use phase ($\boldsymbol{Y}_P, \boldsymbol{Y}_W$). Industries, markets, and the final use phase contain stocks, which are to be interpreted as materials and supply or work in progress for industries, inventories for markets, and in-use stocks for the final use phase, respectively. We consider industrial capital to be part of the final use phase. This allocation of capital follows the tradition of the SNA, where the gross fixed capital formation is part of final demand that leaves the industrial system, and of MFA, where the use phase comprises all anthropogenic stocks. The system variables describe the SEM of one region. The link to other regions or countries is established by introducing trade flows ($\boldsymbol{N}_P$, $\boldsymbol{E}_P$, $\boldsymbol{N}_W$, $\boldsymbol{E}_W$).

We now arrange the three transformation and five distribution activities to a bipartite directed graph (figure 6). This general system structure is the simplest representation of SEM as a bipartite graph that contains the final use phase and two groups of industrial activities, one for converting natural resources into useful products and one for treating waste and EOL products, and that exchanges objects with nature in both directions. This dichotomy of industries into production and waste treatment activities reflects that any economic activity leads to wanted and unwanted output (Baumgärtner et al. 2001). Agents control the system by deploying labor and capital service in industrial activities and by determining market transactions.

The scheme in figure 6 defines which activities are studied and how they are connected. It does not represent a fully specified system definition or even a model, because this would require practitioners to make specific choices, including the location of the boundary between nature and anthroposphere; spatial and temporal boundaries, such as the accounting period or asset and production boundaries (UN 2008); the classification of products and activities; the layers or units that are quantified; and a set of drivers, exogenous variables, and model equations.

The Relation Between the General System Structure and the Accounting Frameworks and Model Families of Socioeconomic Metabolism

We compare figures 2 to 4 with figure 6.

The MFA system in figure 2 is compatible with the general system structure if all interindustry flows are intersected by markets. The waste treatment processes in figure 2 appear now on the left side of the use phase, below the production industries. Trade between regions at all stages is considered as well. The general system structure is more comprehensive, however: Whereas typical MFA systems contain only a few industries of interest, the general system contains all industries and all variables necessary for a complete physical *and* economic description of SEM.

The system structures of standard SUTs, multilayer SUTs, standard I-O models, and the WIO model (figure 3) are compatible with the general system structure. In fact, it was the WIO model and its structure that inspired the development of the general system structure. The general system structure is more comprehensive, however, because the final use phase with capital stocks is not included in SUTs and I-O models. This lack of coverage of I-O systems can be overcome by combining dynamic stock models of the use phase with WIO models (Nakamura et al. 2014; Kagawa et al. 2015; Pauliuk et al. 2014).

Life cycle inventories (LCIs) of product systems contain a description of the flows occurring in production, use, and disposal of the products studied. After including market transactions at all stages, the generalized version of an LCI matches the general system structure in figure 6. In-use stocks are often not modeled explicitly, which is not necessary if only one product is studied. Similar to MFA systems, LCA systems describe subsystems of SEM.

The general system structure of IAMs (figure 4) is a bipartite directed graph comprising industries and product markets; it is therefore included in the general system structure. The same holds for CGE models because they have the same system structure as the SUTs they are constructed from.

A Generalized Accounting Framework for Socioeconomic Metabolism

Proper accounting of SEM should precede modeling (Majeau-Bettez et al. 2014b). Process balances are crucial to test the validity of a data set that describes SEM. Next to the monetary process balance, different physical balances can be imposed for energy, total mass, or mass of carbon (Majeau-Bettez et al. 2014a; Schmidt et al. 2010). We define the *throughput* for production industries $\boldsymbol{g}_I$, product markets $\boldsymbol{q}_P$, waste treatment industries $\boldsymbol{g}_T$, waste markets $\boldsymbol{q}_W$, and the use phase $\boldsymbol{g}_Y$ according to the dashed process cross-sections in figure 6. For a balanced system, the sum of all inputs, the sum of all outputs, and the throughput, must be equal for each node. Net additions to stocks are included and treated as if they were outputs (cf. section 4 of the supporting information on the Web). For each activity, we arrange the input and output flows as generalized SUTs, one for industry output/market input and one for industry input/market output (figure 7). The column sum gives the industry throughput, and the row sum gives the market throughput. The general SUT in figure 7 extends the classical monetary SUTs (European Commission 2008), by adding resources, waste, and emissions. If the system is quantified for different units, for example, mass and monetary value, the SUT can have multiple layers (not shown in figure 7) and different industry and market balances hold simultaneously. The general multilayer SUT was developed from the CREEA hybrid SUTs (Schmidt et al. 2010, 2012). We added the final use phase as a third transformation process group, separated waste treatment activities from the production industries, and included postconsumer waste.

