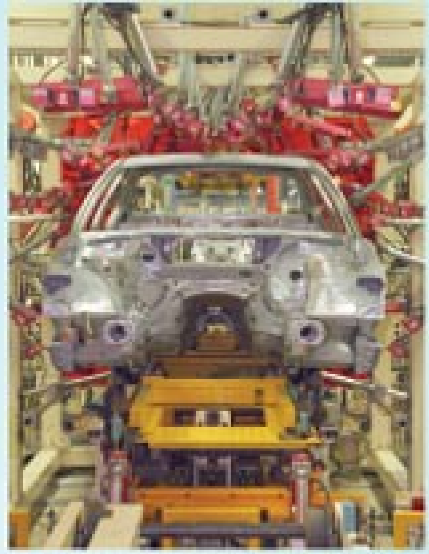
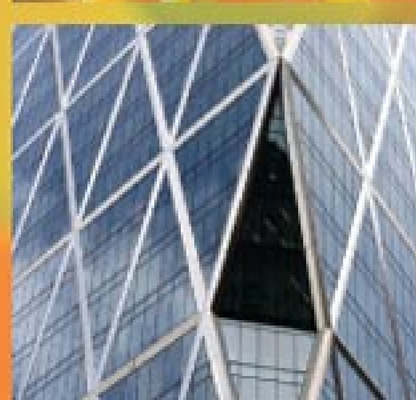
Trigonometry for Engineering Technology

With Mechanical, Civil, and Architectural Applications





Gary Powers

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Trigonometry for Engineering Technology

With Mechanical, Civil, and Architectural Applications

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Preface

This text was developed for use in a Geometry and Trigonometry course for Engineering Technology students. Many technical mathematics textbooks also include all of the math and algebra topics in addition to geometry and trigonometry. In most cases, the students have already had one or more algebra courses, and therefore do not need another expensive textbook with these topics. This text, therefore, is intended to be a low-cost alternative to the all-inclusive textbooks.

At my school we use this text for students who are preparing for careers in the mechanical, architectural, and civil engineering fields. Practice problems from each of these fields are included so that the students can see real-world examples of how the trigonometry functions are used.

This text assumes that the students have solid algebra skills for solving simple equations. In addition, it is assumed that the students have a working knowledge of the Cartesian coordinate system.

The examples in this text are fully-worked problems to illustrate solutions for the students.

The exercises are problems for the students to solve. Answers and solutions to these exercises are provided in the Appendix at the end of the text.

Gary Powers
September 2012

Chapter 1: Introduction

Why Is Trigonometry Important?

You will encounter applications of trigonometry in most engineering fields. This text provides examples of how trigonometry is applied in the mechanical, architectural, and civil fields. Some topics in physics, specifically mechanics, also use trigonometry to solve problems.

If you learn how to use CAD programs (such as AutoCAD), you may find that the software can relieve you of performing trigonometry calculations. You should understand, however, what the software is doing, and be able to do the same calculations manually.

Types of Trigonometry

This text covers the topic of right angle trigonometry. Chapters 12 and 13 also discuss the Laws of Sines and Cosines, which are applied to oblique triangles.

The topic of spherical trigonometry is not presented in this text. We will apply trigonometry only to plane, or flat, surfaces. Spherical trigonometry is applied to spherical surfaces such as the surface of the earth, as shown in Figure 1-1.

Distances between points on the surface of the earth may be calculated using spherical trigonometry. For example, the distance traveled by a ship or aircraft between Boston and Lisbon can be calculated. Additionally, the direction of travel between these two locations may be calculated using spherical trigonometry.

Star maps also use the principles of spherical trigonometry. Stars appear to be located on a "celestial sphere." Their locations in the sky may be described using methods similar to locations on the surface of the earth.

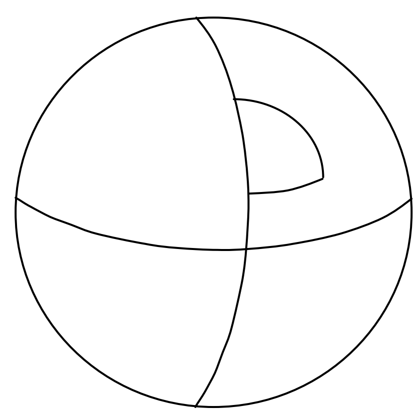


Figure 1-1: Application of spherical trigonometry

Examples of Applications

Mechanical Problem

A metal plate with a bolt-hole circle is shown in Figure 1-2. You have the task of determining the *X* and *Y* coordinates of the centers of each hole. These *X* and *Y* coordinates can then be used in a CNC (computer numerical control) program so that the holes can be drilled on a CNC milling machine.

Trigonometry is used to determine the *X* and *Y* locations of the center of each hole. To perform this calculation, you must know the radius of the bolt-hole circle and the angle to the center of each hole on the circle.

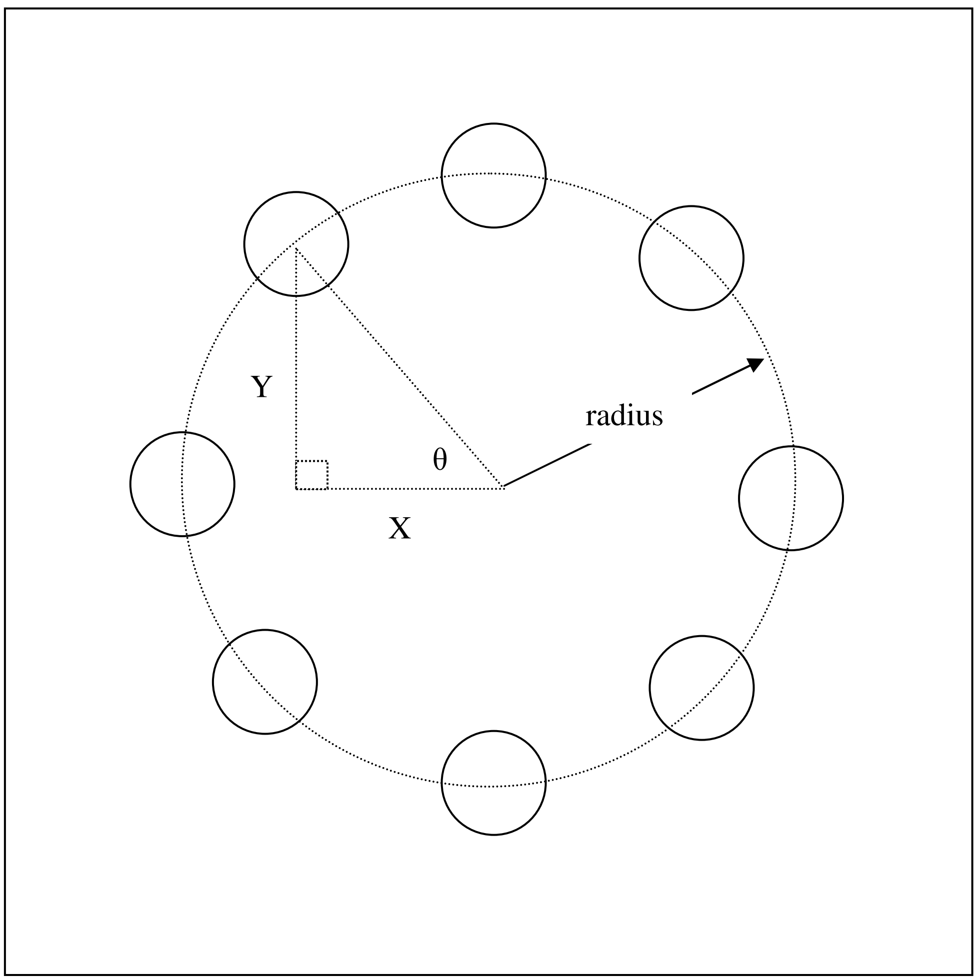


Figure 1-2: Metal plate with a bolt-hole circle

Examples of Applications, continued

Architectural Problem

A sketch of a building and its roof line is shown in Figure 1-3. This sketch represents the side view of a hip roof, which slopes downward towards each wall of a building. The triangle labeled with x, y, and r represents part of the geometry of the roof.

In the sketch, r represents the length of the rafter. The angle, θ, represents the roof angle. The x and y dimensions represent the horizontal and vertical dimensions for that part of the hip roof.

Given the roof angle, trigonometry is used to solve for the length of the unknown dimensions, x, y, and r.

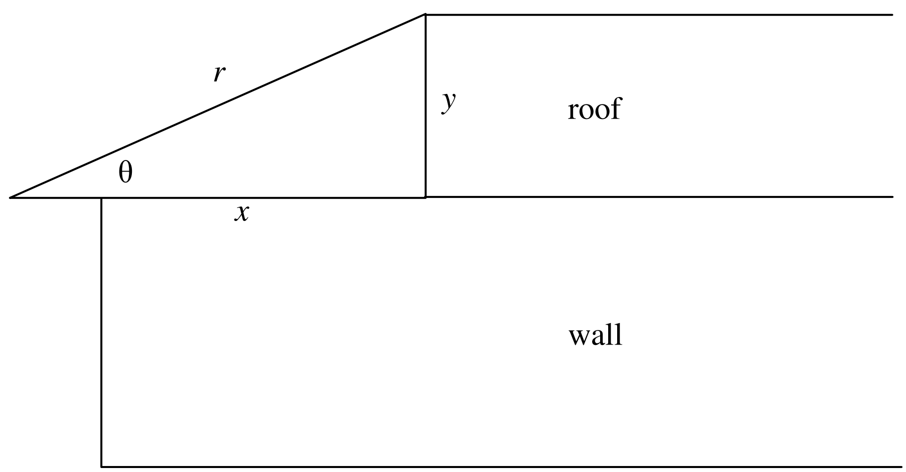


Figure 1-3: Roof line of a building

Examples of Applications, continued

Civil Problem

During a survey, a survey crew measured the slope distance and determined the elevation between the two points *A* and *B*, as shown on the sketch in Figure 1-4. What is the angle of the slope, and what is the horizontal distance between the two points?

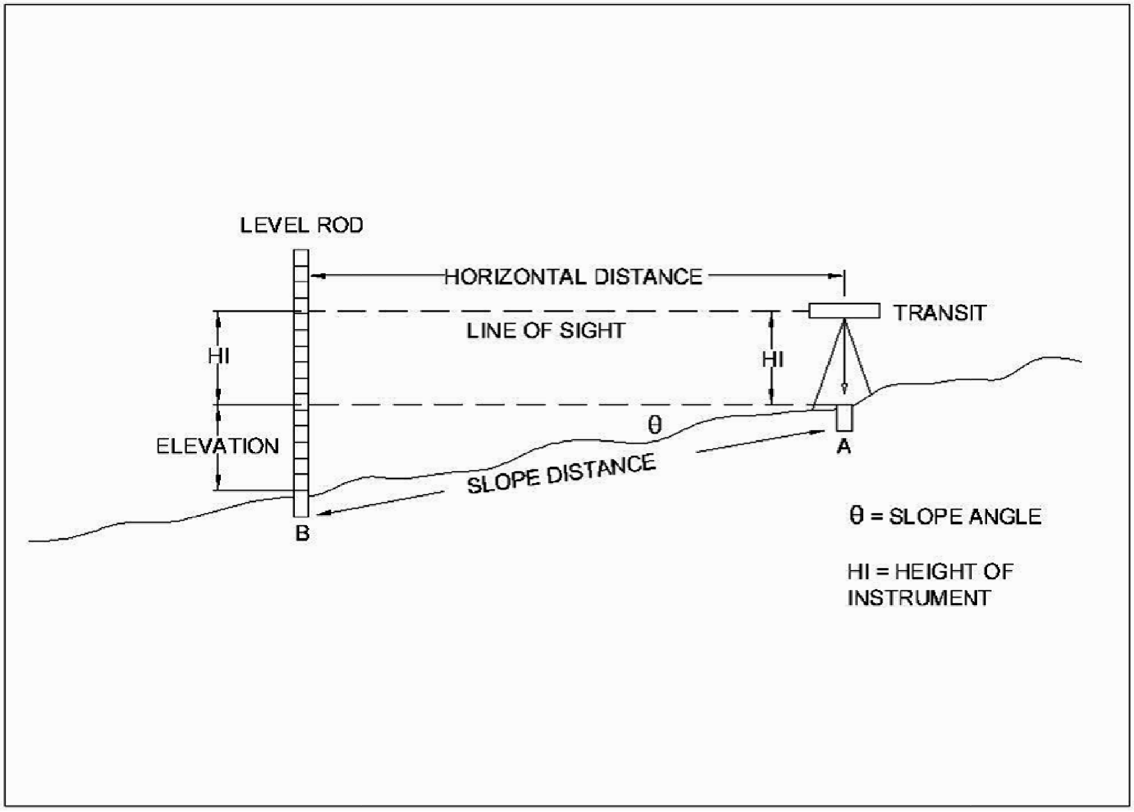


Figure 1-4: Surveying operation

Chapter 2: Angles

Introduction

You must be familiar with angles, how they are measured, and the units used to express angles. Trigonometry requires the use of angles in the problem solution.

What Is an Angle?

An angle expresses the difference in direction between two lines that share the same *vertex*, shown as point A in Figure 2-1. These lines, called *rays* or *vectors*, have a direction, as shown by the arrowhead.

Each vector is labeled using two points that appear on the vector. In Figure 2-1, the two vectors are named vector AB and vector AC. The first letter in the name is the vertex. The name of the vector, therefore, also communicates the direction of the vector.

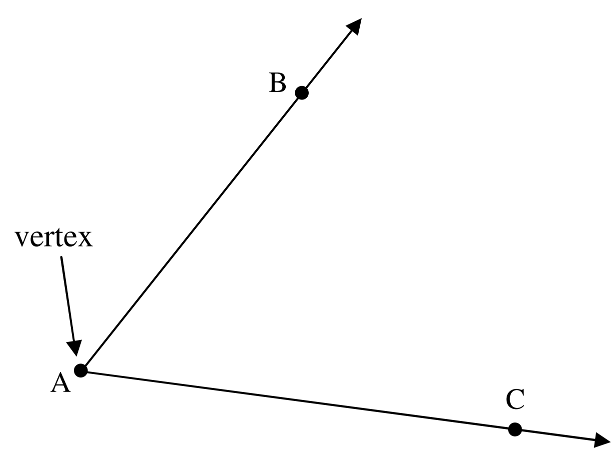


Figure 2-1: The angle between two rays

Angular Units

Angles are expressed in degrees. There are 360 degrees (or 360°) in a complete circle, as shown in Figure 2-2. The symbol for angular degrees, °, is the same symbol we use for temperature degrees.

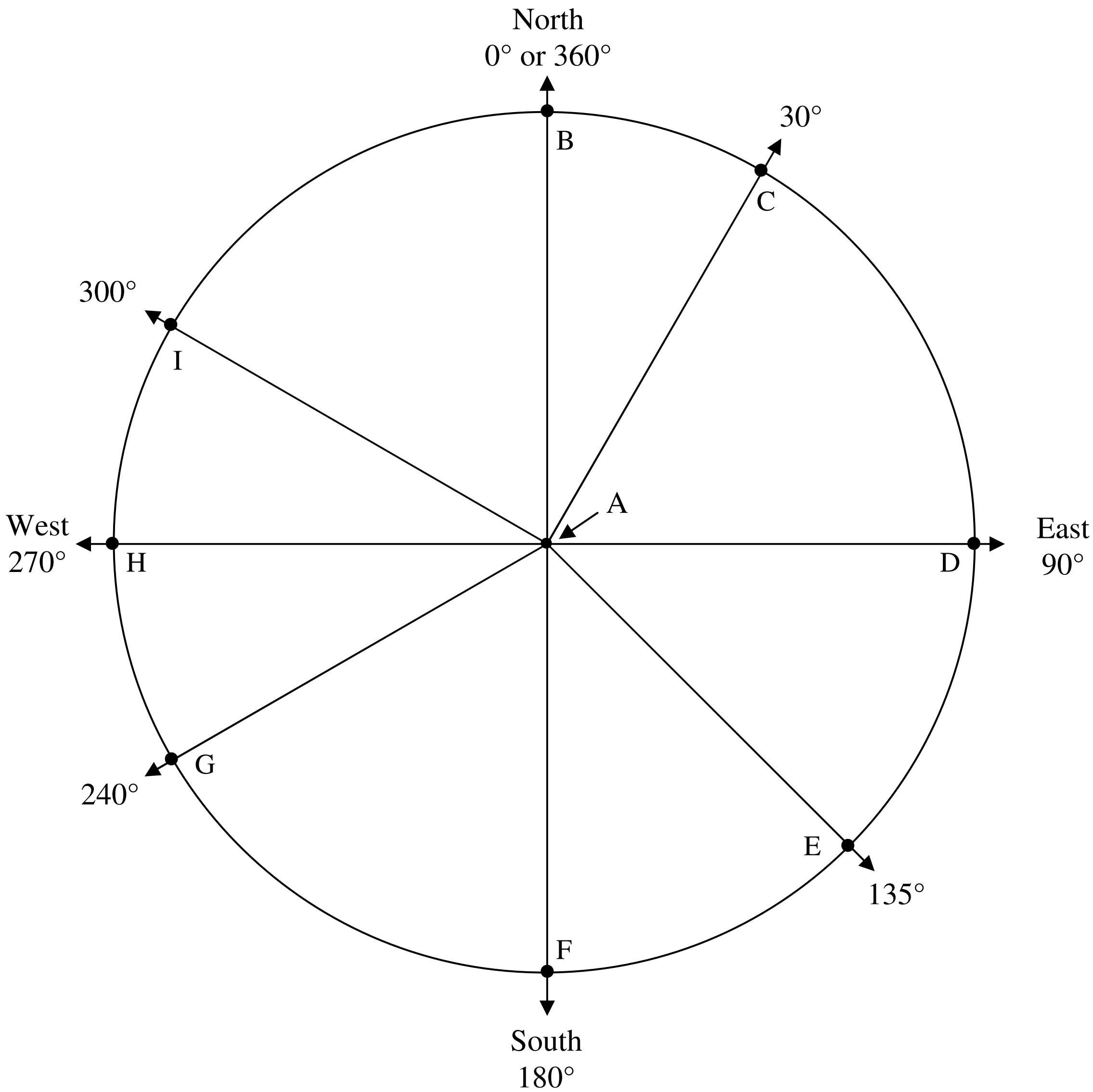


Figure 2-2: Bearing circle with directions

Example

1. Measuring in a clockwise direction, how many degrees are between the North direction (vector AB) and the East direction (vector AD)? See Figure 2-2.
   This angle = 90° – 0° = 90°

2. Measuring in a counterclockwise direction, what is the magnitude of the angle (number of degrees) between vector AG and the South direction (vector AF)?
   This angle = 240° – 180° = 60°

▲

Angular Units, continued

<u>Exercise 2-1 (see Figure 2-2)</u>

1. Measuring in a clockwise direction, how many degrees are between vector AC and the West direction (vector AH)?

2. Measuring in a counterclockwise direction, what is the magnitude of the angle between the East direction (vector AD) and vector AI?

3. Measuring in a clockwise direction, how many degrees are between vector AE and vector AG?

4. Measuring in a counterclockwise direction, what is the magnitude of the angle between the North direction (vector AB) and vector AG?

5. Measuring in a clockwise direction, how many degrees are between vector AI and vector AC?

6. Measuring in a counterclockwise direction, what is the magnitude of the angle between the South direction (vector AF) and vector AE?

Formats for Expressing Degrees

Decimal Degrees

If an angle is not a whole number of degrees, it can be expressed as a decimal. For example, an angle of 47 and one-half degrees can be written as 47.5°.

Fractional Degrees

Partial degrees can also be expressed as fractions. Using the previous example, an angle of 47 and one-half degrees can be written as 47 ½ °.

Degrees, Minutes, and Seconds

Partial degrees can also be expressed in minutes and seconds. This approach is similar to how we express partial hours of time in minutes and seconds.

One degree is divided into 60 minutes and one minute is divided into 60 seconds.

Using the previous example, an angle of 47 and one-half degrees can be written as 47 degrees and 30 minutes. (Remember: one degree is 60 minutes, so one-half degree is 30 minutes.) This amount is typically expressed as 47°30′.

The symbol for minutes is ′ (similar to what we use as a symbol for feet).

The symbol for seconds is ″ (similar to what we use as a symbol for inches).

Naming Angles

In formulas and problem statements, angles are identified by several different types of notations. Here are several notations that you may see. Refer to Figure 2-3.

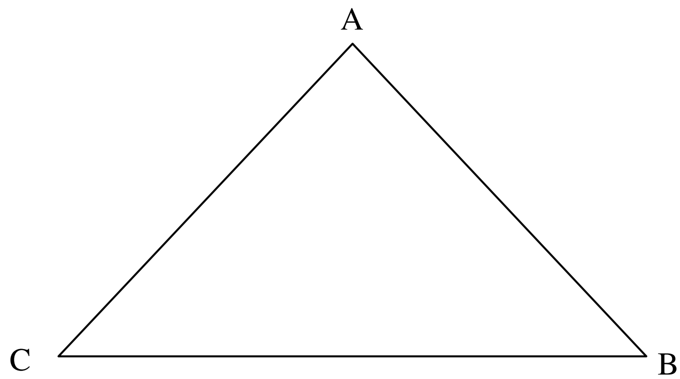


Figure: 2-3: Naming angles in a triangle

In this figure, the letters *A*, *B*, and *C* identify each vertex, or point, where two sides come together.

Pairs of letters are used to identify the *sides* of the triangle. This triangle has three sides named AB, BC, and CA. The sides may also be named BA, CB, and AC.

This triangle has three *internal angles*. They can be named using only the letter of the vertex and the angle symbol, ∠. This triangle has internal angles ∠A, ∠B, and ∠C.

Angle *A*, or ∠A, is the internal angle between sides BA and CA.

Angles may also be named using three letters, with the middle letter being the vertex of the angle. For example, ∠A may also be named ∠CAB or ∠BAC. Either name is acceptable.

Exercise 2-2

1. Referring to Figure 2-4, name all the vertices and sides.

2. What are all of the acceptable ways to name the interior angle, from Figure 2-4, which is formed between sides DE and DF? You should be able to list three different names for this angle.

3. What are all of the acceptable ways to name the interior angle, from Figure 2-4, which is formed between sides EF and DF? You should be able to list three different names for this angle.

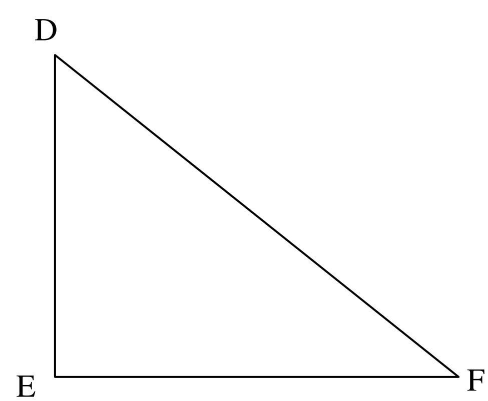


Figure 2-4: Naming angles in a triangle

Naming Angles, continued

An additional method for naming angles, which we will use frequently, uses Greek symbols rather than the letters that identify each vertex.

Greek symbols are used extensively in science and engineering. Table 2-1 lists some of the commonly used symbols.

Table 2-1: Commonly used Greek symbols

Symbol		English Name	Common Use
Upper Case	Lower Case		
Α	α	Alpha	angle, coefficient of thermal expansion
Β	β	Beta	angle
Δ	δ	Delta	symbol for change
Θ	θ	Theta	angle
Λ	λ	Lambda	physics
Π	π	Pi	math constant
Ρ	ρ	Rho	mass density
Σ	σ	Sigma	statistics
Φ	φ	Phi	angle
Ω	ω	Omega	physics, electricity

In the triangle in Figure 2-5, ∠A may also be named θ, by placing this symbol inside the triangle near the vertex *A*.

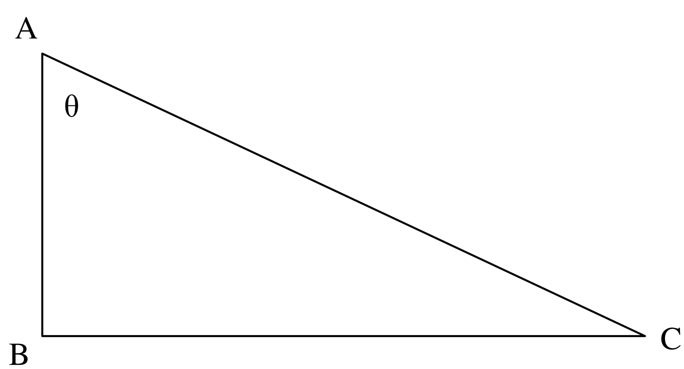


Figure 2-5: Angle identified with a Greek symbol

Measuring Angles

Angles can be measured with a protractor. The accuracy is only to the nearest whole or half degree with an inexpensive plastic protractor. However, this will serve our purposes for learning angle measurement.

Protractors have two scales, and it is easy to use the wrong scale. In Figure 2-6, we see that one scale begins at 0° on the right side and the measurement increases counterclockwise. The other scale begins at 0° on the left side and increases in a clockwise direction. Your choice of which scale to use depends on whether you are measuring an angle in a clockwise or counterclockwise direction.

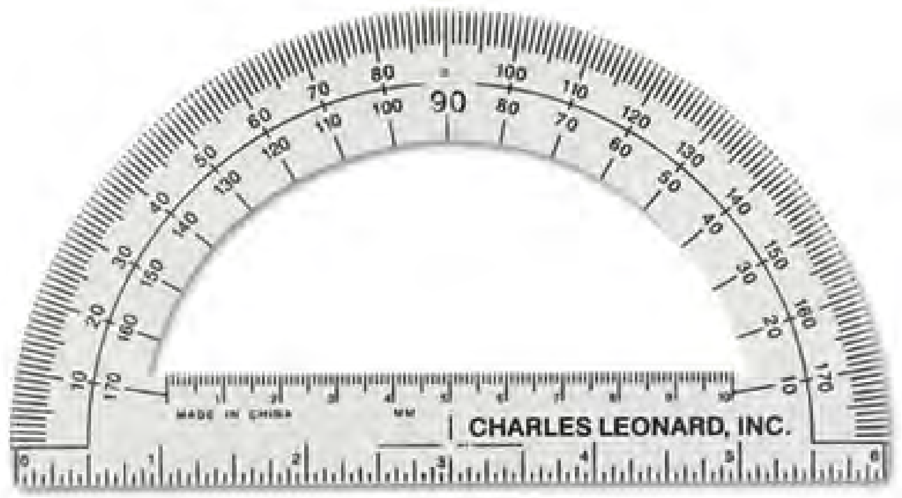


Figure 2-6: Degree scales on a protractor

Example

Measure the angle between the two vectors in Figure 2-7. Your answer should be approximately 64°, read on the inner scale. Your result could be between 63° and 65° degrees, depending on how well you lined up the protractor with the vectors.

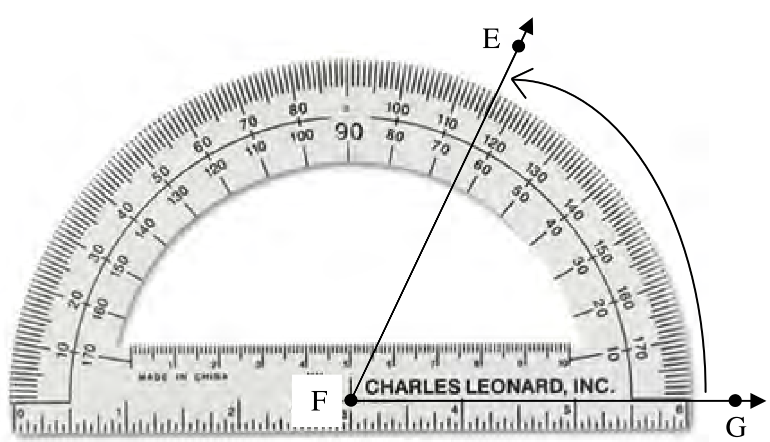


Figure 2-7:Measuring an angle

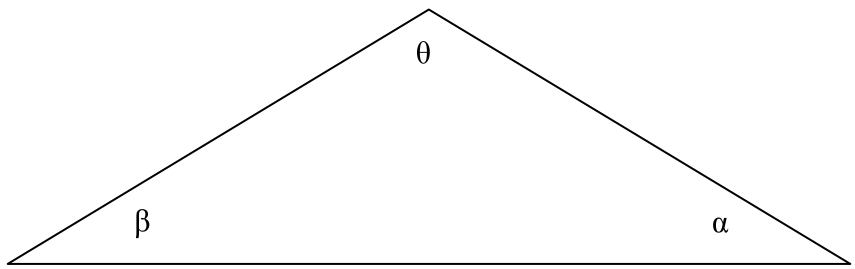
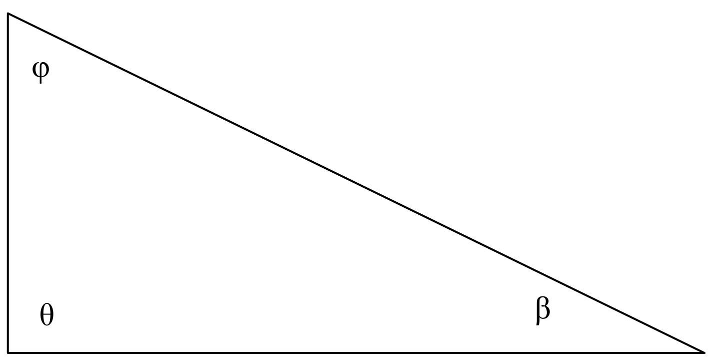
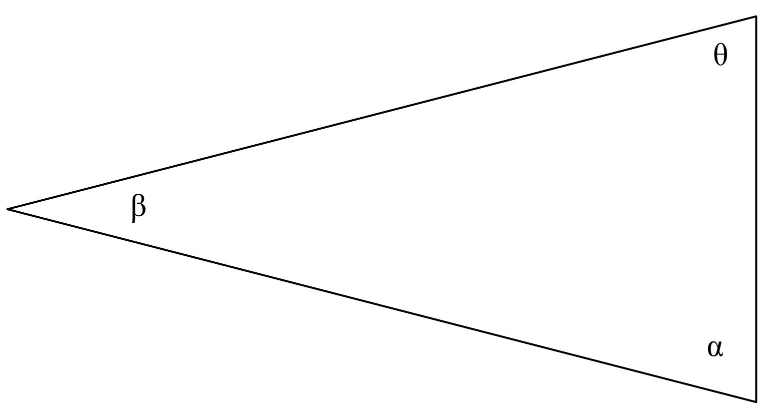
Figure 2-7 shows a counterclockwise measurement with vector FG lined up with 0°. Using the outer scale on the protractor gives an incorrect answer of approximately 116°.

This measurement may also be done in a clockwise direction. You can line up vector FE with 0°on the left side of the protractor. Then, read the angle at the point where vector FG intersects the outer scale. ▲

Measuring Angles, continued

<u>Exercise 2-3</u>

You can also measure angles which are inside a closed figure, such as a triangle. Measure each of these interior angles with a protractor. You may check your answers in Appendix 1. Be sure to include the degree symbol ° with your answers. Express your answers to the nearest whole degree.

	Your measurements
1. 	θ ________
	α ________
	β ________
2. 	θ ________
	φ ________
	β ________
3. 	θ ________
	α ________
	β ________

Using Your Calculator

You can use a number of good, affordable scientific calculators to perform trigonometry calculations. We will focus on two popular models: the Texas Instruments TI-30XA and the TI-30XIIS. We will start with the TI-30XA, shown in Figure 2-8.

The first step is to make sure you are using the correct angular *units*. Units can be in *degrees*, *radians*, or *grads*. In this text, we will usually work in degrees. Radians are featured in Chapter 11, *Graphing Trigonometry Functions*.

Use the DRG key to select the units. As you push the DRG key, you will see the units change at the top of the display. Make sure that the units are in DEG for degrees. You can toggle between degrees/minutes/seconds DMS and decimal degrees DD, by pressing the 2nd key and the appropriate key shown in Figure 2-8.

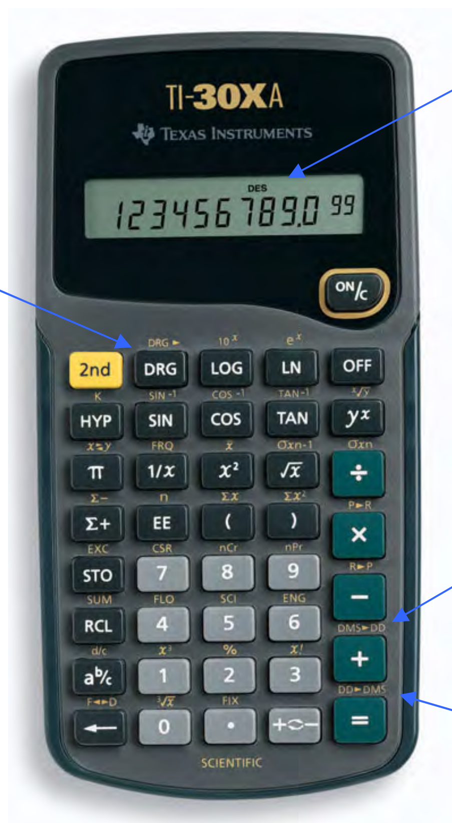


Figure 2-8: Texas Instruments TI-30XA

Reference

For a complete instruction manual on the TI 30XA calculator, visit this Internet site:

http://education.ti.com/educationportal/sites/US/productDetail/us_ti30xa.html?bid=6

Entering and Converting Degrees with the TI-30XA

Entering an angle into the calculator in degrees is an important operation. The angle can initially be entered as either decimal degrees or degrees/minutes/seconds.

Example

Enter an angle in degrees/minutes/seconds (DMS), and then convert to decimal degrees (DD).

- Enter 37°45′12″ into the calculator by typing *37.4512*.
- Next, convert this angle to decimal degrees by using the keys $\boxed{2^{nd}}$ $\boxed{\text{DMS} \blacktriangleright \text{DD}}$.
- The display then changes to 37.75333333, which is the decimal equivalent of 37°45′12″.
- This conversion to decimal degrees is necessary before performing further calculations. ▲

Example

- Enter 15°07′53″ into the calculator by typing 15.0753. (Note that the minutes take *two decimal places*, even if the minutes are less than 10. The same is true of seconds.)
- Next, convert this angle to decimal degrees by using the keys $\boxed{2^{nd}}$ $\boxed{\text{DMS} \blacktriangleright \text{DD}}$.
- The display then changes to 15.13138889, which is the decimal equivalent of 15°07′53″. ▲

All degrees used in a calculation *must be* in decimal degrees. Therefore, you must convert an angle in DMS to DD after entering it into the calculator.

Example

Convert an angle from decimal degrees (DD) to degrees-minutes-seconds (DMS).

Converting an angle from DD to DMS may be necessary if you have performed a calculation, but the answer is required to be in DMS format.

- Enter 43.98° into the calculator by typing 43.98.
- Next, convert this angle to DMS format by using the keys $\boxed{2^{nd}}$ $\boxed{\text{DD} \blacktriangleright \text{DMS}}$.
- The display then changes to 43°58′48″, which is equivalent to 43.98°.
- Convert this DMS format angle back to DD by using the keys $\boxed{2^{nd}}$ $\boxed{\text{DMS} \blacktriangleright \text{DD}}$. ▲

Example

- Enter 121.34° into the calculator by typing 121.34.
- Next, convert this angle to DMS format by using the keys $\boxed{2^{nd}}$ $\boxed{\text{DD} \blacktriangleright \text{DMS}}$.
- What is on the display in DMS format? ________________
  (The display should read 121°20′24″)
- Convert this DMS format angle back to DD by using the keys $\boxed{2^{nd}}$ $\boxed{\text{DMS} \blacktriangleright \text{DD}}$. ▲

Entering and Converting Degrees with the TI-30XIIS

See Figure 2-9 below.

DRG Key

Pressing the DRG key displays DEG RAD GRD. The dashed underline shows which mode is selected. Using the right arrow key, select the DEG mode, and then press ENTER.

Figure 2-9: Texas Instruments TI-30XIIS

Entering and Converting Degrees in the TI-30XIIS, continued

Entering an Angle into the Calculator

One important operation is entering an angle into the calculator in degrees. The angle can be entered as decimal degrees or degrees/minutes/seconds.

Example

Enter an angle in degrees-minutes-seconds (DMS) and convert to decimal degrees (DD).

Enter 37°45′12″.

- Enter 37, then press the angle units key, [° ′ ″].
- Use the right [▶] or left [◀] arrow key to select the degree symbol, then press [ENTER].
- Enter 45, then press the angle units key, [° ′ ″].
- Use the right [▶] or left [◀] arrow key to select the minutes symbol, then press [ENTER].
- Enter 12, then press the angle units key, [° ′ ″].
- Use the right [▶] or left [◀] arrow key to select the seconds symbol, then press [ENTER].
- Press [ENTER] again to see the angle in decimal format: 37.75333333. ▲

Example

Enter an angle in degrees-minutes-seconds (DMS) and convert to decimal degrees (DD).

Enter 15°07′53″.

- Enter 15, then press the angle units key, [° ′ ″].
- Use the right [▶] or left [◀] arrow key to select the degree symbol, then press [ENTER].
- Enter 07, then press the angle units key, [° ′ ″].
- Use the right [▶] or left [◀] arrow key to select the minutes symbol, then press [ENTER].
- Enter 53, then press the angle units key, [° ′ ″].
- Use the right [▶] or left [◀] arrow key to select the seconds symbol, then press [ENTER].
- Press [ENTER] again to see the angle in decimal format: 15.13138889. ▲

Entering and Converting Degrees in the TI-30XIIS, continued

Converting an Angle from Decimal Degrees (DD) to Degrees-Minutes-Seconds (DMS)

You may want to convert an angle from DD to DMS. This may be necessary if you have performed a calculation, but the answer is required to be in DMS format.