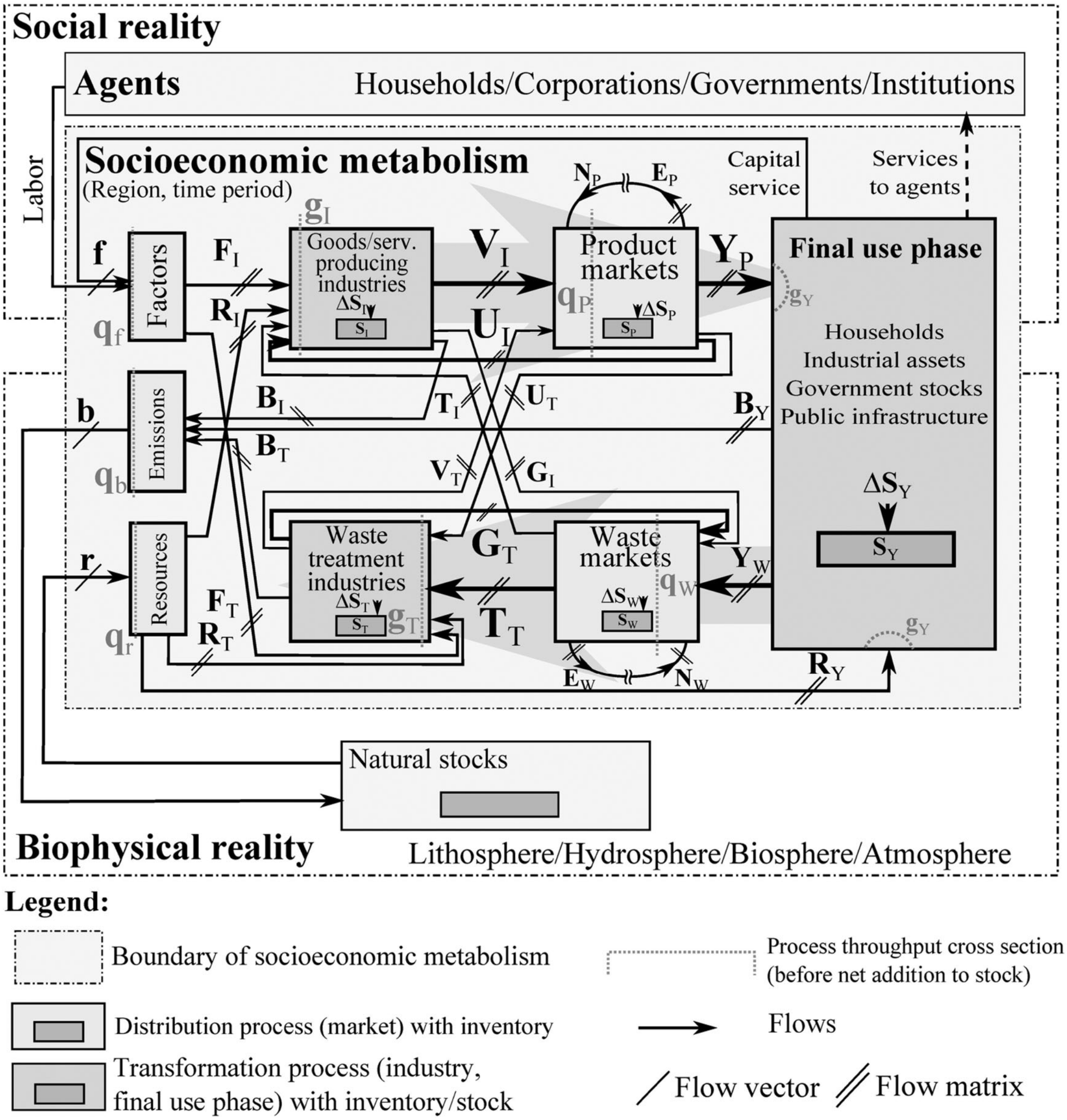

Figure 6 General system structure of socioeconomic metabolism. Each of the boxes with a solid boundary represents a group of activities. The number of activities represented by each box, number of regions, and choice of unit(s) depend on the classification chosen by the modeler. Flows represent vectors, such as the total emissions $\boldsymbol{b}$, or matrices, such as the industry supply table $\boldsymbol{V}_I$. The system structure forms a bipartite directed graph: Each flow connects a dark gray transformation process with a light gray distribution process.