Example

- Enter 43.98° into the calculator by typing 43.98.
- Next, select the units key, ° ′ ″.
- Use the right ▶ key until ▶DMS is displayed.
- Press ENTER twice.
- Observe the DMS format displayed as 43°58′48″.
- Convert this DMS format angle back to DD by selecting the units key, ° ′ ″.
- Use the right ▶ or left ◀ arrow key to select the degree symbol, then press ENTER.
- Press ENTER twice and observe the angle displayed as 43.98. ▲

Example

- Enter 121.34° into the calculator by typing 121.34.
- Next, select the units key, ° ′ ″.
- Use the right ▶ key until ▶DMS is displayed.
- Press ENTER twice.
- Observe the DMS format displayed as 121°20′24″.
- Convert this DMS format angle back to DD by selecting the units key, ° ′ ″.
- Use the right ▶ or left ◀ arrow key to select the degree symbol, then press ENTER.
- Press ENTER twice and observe the angle displayed as 121.34. ▲

Reference

For a complete instruction manual on the TI 30XIIS calculator, visit this Internet site:

http://education.ti.com/educationportal/sites/US/productDetail/us_ti30x_iis.html?bid=6

Adding and Subtracting Angles

You may need to add or subtract angles as part of problem-solving. Adding or subtracting whole degrees is the same as adding numbers. When the angles are expressed in minutes and seconds, however, the operation may be more complicated. Here are some examples.

Example

This is a simple example that is similar to any addition operation.

$$
\begin{array}{rrrr}
 & 27^\circ & 20' & 10'' \\
+ & 15^\circ & 09' & 33'' \\
\hline
 & 42^\circ & 29' & 43''
\end{array}
$$

▲

Example

This example requires some additional steps.

$$
\begin{array}{rrrr}
 & 41^\circ & 37' & 24'' \\
+ & 56^\circ & 46' & 41'' \\
\hline
 & 97^\circ & 83' & 65'' \\
 & & +1' & -60'' \\
\hline
 & 97^\circ & 84' & 05'' \\
 & +1^\circ & -60' & \\
\hline
 & 98^\circ & 24' & 05''
\end{array}
$$

Note: Because the seconds total is greater than 60, the seconds must be simplified by subtracting 60 seconds, and adding one minute to the minutes total.

Note: Because the minutes total is greater than 60, the minutes must be simplified by subtracting 60 minutes, and adding one degree to the degrees total.

▲

Example

This example shows what to do when the angles add up to more than 360°.

$$
\begin{array}{rrrr}
 & 183^\circ & 40' & 10'' \\
+ & 195^\circ & 15' & 25'' \\
\hline
 & 378^\circ & 55' & 35'' \\
- & 360^\circ & & \\
\hline
 & 18^\circ & 55' & 35''
\end{array}
$$

Note: Because one revolution is equal to 360°, you can subtract 360° when the total is more than one revolution.

▲

Adding and Subtracting Angles, continued

Example

This is a simple example that is similar to any subtraction operation.

$$\begin{array}{rrrr} & 27^\circ & 20' & 50'' \\ - & 15^\circ & 09' & 33'' \\ \hline & 12^\circ & 11' & 17'' \end{array}$$

▲

Example

This example requires some additional steps.

$$\begin{array}{rrrr} & 81^\circ & 57' & 24'' \\ - & 56^\circ & 46' & 41'' \\ \hline \end{array}$$

Note: Because 41″ cannot be subtracted from 24″ (and result in a positive answer), you must "borrow" one minute from the minutes column. The borrowed minute is added as 60″ to the seconds column.

$$\begin{array}{rrrr} & 81^\circ & 57' & 24'' \\ & & -1' & +60'' \\ \hline & 81^\circ & 56' & 84'' \\ - & 56^\circ & 46' & 41'' \\ \hline & 25^\circ & 10' & 43'' \end{array}$$

Rewrite the problem

▲

Example

This example illustrates borrowing in the degrees and minutes columns.

$$\begin{array}{rrrr} & 79^\circ & 32' & 29'' \\ - & 41^\circ & 47' & 53'' \\ \hline \end{array}$$

Note: This problem requires borrowing from both the degrees and minutes columns.

$$\begin{array}{rrrr} & 79^\circ & 32' & 29'' \\ & -1^\circ & +60' & \\ \hline & 78^\circ & 92' & 29'' \\ & & -1' & +60'' \\ \hline & 78^\circ & 91' & 89'' \\ - & 41^\circ & 47' & 53'' \\ \hline & 37^\circ & 44' & 36'' \end{array}$$

Rewrite the problem

▲

Adding and Subtracting Angles, continued

Exercise 2-4

Try these addition problems using two different methods. First, calculate the sum using pencil and paper. Then check your answers by performing the addition on your calculator.

1.
$$\begin{array}{rrrr} & 26^\circ & 10' & 17'' \\ + & 15^\circ & 24' & 28'' \\ \hline \end{array}$$

2.
$$\begin{array}{rrrr} & 119^\circ & 25' & 32'' \\ + & 57^\circ & 19' & 37'' \\ \hline \end{array}$$

3.
$$\begin{array}{rrrr} & 123^\circ & 40' & 27'' \\ + & 25^\circ & 34' & 15'' \\ \hline \end{array}$$

4.
$$\begin{array}{rrrr} & 29^\circ & 23' & 42'' \\ + & 45^\circ & 39' & 33'' \\ \hline \end{array}$$

5.
$$\begin{array}{rrrr} & 162^\circ & 50' & 26'' \\ + & 227^\circ & 8' & 38'' \\ \hline \end{array}$$

6.
$$\begin{array}{rrrr} & 0^\circ & 37' & 51'' \\ + & 0^\circ & 22' & 23'' \\ \hline \end{array}$$

7.
$$\begin{array}{rrrr} & 35^\circ & 21' & 12'' \\ & 43^\circ & 13' & 27'' \\ + & 52^\circ & 7' & 19'' \\ \hline \end{array}$$

8.
$$\begin{array}{rrrr} & 106^\circ & 32' & 6'' \\ & 72^\circ & 8' & 34'' \\ + & 13^\circ & 47' & 23'' \\ \hline \end{array}$$

Adding and Subtracting Angles, continued

Exercise 2-5

Try these subtraction problems using two different methods. First, calculate the result using pencil and paper. Then check your answers by performing the subtraction on your calculator.

1.
$$\begin{array}{rrrr} & 126^\circ & 53' & 41'' \\ - & 33^\circ & 29' & 24'' \\ \hline \end{array}$$

2.
$$\begin{array}{rrrr} & 37^\circ & 40' & 30'' \\ - & 25^\circ & 09' & 33'' \\ \hline \end{array}$$

3.
$$\begin{array}{rrrr} & 226^\circ & 20' & 50'' \\ - & 135^\circ & 49' & 33'' \\ \hline \end{array}$$

4.
$$\begin{array}{rrrr} & 78^\circ & 28' & 22'' \\ - & 54^\circ & 37' & 43'' \\ \hline \end{array}$$

5.
$$\begin{array}{rrrr} & 219^\circ & 31' & 17'' \\ - & 187^\circ & 31' & 39'' \\ \hline \end{array}$$

6.
$$\begin{array}{rrrr} & 136^\circ & 0' & 40'' \\ - & 49^\circ & 38' & 53'' \\ \hline \end{array}$$

Chapter 3: Triangles

Right Triangles

Right angle trigonometry is applied to *right triangles*. A right triangle has one interior angle that is exactly 90°. The other two interior angles are each between 0° and 90°. See Figure 3-1 for an example of a right triangle. The right angle is identified with a small square symbol.

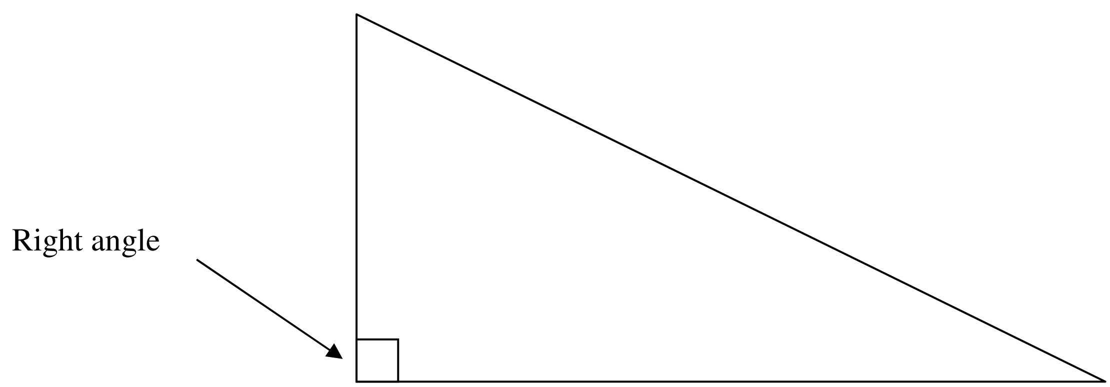
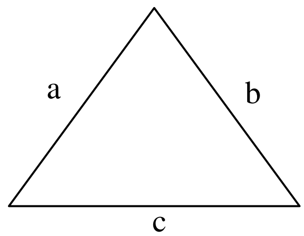
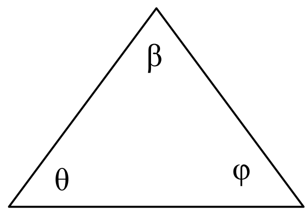
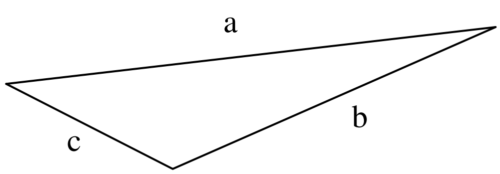
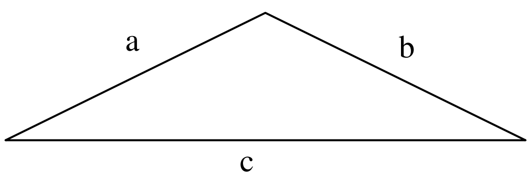
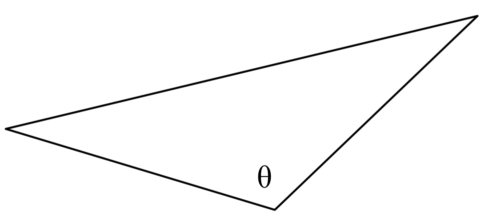
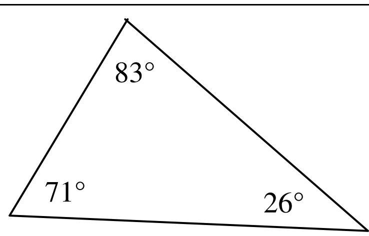


Figure 3-1: Right triangle

Other Types of Triangles

Other types of triangles are shown below. We will work with these triangles in Chapters 12 and 13 as part of the presentation of the Law of Sines and Law of Cosines.

	Equilateral triangle: All three sides are the same length. $a = b = c$
	Equiangular triangle: The three interior angles are equal. $\theta = \varphi = \beta$ Note: This triangle is also equilateral.
	Scalene triangle: The three sides have three different lengths. $a \neq b$ $a \neq c$ $b \neq c$
	Isosceles triangle: Two sides are of equal length. $a = b \neq c$
	Obtuse triangle: One interior angle is greater than 90°. angle $\theta > 90°$
	Acute triangle: All of the interior angles are less that 90°. Note: Obtuse and acute triangles are both types of *oblique* triangles. An oblique triangle does not include a 90° angle.

The Sum of Interior Angles

In solving trigonometry problems, one tool you will use is knowing the sum of interior angles in a *polygon*. A polygon is a *closed figure* with three or more *straight sides*. A triangle is the simplest polygon.

For all polygons, the sum of the interior angles is calculated using this formula:

Σ interior angles = (n – 2) (180°), where n is the number of sides

Note: The Greek symbol Σ is used to stand for *the sum of*.

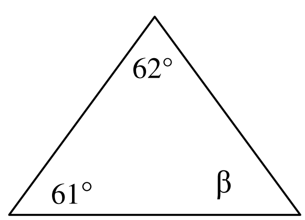
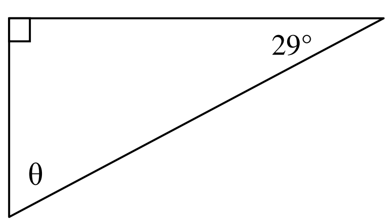
Therefore, in a triangle which has three sides:

$$\Sigma \text{ interior angles} = (3 - 2)\,(180^\circ) = 180^\circ$$

The sum of a triangle's interior angles is 180°.

Example

Using this fact about the sum of interior angles, calculate the following unknown angles:

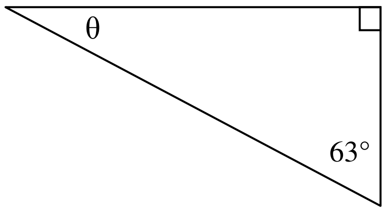
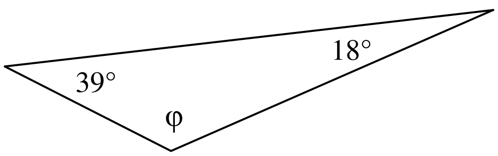
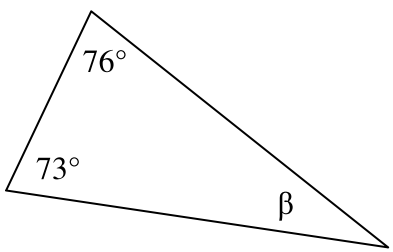
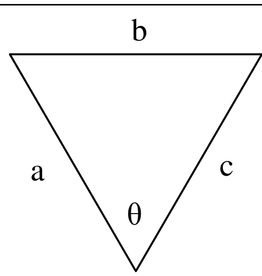
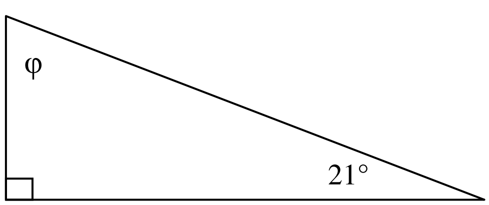
	Find angle β $\beta = 180^\circ - 62^\circ - 61^\circ = 57^\circ$
	Find angle θ The angle with the square is a right angle and measures 90°. $\theta = 180^\circ - 90^\circ - 29^\circ = 61^\circ$

▲

The Sum of Interior Angles, continued

<u>Exercise 3-1</u>

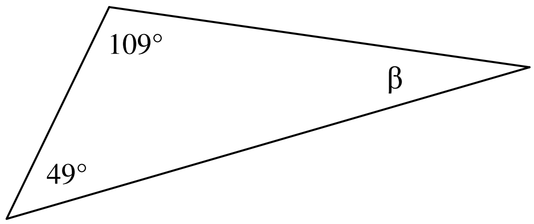
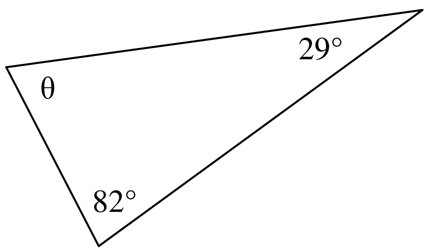
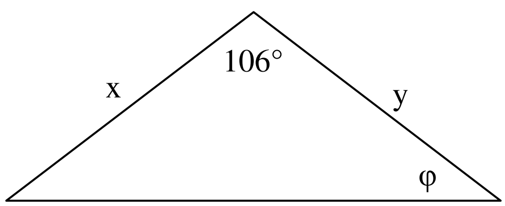
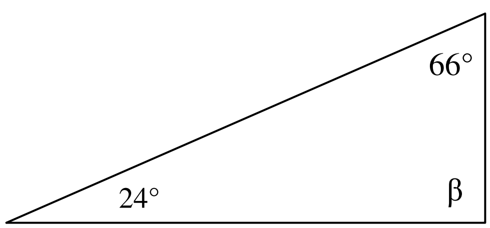
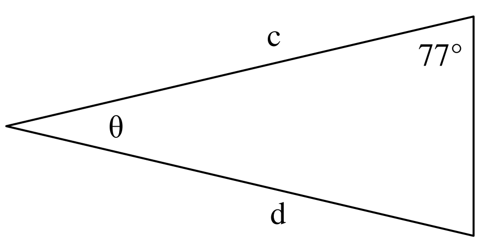
Using your knowledge about the sum of interior angles, calculate the following unknown angles:

1. 	Find angle θ.
2. 	Find angle φ.
3. 	Find angle β.
4. 	Sides a, b, and c are of equal length. Find angle θ.
5. 	Find angle φ.

The Sum of Interior Angles, continued

<u>Exercise 3-2</u>

Using your knowledge about the sum of interior angles, calculate the following unknown angles:

1. 	Find angle β.
2. 	Find angle θ.
3. 	Sides x and y are of equal length. Find angle φ.
4. 	Find angle β.
5. 	Sides c and d are of equal length. Find angle θ.

Pythagorean Theorem

Before we learn the trigonometry functions, let's look at one more tool that you can use to solve right triangle problems.

The *Pythagorean Theorem* is used to solve for the length of an unknown side in a *right triangle* only. You must know the length of the other two sides to use this theorem.

Referring to Figure 3-2, the Pythagorean Theorem formula is:

$$\boxed{c^2 = a^2 + b^2}$$

Side c is the side opposite the right angle and is called the *hypotenuse*. Sides a and b are the *legs*.

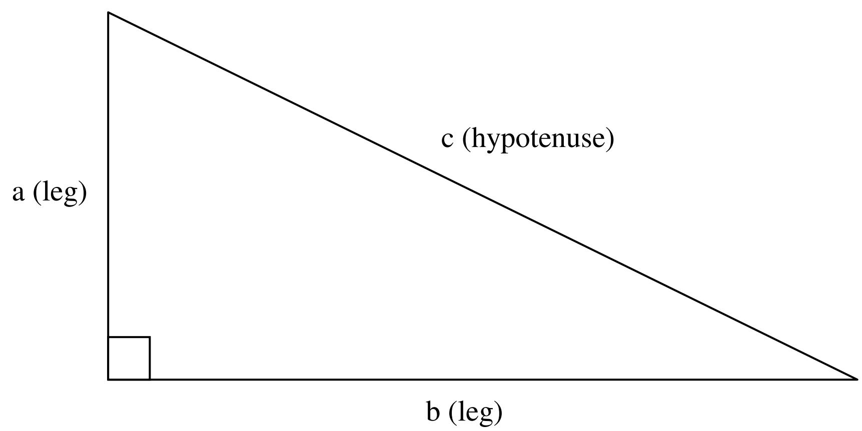


Figure 3-2: Right triangle

Example

In Figure 3-2, if side a has a length of 5.25 inches and side b has a length of 8.85 inches, how long is the hypotenuse, side c? If $c^2 = a^2 + b^2$, then

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{(5.25)^2 + (8.85)^2} = 10.29 \text{ inches}$$

▲

Pythagorean Theorem, continued

Example

In Figure 3-3, if side a has a length of 4.5 cm and side c has a length of 10.5 cm, what is the length of side b?

If $c^2 = a^2 + b^2$, then

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{(10.5)^2 - (4.5)^2} = 9.5 \text{ cm}$$

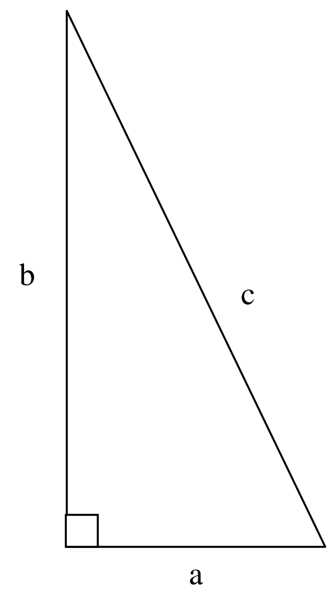


Figure 3-3: Right triangle

▲

Example

In Figure 3-4, if side c has a length of 7.50 inches and side b has a length of 6.71 inches, what is the length of side a?