The Balancing Equations of Socioeconomic Metabolism

Equations (1) to (3) contain the balances of the five distribution process groups: product markets (equation 1); waste markets (equation 2), and the three markets for factors, resources, and emissions (equation 3). In all equations, $\boldsymbol{i}_x$ stands for a summation vector of ones with length x.

$$\boldsymbol{U}_I \cdot \boldsymbol{i}_I + \boldsymbol{U}_T \cdot \boldsymbol{i}_T + \mathbf{Y}_P \cdot \boldsymbol{i}_Y + \boldsymbol{E}_P \cdot \boldsymbol{i}_M + \boldsymbol{\Delta S}_P = \boldsymbol{q}_p = \mathbf{V}_I \cdot \boldsymbol{i}_I + \mathbf{V}_T \cdot \boldsymbol{i}_T + \mathbf{N}_P \cdot \boldsymbol{i}_M \tag{1}$$

$$\boldsymbol{T}_I \cdot \boldsymbol{i}_I + \boldsymbol{T}_T \cdot \boldsymbol{i}_T + \boldsymbol{E}_W \cdot \boldsymbol{i}_M + \boldsymbol{\Delta S}_W = \boldsymbol{q}_W = \boldsymbol{G}_I \cdot \boldsymbol{i}_I + \boldsymbol{G}_T \cdot \boldsymbol{i}_T + \mathbf{Y}_W \cdot \boldsymbol{i}_Y + \mathbf{N}_W \cdot \boldsymbol{i}_M \tag{2}$$

$$\begin{aligned} \boldsymbol{F}_I \cdot \boldsymbol{i}_I + \boldsymbol{F}_T \cdot \boldsymbol{i}_T &= \boldsymbol{q}_f = \boldsymbol{f} \\ \boldsymbol{R}_I \cdot \boldsymbol{i}_I + \boldsymbol{R}_T \cdot \boldsymbol{i}_T + \boldsymbol{R}_Y \cdot \boldsymbol{i}_Y &= \boldsymbol{q}_r = \boldsymbol{r} \\ \boldsymbol{B}_I \cdot \boldsymbol{i}_I + \boldsymbol{B}_T \cdot \boldsymbol{i}_T + \boldsymbol{B}_Y \cdot \boldsymbol{i}_Y &= \boldsymbol{q}_b = \boldsymbol{b} \end{aligned} \tag{3}$$

The second group of equations contains the balance of the three transformation process groups: production industries