If $c^2 = a^2 + b^2$, then

$$a = \sqrt{c^2 - b^2}$$

$$a = \sqrt{(7.50)^2 - (6.71)^2} = 3.35 \text{ inches}$$

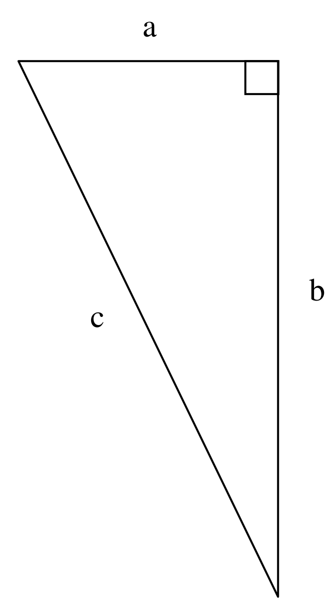


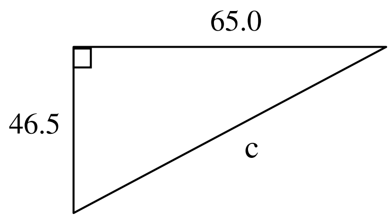
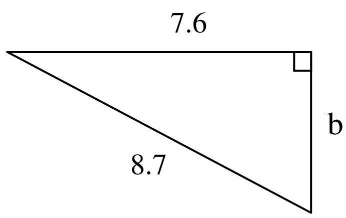
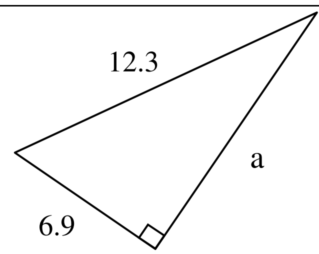
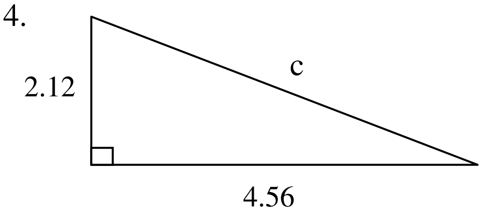
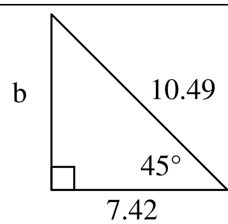
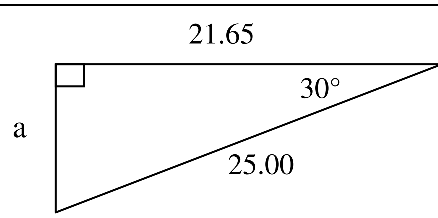
Figure 3-4: Right triangle

▲

Pythagorean Theorem, continued

<u>Exercise 3-3</u>

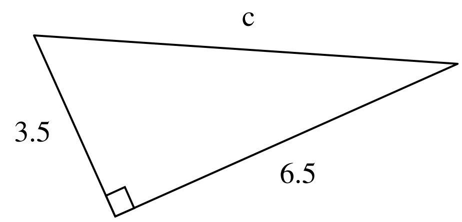
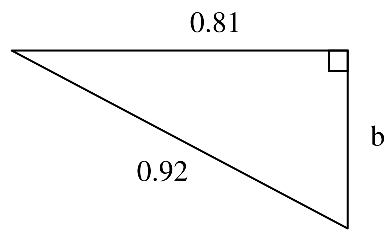
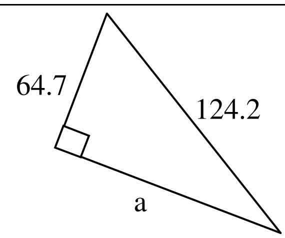
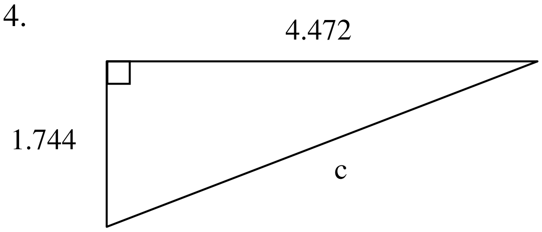
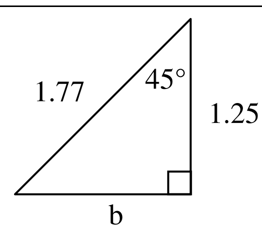
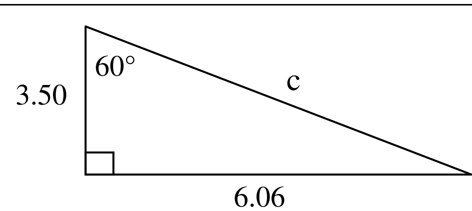
Solve for the unknown sides in these triangles. You will need to manipulate the basic Pythagorean Theorem formula to solve for the unknowns. Make an *estimate* before performing the calculation. Your estimate will help you to decide if your answer is reasonable.

Your estimate of side c	1. 	Solve for side c.
Your estimate of side b	2. 	Solve for side b.
Your estimate of side a	3. 	Solve for side a.
Your estimate of side c	4. 	Solve for side c.
Your estimate of side b	5. 	Solve for side b. Note the result you get with a 45° internal angle.
Your estimate of side a	6. 	Solve for side a. Note the result you get with a 30° internal angle.

Pythagorean Theorem, continued

Exercise 3-4

Solve for the unknown sides in these triangles. You will need to manipulate the basic Pythagorean Theorem formula to solve for the unknowns. Make an estimate before performing the calculation.

Your estimate of side c	1. 	Solve for side c.
Your estimate of side b	2. 	Solve for side b.
Your estimate of side a	3. 	Solve for side a.
Your estimate of side c	4. 	Solve for side c.
Your estimate of side b	5. 	Solve for side b. Note the result you get with a 45° internal angle.
Your estimate of side c	6. 	Solve for side c Note the result you get with a 60° internal angle.

Chapter 4: Preparation for Right Angle Trigonometry

Similar Triangles

Right angle trigonometry is based on the fact that *similar* triangles have *proportional* sides. What does that mean? Let's first look at similar triangles.

When two triangles are similar, their internal angles are the same, but the triangles may be of different sizes. In Figure 4-1, the two triangles are similar, but triangle DEF is larger than triangle ABC. These two triangles have the same *shape*.

> Note: These triangles are named by using the *vertex* labels. For example, ABC refers to the vertices of triangle ABC.

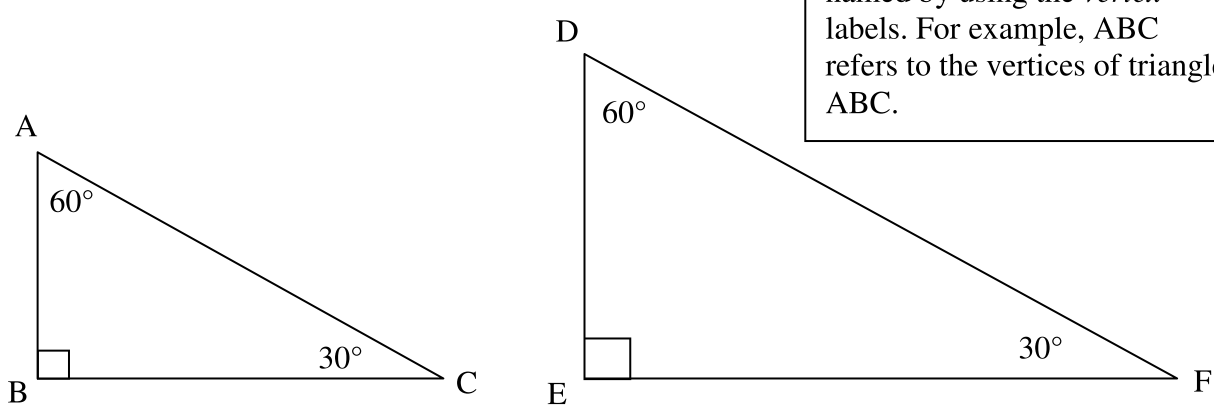


Figure 4-1: Similar right triangles

Corresponding Sides

Similar triangles have *corresponding sides*.

Corresponding sides appear in the *same position* in similar triangles, as seen in Figure 4-2.

Example

In Figure 4-2, side AB in triangle ABC corresponds with side DE in triangle DEF. Both of these sides are opposite the 30° angle in each triangle.

Side FE in triangle DEF corresponds with side CB in triangle ABC. Both of these sides are opposite the 60° angle in each triangle.

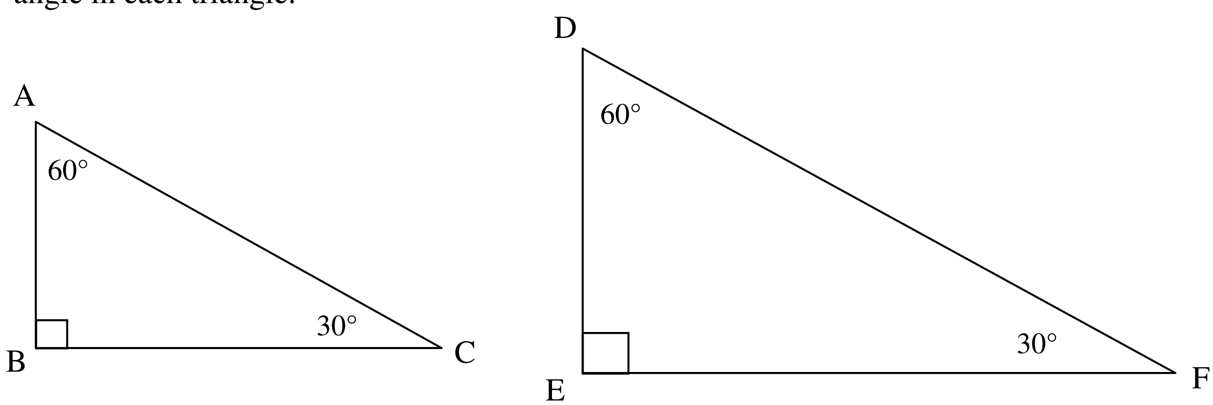


Figure 4-2: Corresponding sides

▲

Exercise 4-1:

See Figure 4-3. Assume that these two triangles are similar.

1. Which side in triangle MNO corresponds with side PR in triangle PQR? ____________ Why?

2. Which side in triangle PQR corresponds with side MN in triangle MNO? ___________ Why?

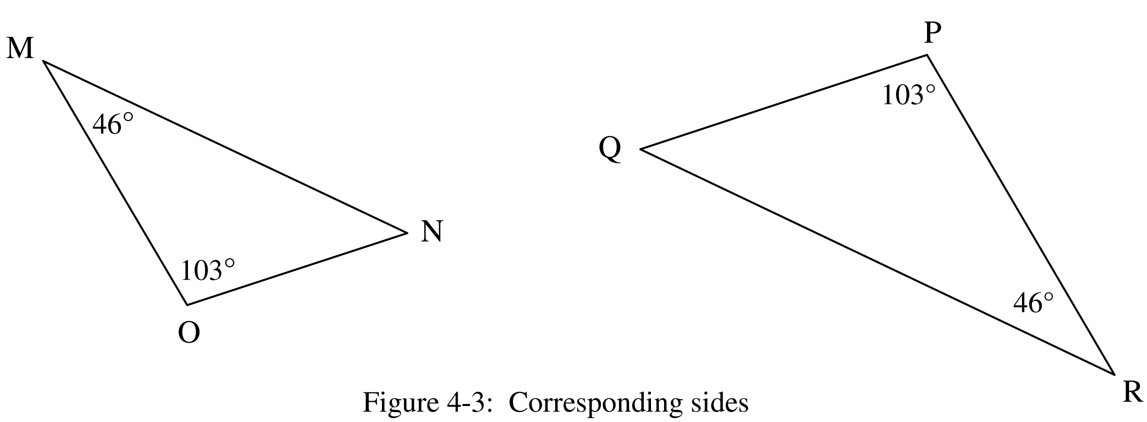


Figure 4-3: Corresponding sides

Corresponding Sides, continued

Exercise 4-2:

See Figure 4-4. Assume that these two triangles are similar.

1. Which side in triangle RST corresponds with side XY in triangle XYZ? ____________ Why?
2. Which side in triangle XYZ corresponds with side ST in triangle RST? ____________ Why?

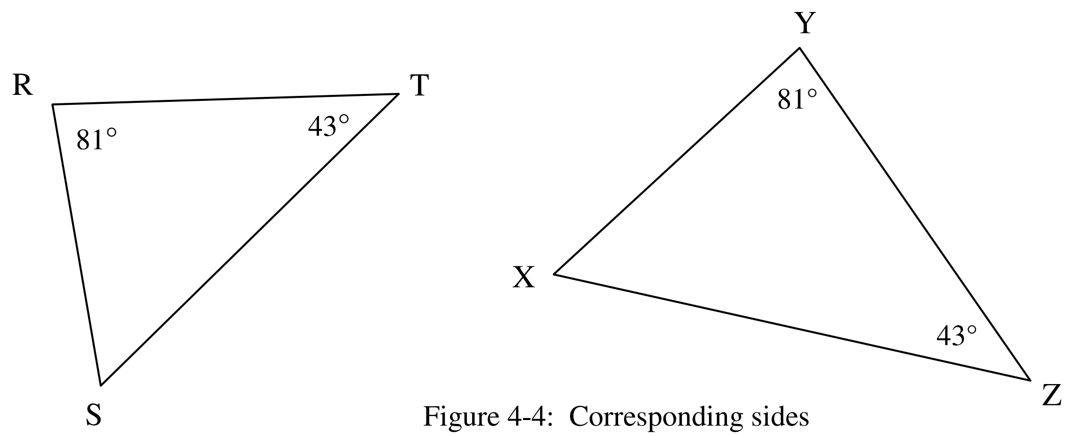


Figure 4-4: Corresponding sides

Exercise 4-3:

1. See Figure 4-5 below. Are these two triangles similar? Explain your answer.
2. Which side in triangle TUV corresponds with side JK in triangle JKL? ____________ Why?
3. Which side in triangle JKL corresponds with side TU in triangle TUV? ____________ Why?

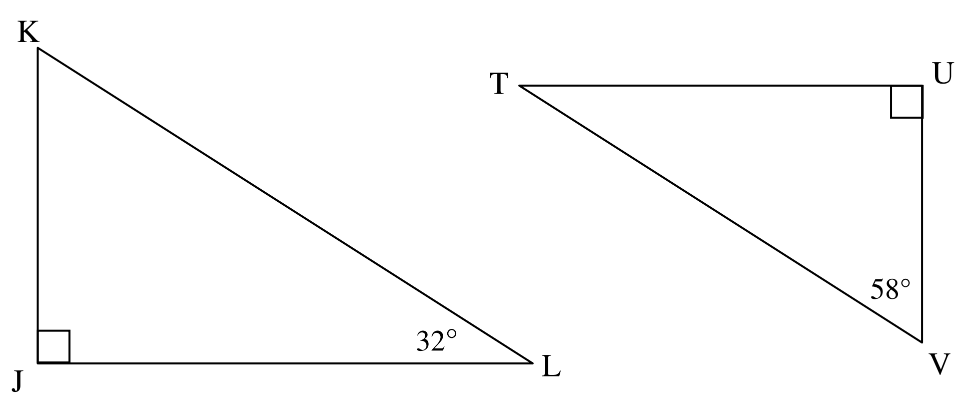


Figure 4-5: Corresponding sides

<u>Exercise 4-4</u>:

1. See Figure 4-6 below. Are these two triangles similar? Explain your answer.

2. Which side in triangle DEF corresponds with side GH in triangle GHI? ____________ Why?

3. Which side in triangle GHI corresponds with side DF in triangle DEF? ____________ Why?

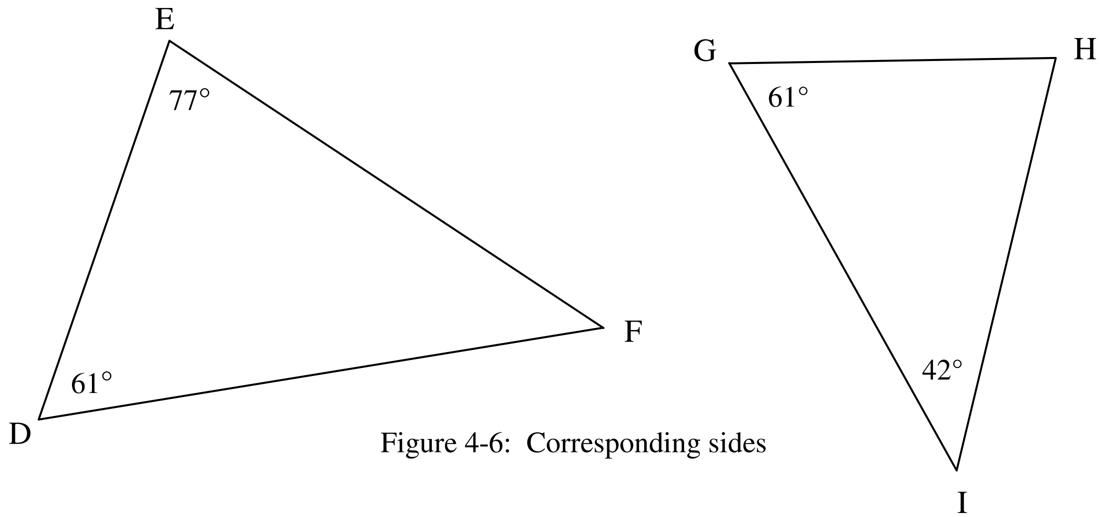


Figure 4-6: Corresponding sides

<u>Exercise 4-5</u>:

1. See Figure 4-7 below. Are these two triangles similar? Explain your answer.

2. Which side in triangle MNO corresponds with side AB in triangle ABC? ___________ Why?

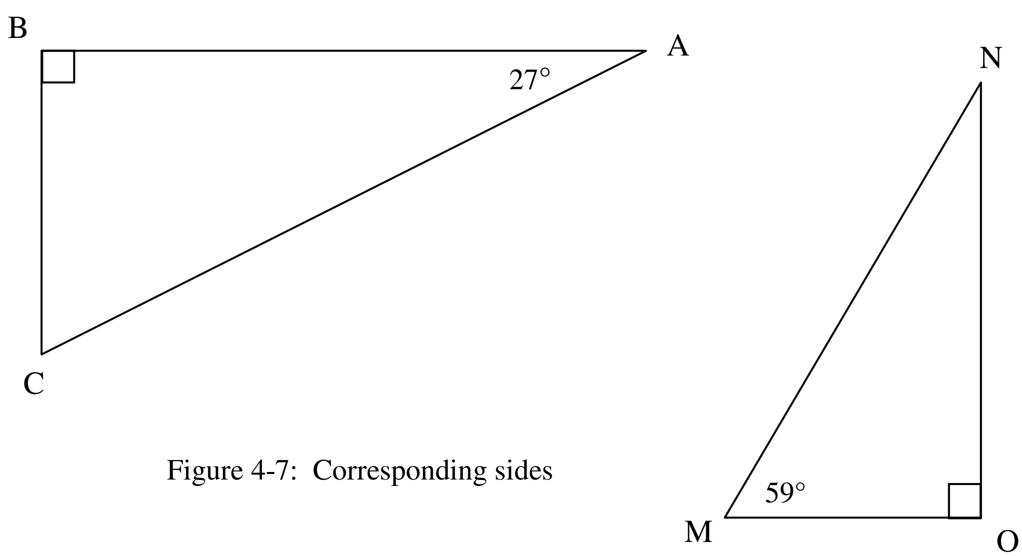


Figure 4-7: Corresponding sides

Proportional Sides

A proportion is a *ratio* that compares two quantities. We will use proportions to compare the lengths of sides of two similar right triangles.