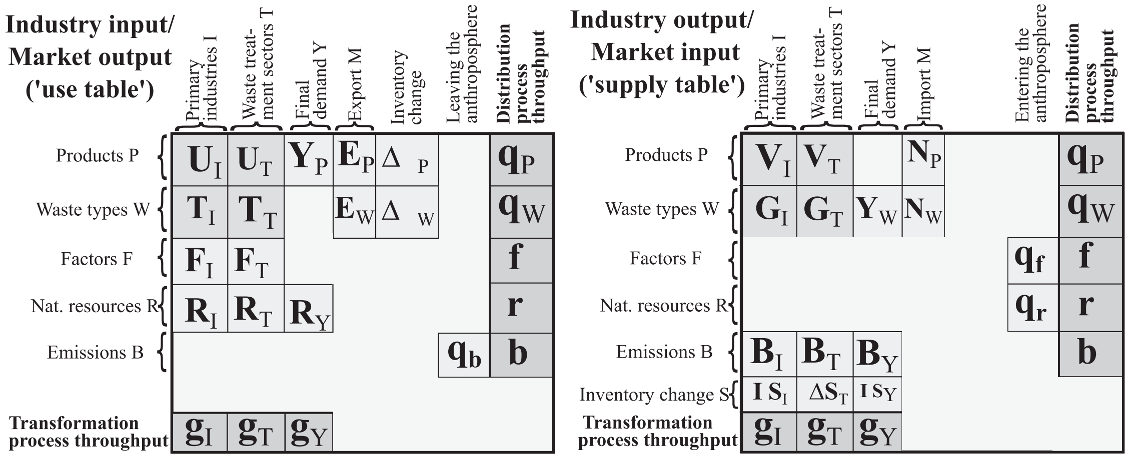

Figure 7 General accounting framework (supply and use tables; SUTs) for socioeconomic metabolism. This framework generalizes the CREEA hybrid SUT (Schmidt et al. 2010, 2012). The general accounting framework, the general system structure in figure 6, and the balancing equations (1) to (6) are equivalent representations of the bipartite system structure. Three groups of transformation activities (column account) and five groups of market activities (row accounts) are included. Note that, in general, none of the matrices in the general accounting framework are square. All symbols are introduced in table 1. The resolution of products, industries, final demand categories, and so on, and the choice of units depend on the classification chosen by the practitioner. The framework can be extended by satellite accounts that do not enter the industry balance, such as noise emissions or labor statistics. Prices and other interlayer coefficients are inhomogeneous in general (Weisz and Duchin 2006; Merciai and Heijungs 2014).

(equation 4); waste treatment industries (equation 5); and the final use phase (equation 6). Equations (4) to (6) are meaningful only when a common unit is used across the entire SUT.

$$\boldsymbol{i}_P^T\cdot\boldsymbol{U}_I + \boldsymbol{i}_W^T\cdot\boldsymbol{T}_I + \boldsymbol{i}_F^T\cdot\boldsymbol{F}_I + \boldsymbol{i}_R^T\cdot\boldsymbol{R}_I = \boldsymbol{g}_I = \boldsymbol{i}_P^T\cdot\boldsymbol{V}_I + \boldsymbol{i}_W^T\cdot\boldsymbol{G}_I + \boldsymbol{i}_B^T\cdot\boldsymbol{B}_I + \boldsymbol{i}_S^T\cdot\boldsymbol{\Delta S}_I \quad (4)$$

$$\boldsymbol{i}_P^T\cdot\boldsymbol{U}_T + \boldsymbol{i}_W^T\cdot\boldsymbol{T}_T + \boldsymbol{i}_F^T\cdot\boldsymbol{F}_T + \boldsymbol{i}_R^T\cdot\boldsymbol{R}_T = \boldsymbol{g}_T = \boldsymbol{i}_P^T\cdot\boldsymbol{V}_T + \boldsymbol{i}_W^T\cdot\boldsymbol{G}_T + \boldsymbol{i}_B^T\cdot\boldsymbol{B}_T + \boldsymbol{i}_S^T\cdot\boldsymbol{\Delta S}_T \quad (5)$$

$$\boldsymbol{i}_P^T\cdot\boldsymbol{Y}_P + \boldsymbol{i}_R^T\cdot\boldsymbol{R}_Y = \boldsymbol{g}_Y = \boldsymbol{i}_W^T\cdot\boldsymbol{Y}_W + \boldsymbol{i}_B^T\cdot\boldsymbol{B}_Y + \boldsymbol{i}_S^T\cdot\boldsymbol{\Delta S}_Y \quad (6)$$

Note that equations (1) to (6) are vector equations. Equation (1), for example, represents #*P* equations, one for each product group.

The general system structure and the general SUT are different representations of the balancing equations of the activities (equations 1 to 6). All three representations are equivalent. The balancing equations (1) to (6) can be read from the general system structure in figure 6 and from the general SUT in figure 7. Vice versa, the general system structure and the general SUT can be constructed from the balancing equations: Writing the balancing equations in tabular form leads to the SUT, and interpreting each equation as the balance of a process will, after rearrangement, leads to the general system definition of SEM (figure 6).

This equivalence facilitates the understanding, construction, and comparison of different models, given that practitioners can choose between graphical system definitions, SUTs, and the balancing equations without losing information.

How Can the Concepts Developed Here be Used in Future Work?

Working Across Different Model Families

Accounting and modeling of SEM is becoming more complex, given that physical and economic aspects are quantified in parallel and industry and product resolution increases. Comparing the structure of different accounting frameworks and models allows practitioners to better understand the fundamental commonalities or differences between them. A graphical representation of the system structure of a model can help to achieve this goal because it reveals the function of the different process groups and makes implicit processes explicit. This may prove especially helpful when working across fields that follow a different accounting and modeling tradition (e.g., MFA and IAM models). Section 5 of the supporting information on the Web provides an example where confusion about the system structure led to the development of a flawed model framework.