Proportional sides is the key relationship that makes right angle trigonometry work. Similar triangles have proportional sides because similar triangles have the same shape.

Using Figure 4-8 below, we will first create ratios that compare the length of sides.

In triangle ABC, we can compare the length of two sides using this ratio: $\dfrac{\text{the length of side AB}}{\text{the length of side AC}}$

In triangle DEF, we can compare the length of two sides using this ratio: $\dfrac{\text{the length of side DE}}{\text{the length of side DF}}$

Recognize that side AB in triangle ABC corresponds to side DE in triangle DEF, and that side AC in triangle ABC corresponds to side DF in triangle DEF.

The ratio that compares two sides of a triangle equals the ratio that compares the *corresponding* two sides of a *similar* triangle. Because these two triangles are similar, we can write the following proportion that sets these two ratios equal to each other:

$$\frac{\text{the length of side AB}}{\text{the length of side AC}} = \frac{\text{the length of side DE}}{\text{the length of side DF}}$$

These relationships would also be true for any other triangle which is similar to these two triangles.

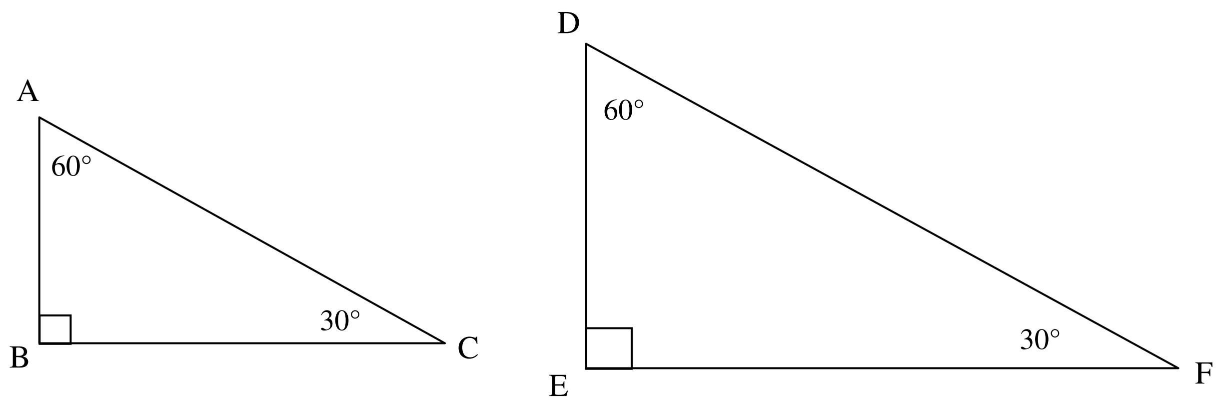


Figure 4-8: Similar triangles

Example

Other ratios and proportions are possible for these two triangles in Figure 4-8.
The following proportion is also true for these two triangles because the proportion compares the ratios of corresponding sides.

$$\frac{\text{the length of side BC}}{\text{the length of side AC}} = \frac{\text{the length of side EF}}{\text{the length of side DF}}$$

▲

Proportional Sides, continued

Example

If the corresponding sides of two triangles are proportional, you can conclude that the two triangles are similar. In Figure 4-9, are the two triangles similar? First, calculate the ratios of the corresponding sides.

We do not know the values of the internal angles for these two triangles. We will depend on the shape of these two triangles to recognize that side AC corresponds to side DF, and side BC corresponds to side.EF.

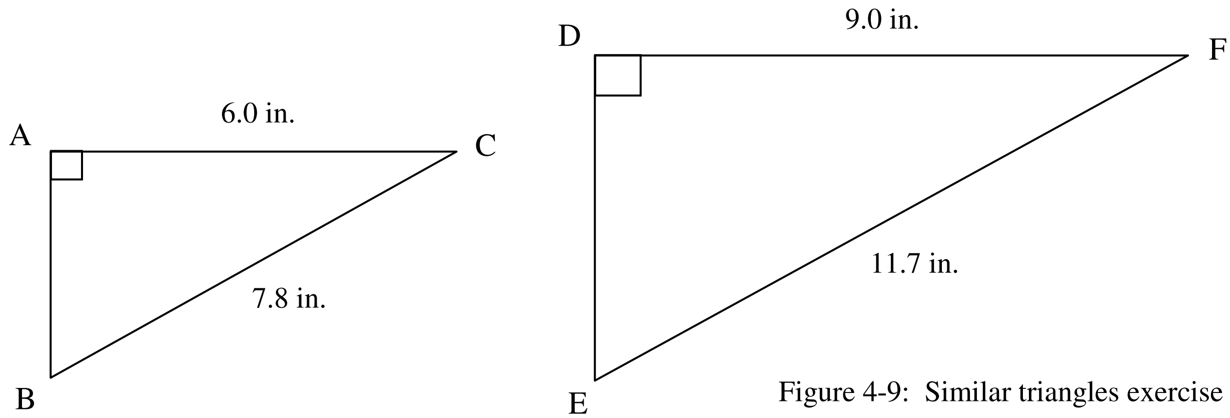


Figure 4-9: Similar triangles exercise

In triangle ABC:

$$\frac{AC}{BC} = \frac{6.0}{7.8} = 0.769$$

In triangle DEF:

$$\frac{DF}{EF} = \frac{9.0}{11.7} = 0.769$$

You can conclude that these two triangles are similar because the corresponding sides are proportional, as expressed in the following proportion:

$$\frac{AC}{BC} = \frac{DF}{EF}$$

The following proportions may also be used to determine if the two triangles are proportional.

$$\frac{AC}{AB} = \frac{DF}{DE}$$

$$\frac{AB}{BC} = \frac{DE}{EF}$$

The Pythogorean Theorem may be used to calculate the length of the unknown sides, AB and DE.
▲

Proportional Sides, continued

Exercise 4-6

Are the two triangles in Figure 4-10 similar? Use a proportion to determine the answer.

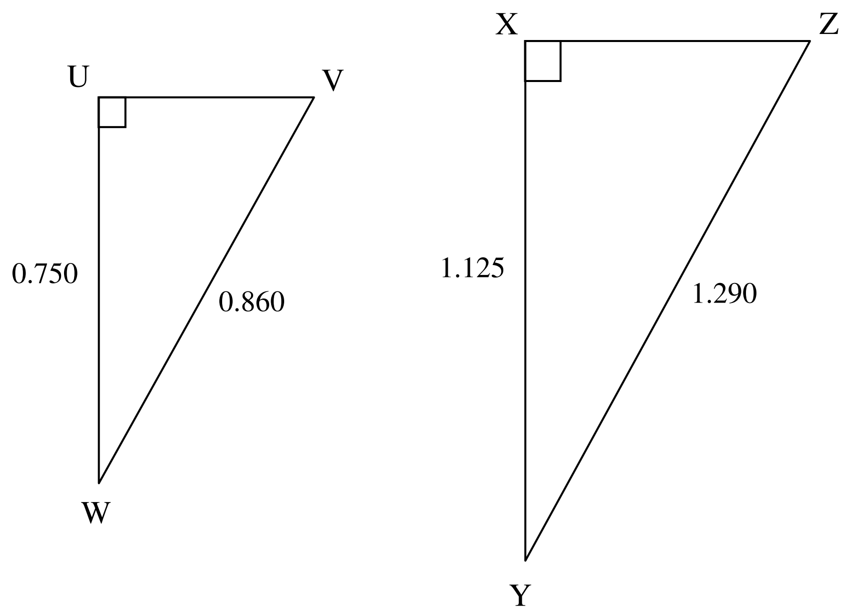


Figure 4-10: Similar triangles exercise

Exercise 4-7

Are the two triangles in Figure 4-11 similar? Use a proportion to determine the answer.

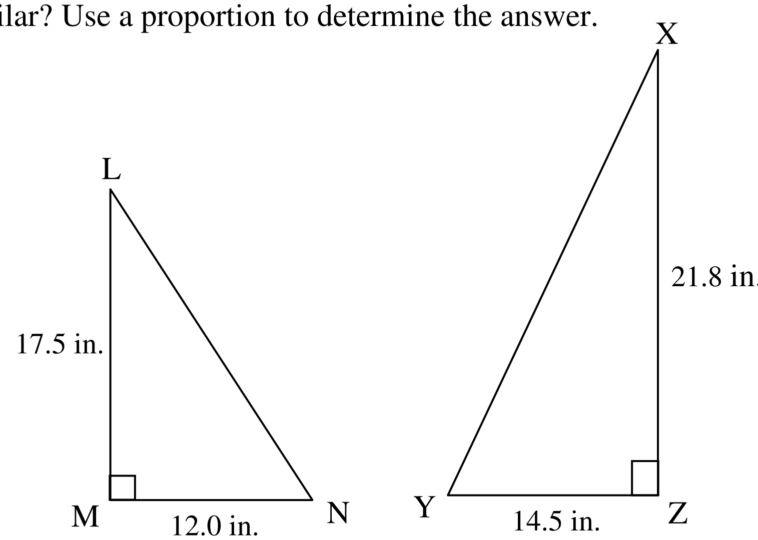


Figure 4-11: Similar triangles exercise

Proportional Sides, continued

Exercise 4-8

Use the Pythogorean Theorem to calculate the length of sides LN and XY in Figure 4-12. Then use a proportion that incorporates sides LN and XY to determine if the two triangles are similar.

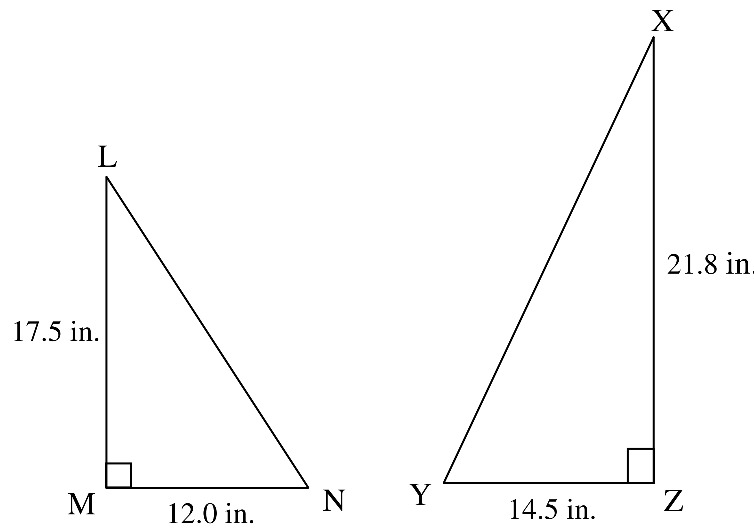


Figure 4-12: Similar triangles exercise

Exercise 4-9

Are the two triangles in Figure 4-13 similar? Use a proportion to determine the answer.

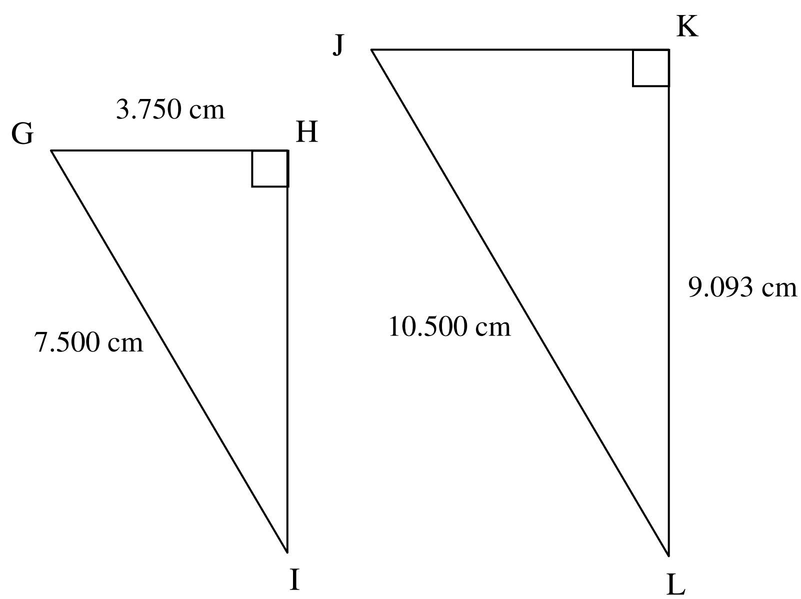


Figure 4-13: Similar triangles exercise

Naming Sides in a Right Triangle

There is one last topic to cover before we discuss the trigonometry functions. Sides of a right triangle are named based on which internal angle we are using in problem solving.

The one exception to this is the *hypotenuse* which is always the side opposite the right angle.

The one exception to this is the *hypotenuse* which is always the side opposite the right angle.

Referring to Figure 4-14, we can name the other sides based on the selection of an internal angle.

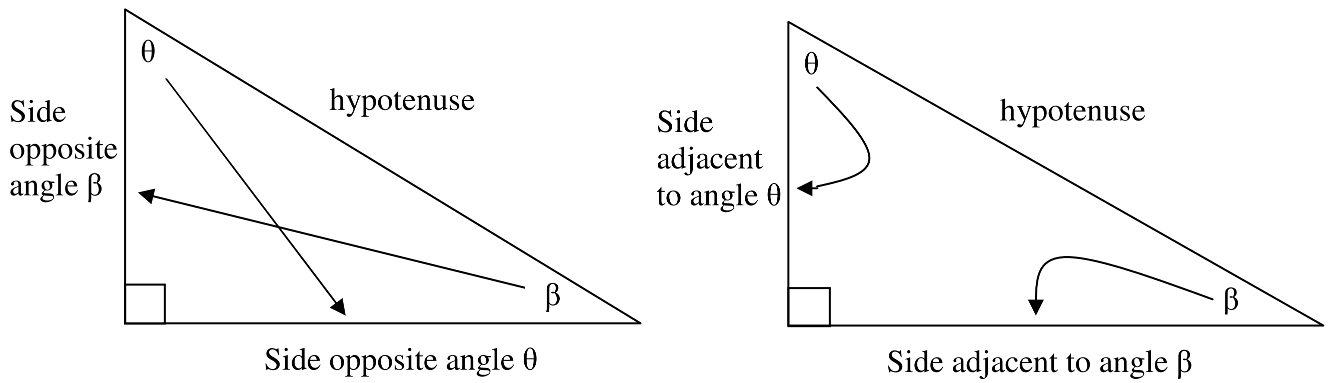


Figure 4-14: Naming sides

Example

Name the opposite and adjacent sides in the right triangles, in Figure 4-15 below.

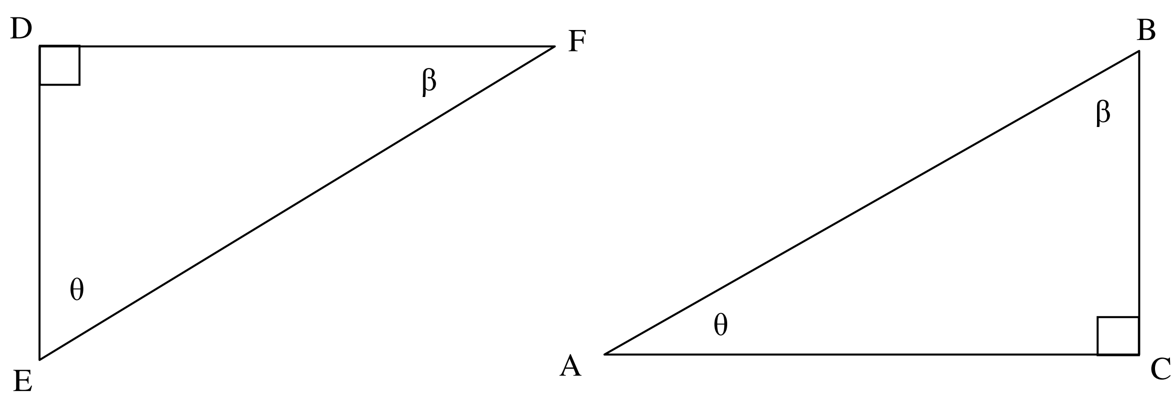


Figure 4-15: Naming sides

In triangle DEF,		In triangle ABC,	
The side opposite angle θ is side	DF	The side opposite angle θ is side	BC
The side opposite angle β is side	DE	The side opposite angle β is side	AC
The side adjacent to angle θ is side	DE	The side adjacent to angle θ is side	AC
The side adjacent to angle β is side	DF	The side adjacent to angle β is side	BC

▲

Naming Sides in a Right Triangle, continued

Exercise 4-10

In Figure 4-16 below, name the opposite and adjacent sides in these right triangles.

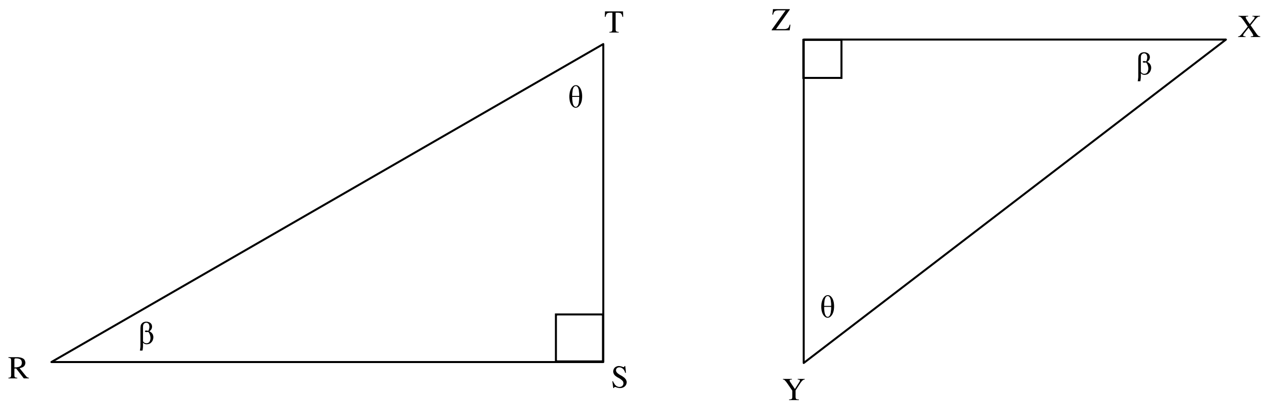


Figure 4-16: Naming sides

In triangle RST,		In triangle XYZ,	
1. The side opposite angle θ is side		5. The side opposite angle θ is side	
2. The side adjacent to angle β is side		6. The side opposite angle β is side	
3. The side adjacent to angle θ is side		7. The side adjacent to angle θ is side	
4. The side opposite angle β is side		8. The side adjacent to angle β is side	

Exercise 4-11

In Figure 4-17 below, name the angles which are related to the opposite and adjacent sides.

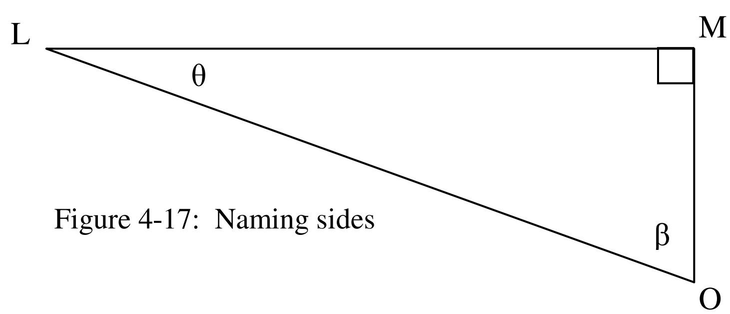
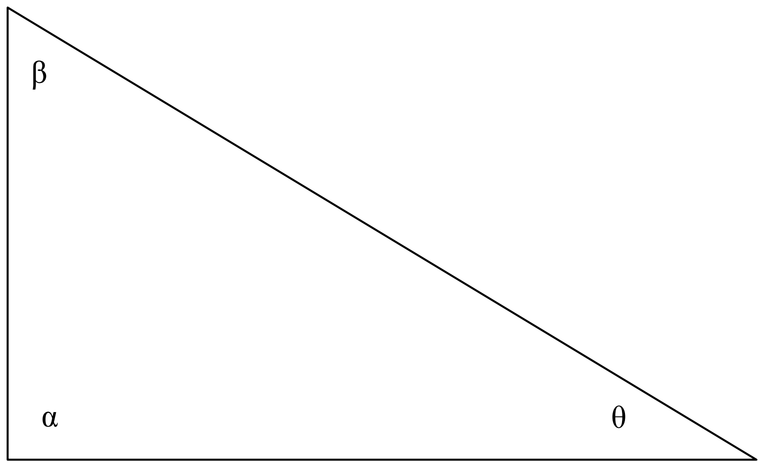
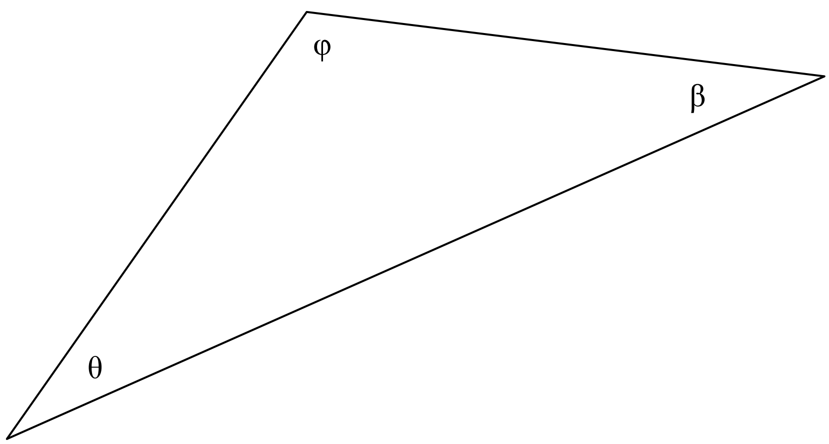
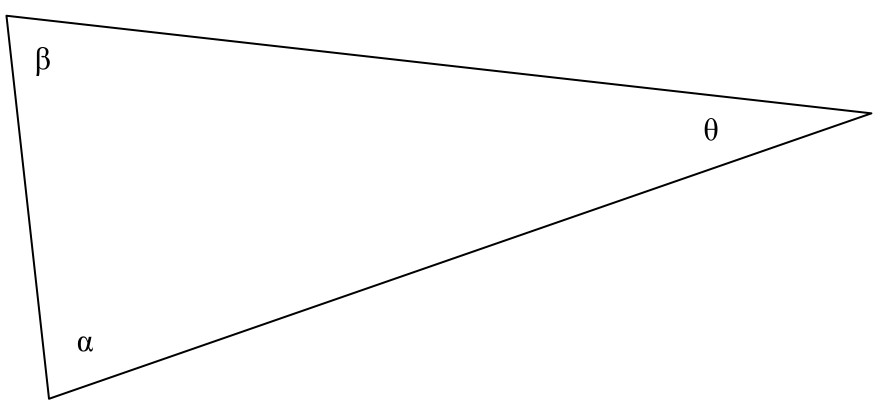


Figure 4-17: Naming sides

side	angle
1. Side LM is opposite which angle?	
2. Side LM is adjacent to which angle?	
3. Side MO is opposite which angle?	
4. Side MO is adjacent to which angle?	

Chapters 2–4, Practice Worksheet

Measure each of these interior angles with a protractor. Be sure to include the degree symbol, °, with your answers. Express your answers to the nearest whole degree.

	Your measurements
1. 	a. $\theta =$
	b. $\alpha =$
	c. $\beta =$
2. 	a. $\theta =$
	b. $\varphi =$
	c. $\beta =$
3. 	a. $\theta =$
	b. $\alpha =$
	c. $\beta =$

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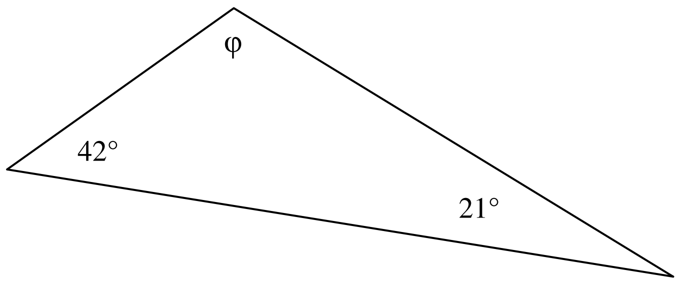
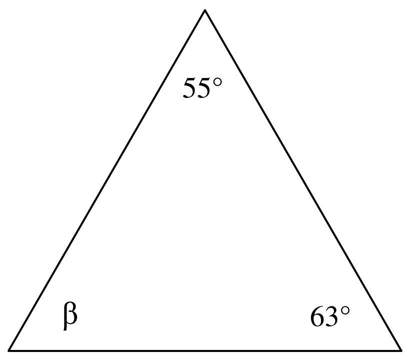
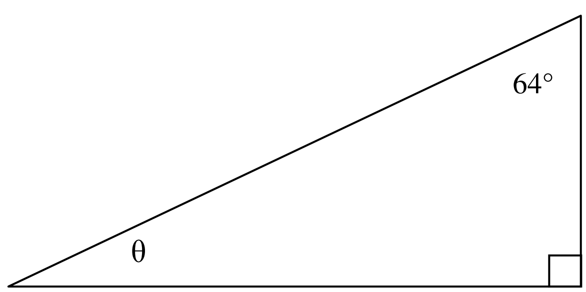
Chapters 2–4, Practice Worksheet, continued

Calculate the following unknown angles.

4. 	Find angle φ
5. 	Find angle β
6. 	Find angle θ

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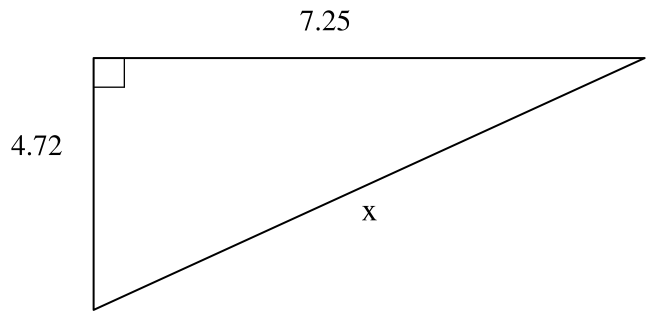
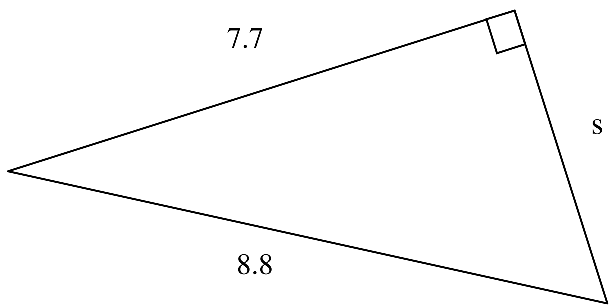
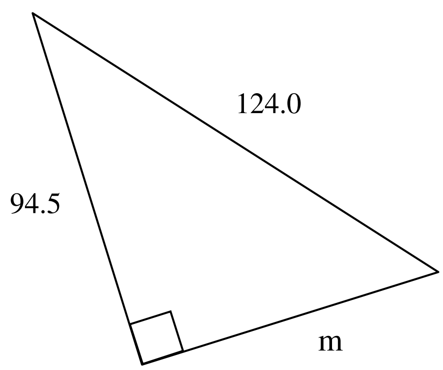
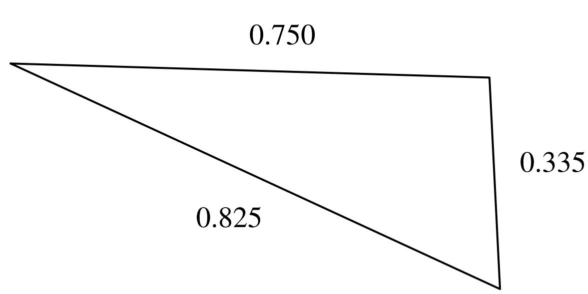
Chapters 2–4, Practice Worksheet, continued

Use the Pythagorean Theorem to solve for the unknown sides in these triangles. Estimate the answer before doing the calculations.

7. 	Solve for side x Your estimate: ______________ Calculated value ______________
8. 	Solve for side s Your estimate: _______________ Calculated value ______________
9. 	Solve for side m Your estimate: _______________ Calculated value ______________
10. 	Is this triangle a right triangle? Justify your answer mathematically.

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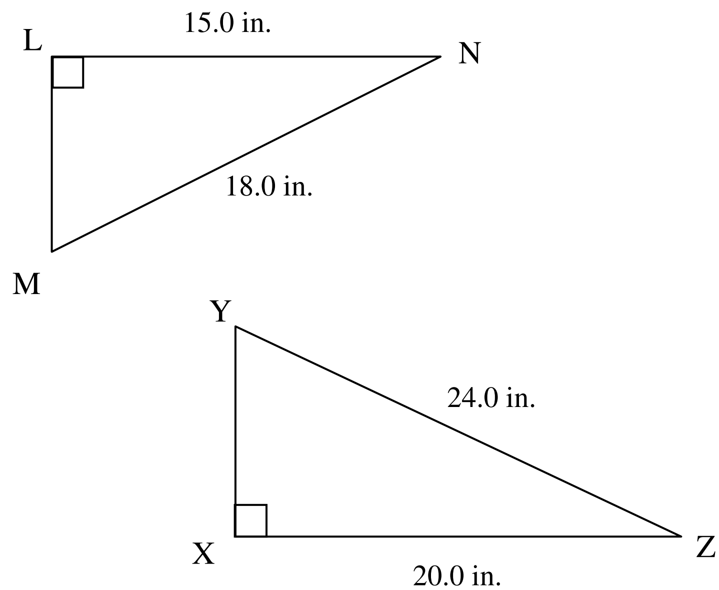
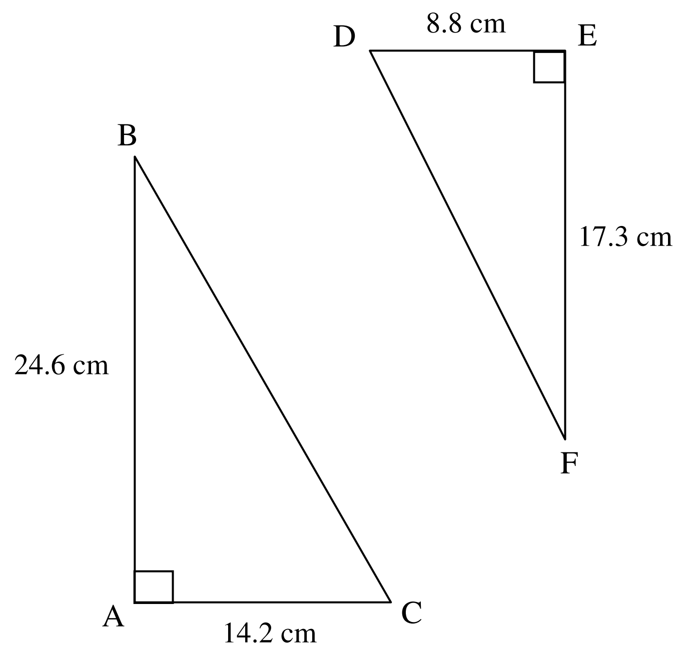
Chapters 2–4, Practice Worksheet, continued

11. Are these two triangles similar? Why or why not?



12. Are these two triangles similar? Why or why not?



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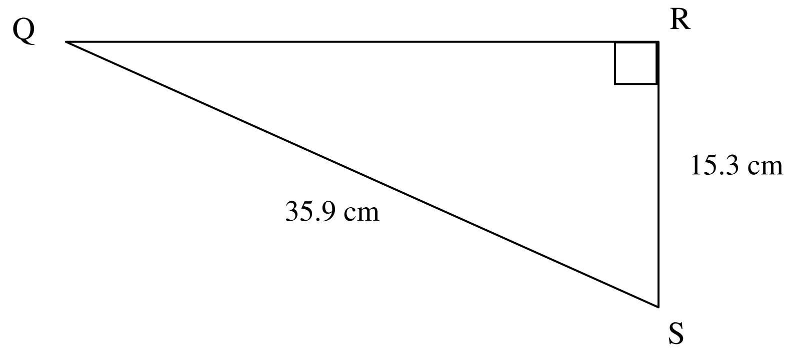
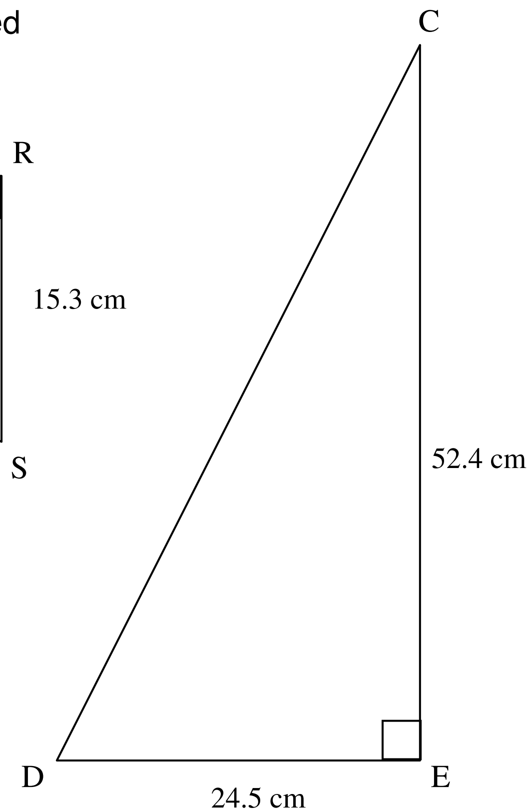
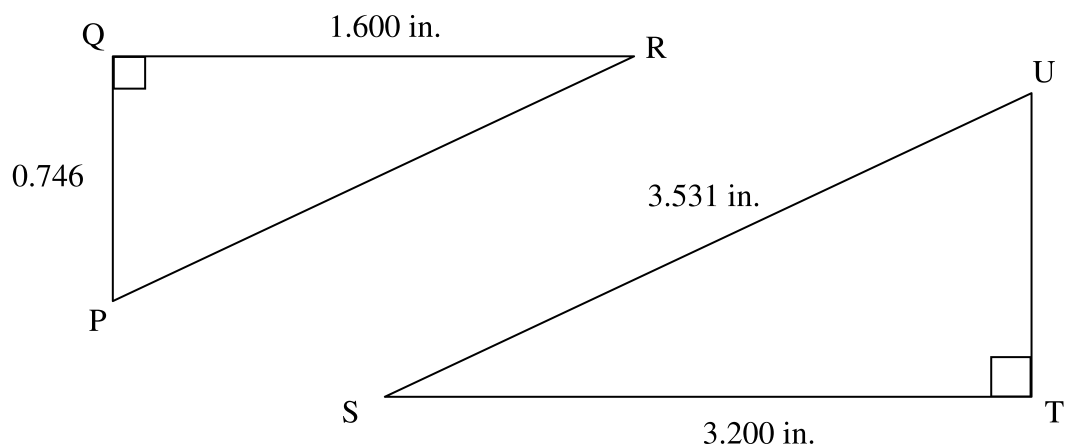
Chapters 2–4, Practice Worksheet, continued

13. Are these two triangles similar? Why or why not?





14. Are these two triangles similar? Why or why not?



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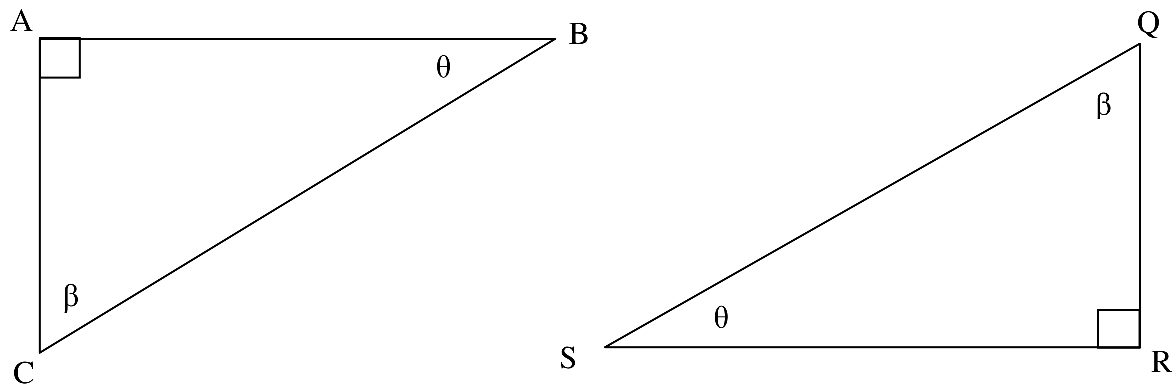
Chapters 2–4, Practice Worksheet, continued

15. Name the opposite and adjacent sides in these right triangles.



In triangle ABC,		In triangle QRS,	
1. The side adjacent to angle θ is side		5. The side adjacent to angle θ is side	
2. The side adjacent to angle β is side		6. The side adjacent to angle β is side	
3. The side opposite angle θ is side		7. The side opposite angle θ is side	
4. The side opposite angle β is side		8. The side opposite angle β is side	

16. Using your calculator, convert 14°55′11″ to decimal degrees (write down all decimals displayed on your calculator).

17. Using your calculator, convert 131.18° to DMS format, to the nearest whole second.

18. Add these two angles by hand and then check the result with your calculator.

$$\begin{array}{rrrr} & 36° & 40' & 37'' \\ + & 115° & 14' & 25'' \\ \hline \end{array}$$

19. Subtract these two angles by hand and then check the result with your calculator.

$$\begin{array}{rrrr} & 107° & 15' & 31'' \\ - & 67° & 39' & 07'' \\ \hline \end{array}$$

Chapter 5: Functions of Trigonometry

We will now learn the trigonometry functions that are used to solve right triangle problems. The tools that we learned in Chapter 4 will be invaluable to using these functions successfully.

The trigonometry functions express ratios of the length of sides. These functions are *sine*, *cosine*, and *tangent*.

Sine Function

Let's start with one of the functions, the *sine* (see Figure 5-1). The sine of a given angle is the ratio of the length of the opposite side to the length of the hypotenuse. Sine is typically abbreviated as *sin*.