Use of Clear Nomenclature

The graphical representation of the system structure reveals how the processes in the accounting frameworks and models are connected to one another. Modelers can define terms for certain process groups and flows by referring to the explicit system structure. This is helpful for the development and use of clear terminology, and we provide two examples:

1. In bipartite systems, supply and use are relative, and more specific terms should be used, for example, "industry supply" for the traditional supply table, to avoid confusion with flows that are supplied by markets to industries.
2. Balancing equations should be named after the processes they apply to and which unit they are in: "market balance in monetary units," "industry balance for total mass," and so on. Terms such as "material balance" for the market balance in monetary units or "financial balance" for the industry balance in monetary units, as used by Jansen and ten Raa (1990), may lead to confusion, especially in multilayer accounting.

Specification of Accounting Rules

In section 3 of the supporting information on the Web, we show that explicit graphs can help to define valid accounting rules for waste generation. They also help to understand the differences between the many approaches for dealing with waste in physical I-O models that have been proposed by Hubacek and Giljum (2003), Suh (2004a), Dietzenbacher (2005), Xu and Zhang (2009), and Dietzenbacher and colleagues (2009) and that were formalized by Majeau-Bettez and colleagues (2014a). The main finding is that for models that include waste, by-products, or emissions, it is important to distinguish between industry *throughput* and usable *output* when defining technical coefficients.

When stocks or inventories are present, it matters whether additions to stock are accounted for at the input or output side of a process, and both methods will lead to different accounting frameworks. In bipartite graphs and SUTs, there are two ways of accounting for inventories: They can be placed in industries or on markets. Both ways of locating inventories are possible and they can be used in parallel, thus allowing for a more complete description of industrial metabolism than a single method could do (cf. section 4 of the supporting information on the Web).

Interdisciplinary Research and the Use of Multilayer Supply and Use Tables as a Common Database for Socioeconomic Metabolism

Bipartite graphs contain both industrial and market activities. We therefore believe that they are better suited than directed graphs to serve as a common foundation for integrating the different modeling families and for interdisciplinary research on SEM. Industries and other transformation activities are commonly studied by engineers and economists, and markets are studied by economists, psychologists, and sociologists. Moreover, a directed graph with traceable industry-industry transactions can always be transformed into a bipartite graph by adding auxiliary market nodes that interrupt each interindustry flow. Depending on the research question, these auxiliary markets for individual flows can be further aggregated to commodity markets.

Accounting of SEM in SUTs allows for balanced recording of joint production for any number of layers (physical, monetary), and the system structure is necessarily bipartite. Generalized SUTs, such as the one shown in figure 6, allow for sound integration of core IE concepts such as co-production (industrial symbiosis) and recycling into environmental-economic accounting and modeling. Widespread application of multilayer SUTs could facilitate data exchange between different fields. Establishing SUTs as a common accounting framework would require a change of common practice in MFA and LCA, though SUTs have already been promoted for LCA (Suh et al. 2010), and they were implemented in the latest version of ecoinvent (ecoinvent Center 2014b).

Conclusion

We see the main contribution of this work in offering a general structure of SEM that can form a common ground for researchers from different scientific disciplines. Starting from the general structure and the accounting framework, many different models can be built. Models will differ in level of aggregation, production functions, market mechanisms, and so on. Model choice should always be guided by the research question, but the system structure and the core database should be common to all models. The framework presented here is agnostic of the research question; it is a general framework on which the different models can be grounded.

The graph approach to SEM helps to clarify a number of seemingly unrelated issues from a system structure perspective in a clear and intuitive manner. This includes the role of the implicit markets in I-O models, distinction of useful process output and process throughput in accounting frameworks, and identification of two complementary ways of recording inventory changes.

The equivalence of process balancing equations, general system structure, and SUTs may strengthen the modeling community of IE, because it allows practitioners to compare their ideas with already existing approaches in a simple way. Thus, reinvention, misconception, and pathological modeling could be avoided.

The common system structure of SEM forms a theoretical basis for integrating IE concepts in more mainstream energy and economic modeling. Vice versa, the explicit markets enable IE practitioners to apply more realistic modeling of market mechanisms.

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Additional Supporting Information may be found in the online version of this article at the publisher's web site:

Supporting Information S1: This supporting information contains more detailed explanation of the graph approach, especially for input-ouput models, as well as a collection of applications.