Example

$$\text{sine }\theta = \sin\theta = \frac{\text{side opposite angle }\theta}{\text{hypotenuse}} = \frac{\text{length of AB}}{\text{length of AC}} = \frac{6.0\text{ in.}}{12.0\text{ in.}} = 0.5$$

Note: The units for the length of the sides cancel out during the calculation. Therefore, the numerical value for the sine is a unit-less number. This number expresses the ratio of the lengths of the sides. The length of the sides must be expressed in the same units. If the lengths are in different units, you will need to convert the units so that they are the same.

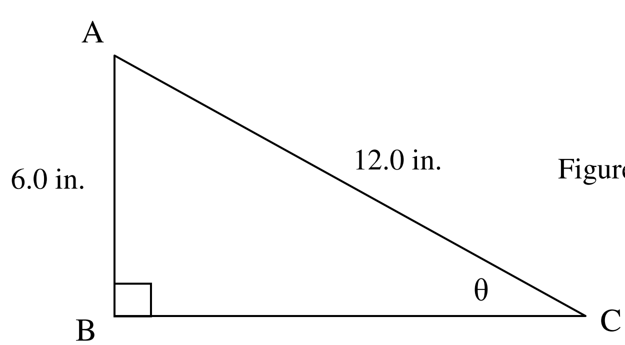


Figure 5-1: Sine function

▲

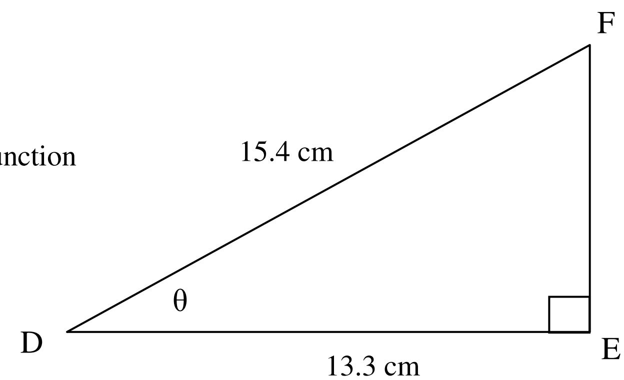
Cosine Function

Let's now look at another function, the *cosine* (see Figure 5-2). The cosine of a given angle is the ratio of the length of the adjacent side to the length of the hypotenuse. Cosine is typically abbreviated as *cos*.

Example

$$\text{cosine } \theta = \cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}} = \frac{\text{length of DE}}{\text{length of DF}} = \frac{13.3 \text{ cm}}{15.4 \text{ cm}} = 0.86$$

Figure 5-2: Cosine function



▲

Example

You may need to convert units if the lengths of the sides are given in different units. In Figure 5-3 below, calculate the value of the cosine of angle β. Convert units before calculating the cosine value.

You must convert the feet to inches or the inches to feet. In this example, the length of side JK is converted to feet:

$$\text{length of side JK} = 9.0 \text{ in.} \quad \text{X} \ \frac{1 \text{ ft.}}{12 \text{ in.}} = 0.75 \text{ ft.}$$

$$\text{cosine } \beta = \cos \beta = \frac{\text{side adjacent to angle } \beta}{\text{hypotenuse}} = \frac{\text{length of JK}}{\text{length of KL}} = \frac{0.75 \text{ ft.}}{1.70 \text{ ft.}} = 0.44$$

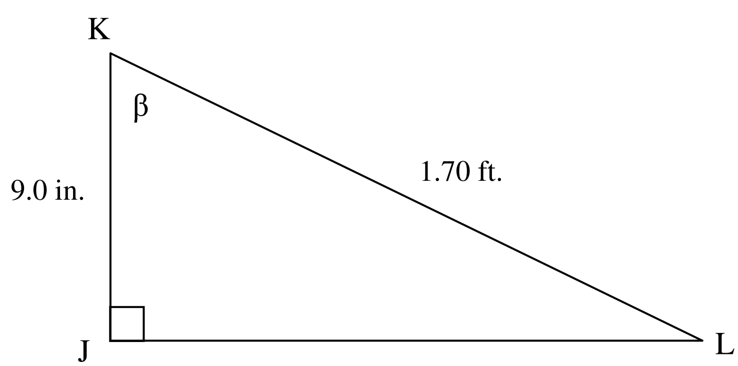


Figure 5-3: Cosine function

▲

Cosine Function, continued

Exercise 5-1

Using the triangle in Figure 5-4, determine the value of the sine and cosine functions by filling in the table.

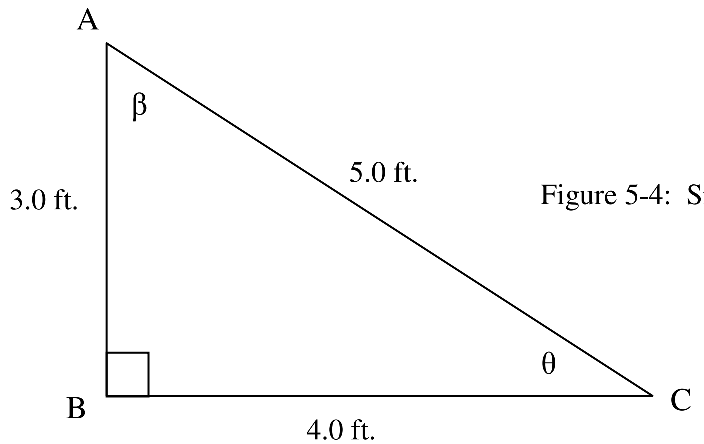


Figure 5-4: Sine and cosine functions

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	sin β	=	$\frac{\text{side opposite angle } \beta}{\text{hypotenuse}}$	=	$\frac{BC}{AC}$	=	______	=	
2.	cos β	=	______	=	______	=	______	=	
3.	sin θ	=	______	=	______	=	______	=	
4.	cos θ	=	______	=	______	=	______	=	

Cosine Function, continued

<u>Exercise 5-2</u>

Using the triangle in Figure 5-5, determine the value of the sine and cosine functions.

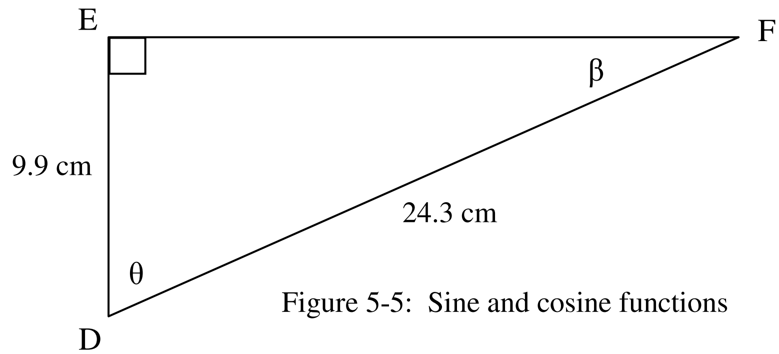


Figure 5-5: Sine and cosine functions

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	cos β	=	__________	=	______	=	______	=	
2.	sin θ	=	__________	=	______	=	______	=	

<u>Exercise 5-3</u>

Using the triangle in Figure 5-6, determine the value of the sine and cosine functions.

Note: one inch is equal to 2.54 centimeters.

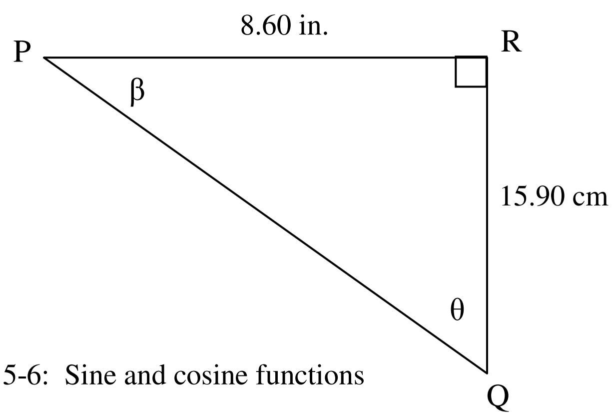


Figure 5-6: Sine and cosine functions

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	sin β	=	__________	=	______	=	______	=	
2.	cos θ	=	__________	=	______	=	______	=	

Tangent Function

Now let's look at yet another function, the *tangent* (see Figure 5-7). The tangent of a given angle is the ratio of the length of the opposite side to the length of the adjacent side. Tangent is abbreviated as *tan*.

Example

$$\text{tangent } \theta = \tan\theta = \frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta} = \frac{\text{length of EF}}{\text{length of DF}} = \frac{8.5 \text{ in.}}{14.7 \text{ in.}} = 0.58$$

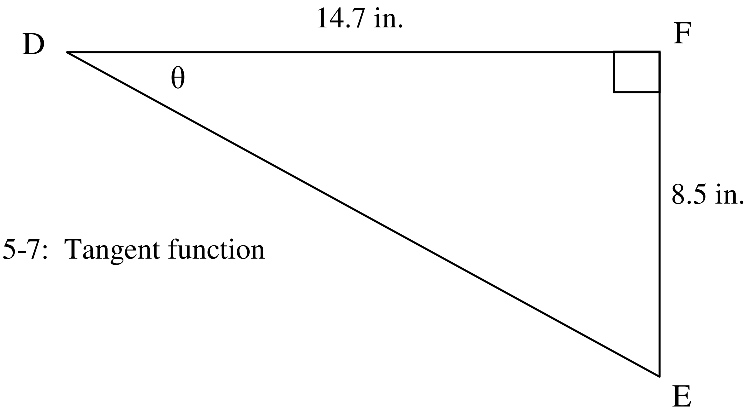


Figure 5-7: Tangent function

▲

<u>Exercise 5-4</u>

Determine the value of the tangent functions in Figure 5-8 for both angle θ and angle β.

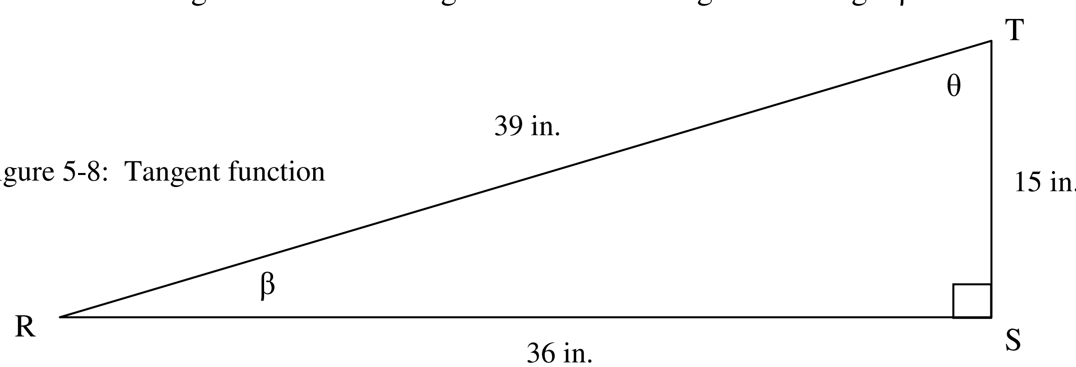


Figure 5-8: Tangent function

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	$\tan\theta$	=	$\dfrac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$	=	________	=	________	=	
2.	$\tan\beta$	=	________________	=	________	=	________	=	

Note: What do you observe about these two tangent functions? How are they different?

Tangent Function, continued

Exercise 5-5

Determine the value of the tangent functions in Figure 5-9 for both angle θ and angle β.

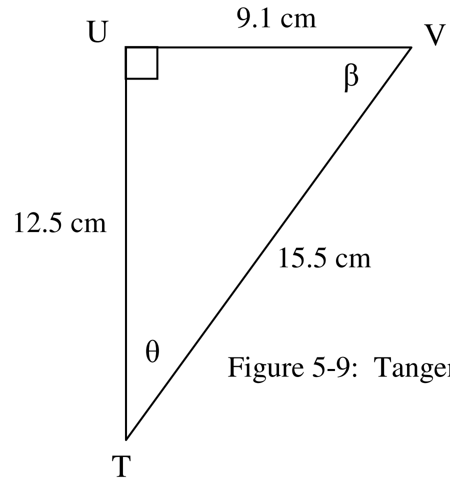


Figure 5-9: Tangent function

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	tan θ	=	__________	=	______	=	______	=	
2.	tan β	=	__________	=	______	=	______	=	

Exercise 5-6

Determine the value of the tangent functions in Figure 5-10 for both angle θ and angle β.

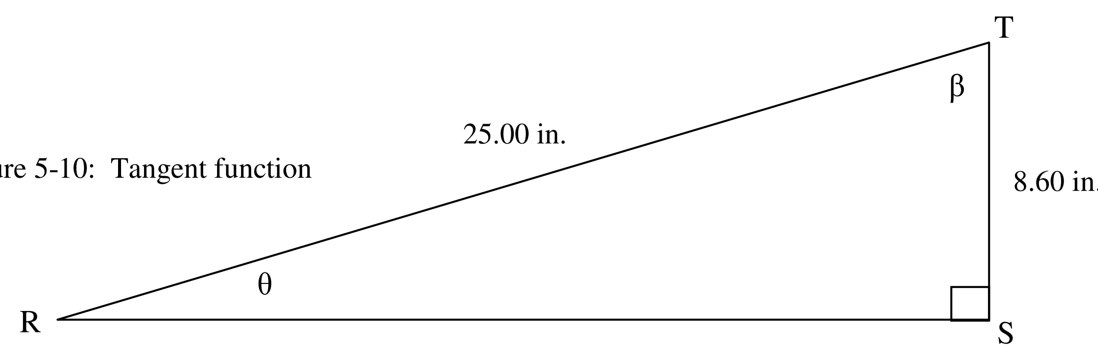


Figure 5-10: Tangent function

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	tan β	=	__________	=	______	=	______	=	
2.	tan θ	=	__________	=	______	=	______	=	

Function Practice

Exercise 5-7

Determine the value of the trigonometry functions in Figure 5-11.

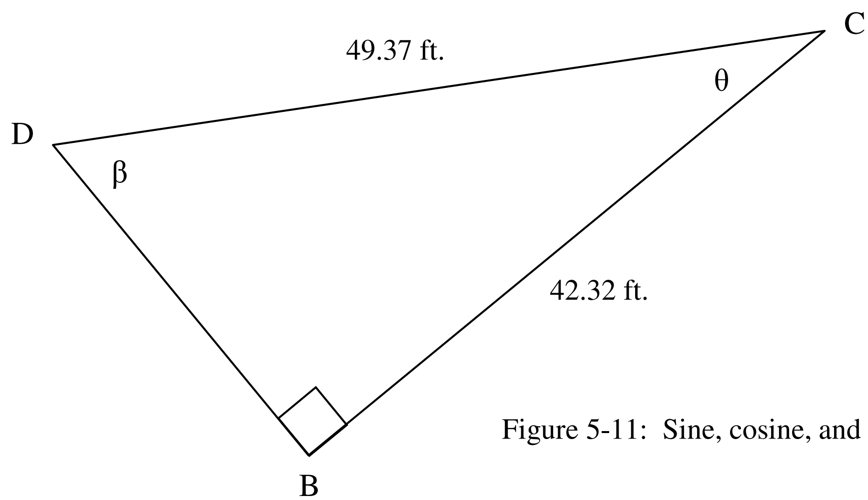


Figure 5-11: Sine, cosine, and tangent functions

1. $\sin \beta$ =

2. $\cos \beta$ =

3. $\tan \beta$ =

4. $\sin \theta$ =

5. $\cos \theta$ =

6. $\tan \theta$ =

Function Practice

Exercise 5-8

Determine the value of the trigonometry functions in Figure 5-12.

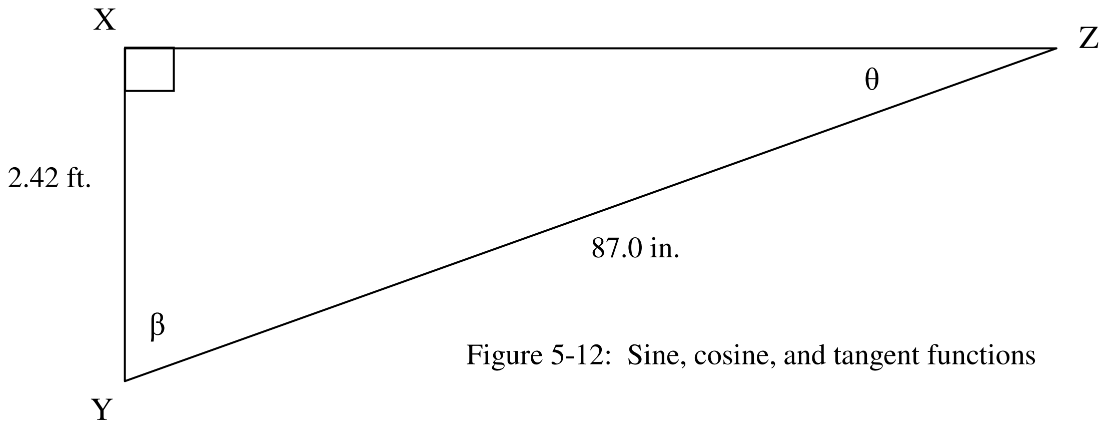


Figure 5-12: Sine, cosine, and tangent functions

1. $\tan \theta$ =

2. $\cos \beta$ =

3. $\sin \theta$ =

4. $\tan \beta$ =

5. $\cos \theta$ =

6. $\sin \beta$ =

Summary of Trigonometry Functions

The following is a summary of the three trigonometry functions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{OPP}}{\text{HYP}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{ADJ}}{\text{HYP}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{OPP}}{\text{ADJ}}$$

A memory aid, or mnemonic, is often used to help remember these functions. This mnemonic is SOHCAHTOA, which means as follows:

SOH (stands for sine – opposite – hypotenuse)

CAH (stands for cosine – adjacent – hypotenuse)

TOA (stands for tangent – opposite – adjacent)

Other Trigonometric Functions

There are three other trigonometry functions that we will not use in this text. You should be aware of these functions because you may encounter them in other texts or technical documents.

$$\text{secant}\ \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\text{HYP}}{\text{OPP}}$$

The secant function is the *reciprocal* of the sine function.

$$\text{cosecant}\ \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\text{HYP}}{\text{ADJ}}$$

The cosecant function is the *reciprocal* of the cosine function.

$$\text{cotangent}\ \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\text{ADJ}}{\text{OPP}}$$

The cotangent function is the *reciprocal* of the tangent function.

Chapter 6: Solving for Unknown Sides

Introduction

If we know the length of two sides of a right triangle, we can use the Pythagorean Theorem to find the length of the third side. What should we do if we know the length of only one side? Our first application of trigonometry is to solve for an unknown side in a right triangle. To do this, you must know the value of one angle (other than the right angle) and the length of one side.

Figure 6-1 shows a triangle which has one known side (the hypotenuse) and one known interior angle. Using the 30° interior angle, which trigonometry function can we use to solve for the unknown side, *a*?

The sine function relates the unknown side a to the hypotenuse as follows:

$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{5.0 \text{ ft.}}$$

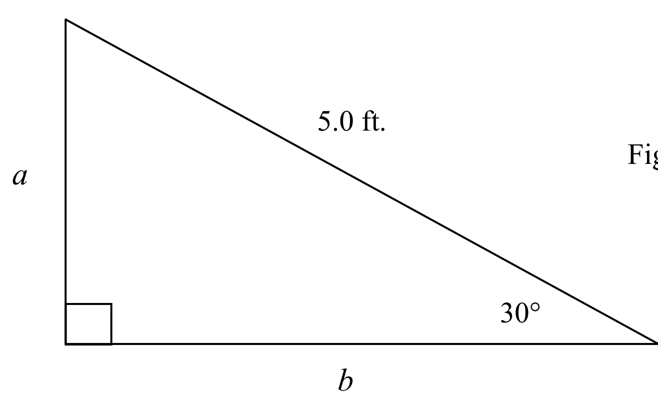


Figure 6-1: Right triangle

Let's continue solving this problem in the next section.

Solving for an Unknown Side

Example

Here are the steps to solve for the length of an unknown side:

1. Determine which unknown side you need to calculate. Do you have enough information to solve for this unknown?

   In this example, in Figure 6-2, we are solving for the length of side *a*, and we already know one interior angle and the length of one side, which is sufficient for right triangles.

2. Next, decide which interior angle you will use as a reference angle. In this example, we can use the 30° interior angle, which is given. However, you can choose to first solve for the other interior angle and use it, rather than the given angle. In this case, the other interior angle is 180° – 90° – 30° = 60°.

3. Decide which trigonometry function to use. Pick the function that relates both the unknown and known sides. As already shown, the sine function is the correct choice for this triangle, with the 30° angle as your reference angle. Create an equation using this function.

$$\sin 30° = \frac{a}{5.0 \text{ ft.}}$$

4. Use algebra to solve for the unknown in the equation.

$$a = (5.0 \text{ ft.})(\sin 30°)$$

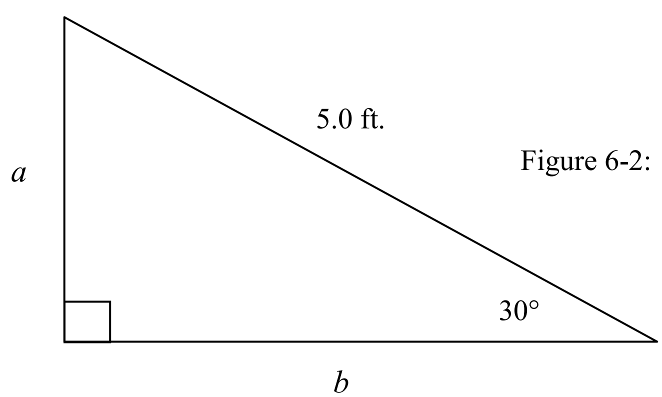


Figure 6-2: Right triangle

▲

Create an Equation to Solve for an Unknown Side

Before we continue solving this problem, we will practice the important step of creating an equation to solve for an unknown side in a right triangle.

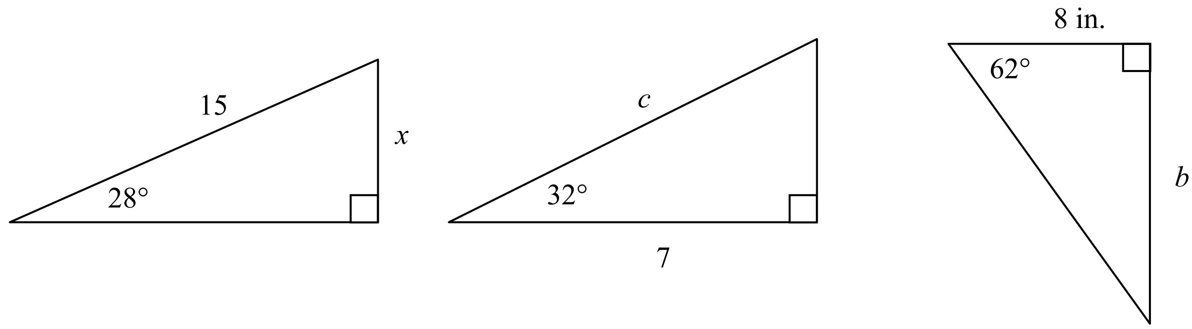
After creating an equation, use algebra to solve for the unknown.

We will use the following abbreviations in this example:

opposite side = OPP
adjacent side = ADJ
hypotenuse = HYP

Example

For each triangle, write an equation which relates the known information to the unknown information. Use the appropriate trigonometry function. Then, use algebra to solve for the unknown.



$$\sin 28° = \frac{\text{OPP}}{\text{HYP}} \qquad \cos 32° = \frac{\text{ADJ}}{\text{HYP}} \qquad \tan 62° = \frac{\text{OPP}}{\text{ADJ}}$$

$$\sin 28° = \frac{x}{15} \qquad \cos 32° = \frac{7}{c} \qquad \tan 62° = \frac{b}{8 \text{ in.}}$$

$$x = (15)(\sin 28°) \qquad c = \frac{7}{\cos 32°} \qquad b = (8 \text{ in.})(\tan 62°)$$

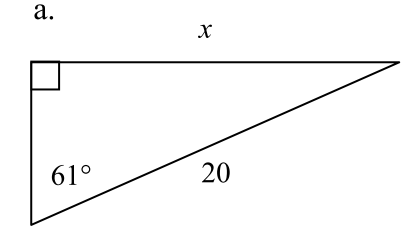
▲

Create an Equation to Solve for an Unknown Side

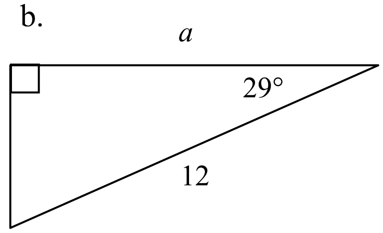
Exercise 6-1

For each triangle, write an equation that relates the known information to the unknown information. Use the appropriate trigonometry function. Then, use algebra to solve for the unknown, as shown in the examples on the previous page.

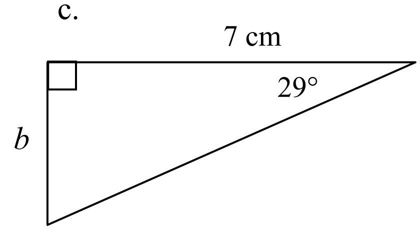
a.



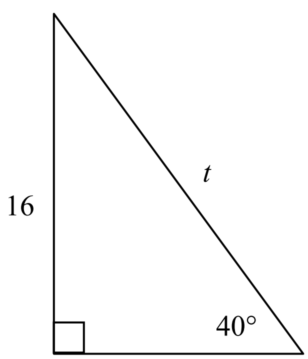
b.



c.



d.



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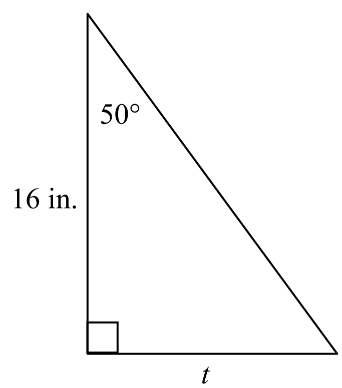
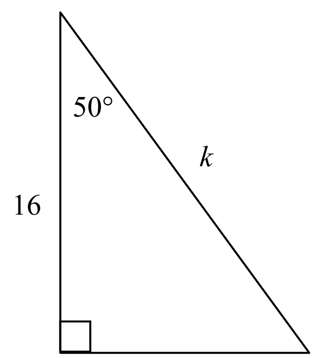
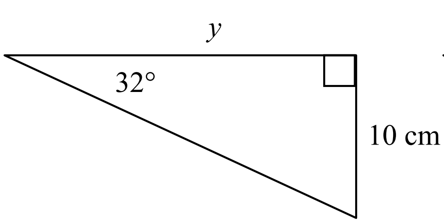


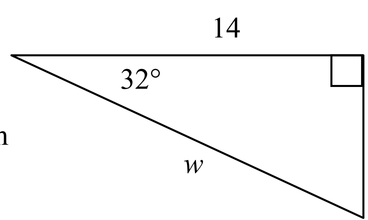
f.



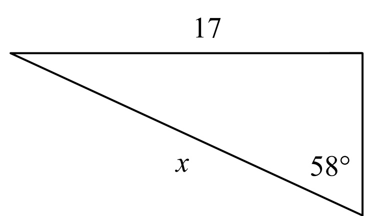
g.



h.



i.



Solving for an Unknown Side

We will now continue solving the earlier example (see Figure 6-3).

1. The correct equation to solve this problem is:

$$\sin 30° = \frac{a}{5.0 \text{ ft.}}$$

2. Use algebra to solve for the unknown in the equation.

$$a = (5.0 \text{ ft.})(\sin 30°)$$

3. Use your calculator to solve for the length of side *a*. The answer to this problem has the units of feet. The answer from the calculator will not have units, so you will need to remember to add the appropriate units to your answer.

4. Your scientific calculator has keys for the three trigonometry functions: sine, cosine, and tangent. Using a Texas Instruments calculator, your key strokes will be similar to:

Calculator model	Keystrokes
TI – 30XA	5.0 X 30 SIN = 2.5
TI – 30XIIS	5.0 X SIN (30) = 2.5

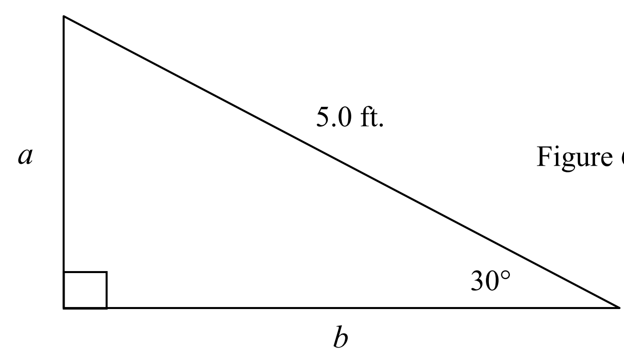


Figure 6-3: Right triangle

▲

Other calculator models may have slightly different procedures to enter the information and perform the calculation.

In the previous example, you used your calculator to find the value of the sine of 30°. You found that sin 30° = 0.5. What does that mean?

For *every* right triangle with a 30° interior angle, the *ratio* of the opposite side to the hypotenuse is 0.5. In this example, we found that the opposite side *a* had a length of 2.5 ft., or half as long as the hypotenuse.

Trigonometry Functions in Your Calculator

Example

Your scientific calculator can determine the value of the three trigonometry functions for any angle. Remember to place your calculator in the degree mode.

Using a Texas Instruments calculator, your key strokes will be similar to the following:

	Calculator model	Keystrokes
sin 28°	TI – 30XA	28 SIN (display shows 0.469471563)
	TI – 30XIIS	SIN 28 Enter (display shows 0.469471563)
cos 9°	TI – 30XA	9 COS (display shows 0.987688341)
	TI – 30XIIS	COS 9 Enter (display shows 0.987688341)
tan 17.5°	TI – 30XA	17.5 TAN (display shows 0.315298789)
	TI – 30XIIS	TAN 17.5 Enter (display shows 0.315298789)

▲

Exercise 6-2

Practice using your calculator to determine the value of trigonometry functions for specific angles.

sin 45° = 0.707106781

cos 40° = ____________

tan 53° = ____________

sin 8° = ____________

cos 79° = ____________

tan 82° = ____________ What does this result mean? Sketch a right triangle with an 82° interior angle to see what this result means.

cos 120° ____________ Why is this result negative?

Note: When solving trigonometry problems, use the value of the function provided by your calculator. Avoid writing down this value on paper, and then re-entering a rounded value.

For example, if you must use the tangent of 37° in your problem solution, your calculator will give a value of tan 37° = 0.75355405. When this value is displayed in your calculator, you should use it directly in the problem solution. If you write the value down on paper and then re-enter it as a rounded number, 0.754, your final answer will be inexact due to this rounding.

Positive and Negative Trigonometry Values

On the previous page, you discovered that the value of the cosine of 120° is a negative value. The following pair of exercises will help you understand why trigonometry values are positive or negative.

Exercise 6-3

Calculate the values of these trigonometry values:

cos 120° = ______________________

sin 120° = ______________________

tan 120° = ______________________

cos 235° = ______________________

sin 235° = ______________________

tan 235° = ______________________

cos 320° = ______________________

sin 320° = ______________________

tan 320° = ______________________

Exercise 6-4

Next, using your protractor, draw these three angles on the Cartesian coordinate system on the next page. The first angle is done for you.

After the line representing the angle is drawn, sketch the right triangle, with the line as the hypotenuse.

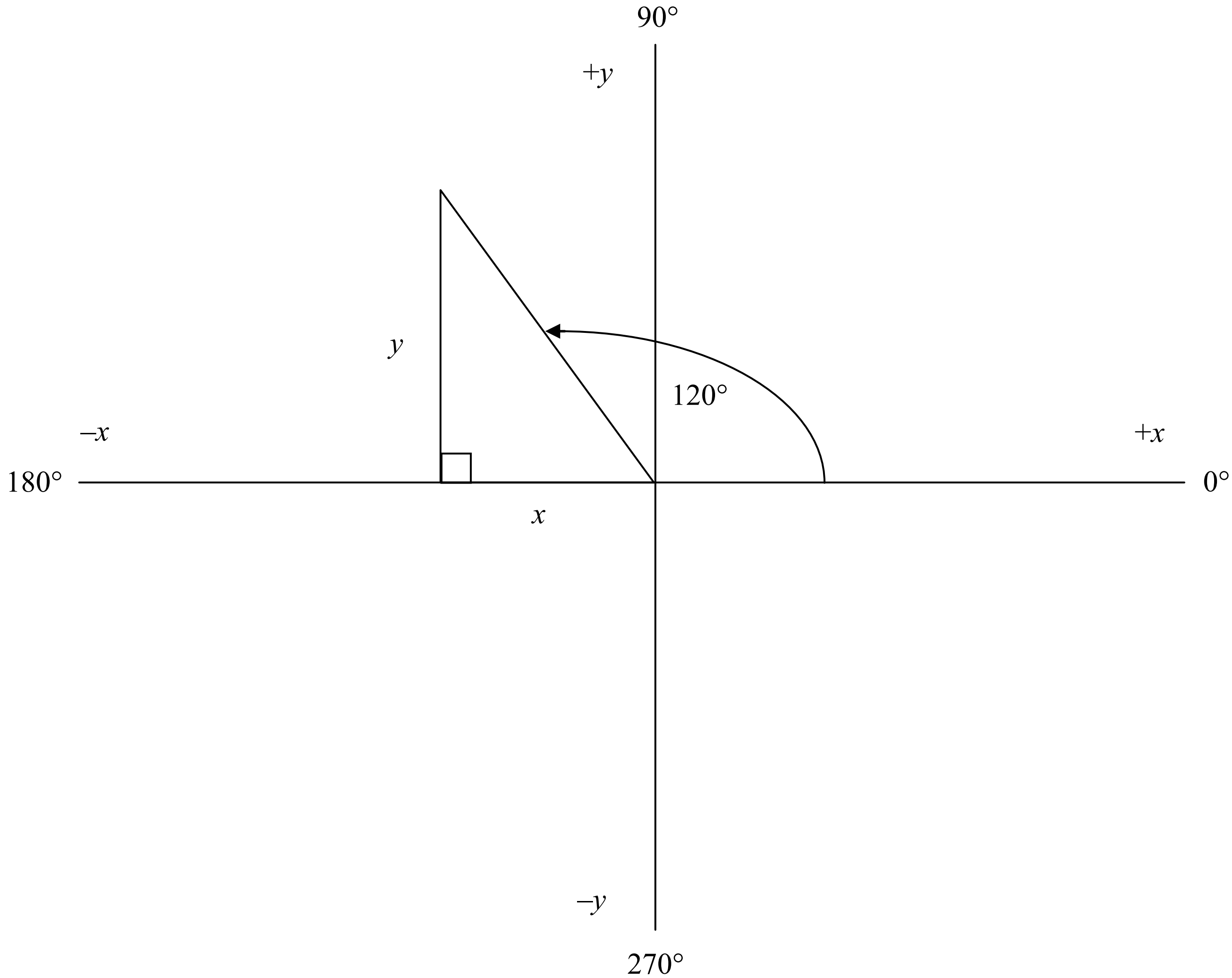
The value of cos 120° is negative because the x value in the right triangle is negative. The x value in this triangle is the *adjacent side* in the cosine calculation. The hypotenuse has an absolute value (always positive). In turn, cos 120° has a negative value because the cosine = adjacent/hypotenuse.

Summary of determining the sign of the trigonometry function:

	is positive when:	is negative when:
sine	y value is positive	y value is negative
cosine	x value is positive	x value is negative
tangent	x and y values have the same sign	x and y values have a different sign

Positive and Negative Trigonometry Values, continued

Exercise 6-4, continued



Using your sketch of the angle 235°, explain why the sine of 235° has a negative value.

Using your sketch of the angle 320°, explain why the tangent of 320° has a negative value.

Alternate Solution to Solving for an Unknown Side

Let's return to the problem we solved earlier in the chapter, as shown in Figure 6-4.

You may choose to use a different interior angle as your reference angle. We have enough information to solve for angle θ. Remember that the sum of the interior angles of a triangle is equal to 180°. Therefore:

Angle $\theta = 180° - 90° - 30° = 60°$

Example

Using the 60° interior angle as your reference angle:

$$\cos 60° = \frac{\text{Adjacent}}{\text{hypotenuse}} = \frac{a}{5.0 \text{ ft.}}$$

Use algebra to solve for the unknown in the equation.

$$a = (5.0 \text{ ft.})(\cos 60°) = 2.5 \text{ ft.}$$

You can see that this method results in the same answer for the length of side *a*. This happens because the value of the cosine of 60° is the same as the value of the sine of 30°. Sine and cosine are *cofunctions*, as discussed in the next section.

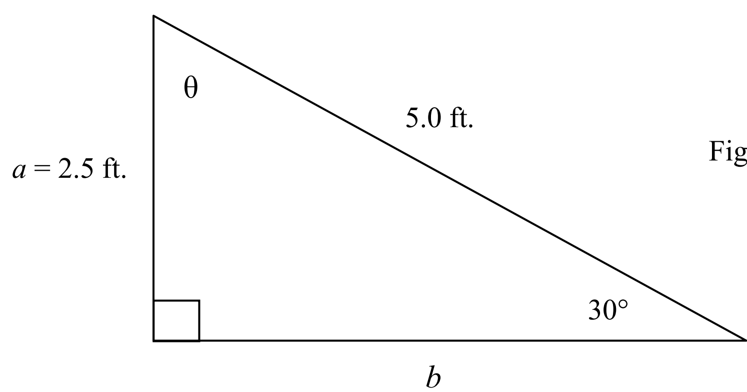


Figure 6-4: Right triangle

▲

Cofunctions

The sine and cosine functions are called cofunctions. In Figure 6-5, the cosine of angle β is equal to a/c. Additionally, the sine of angle θ is equal to a/c. Therefore, the value of cos β is equal to the value of sin θ.

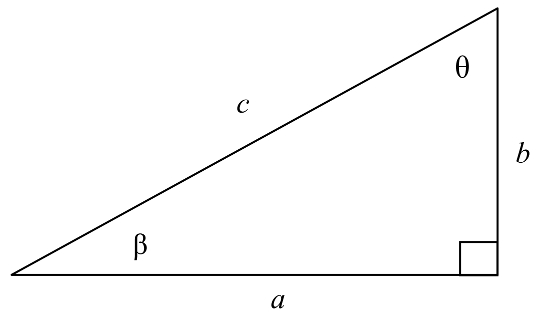


Figure 6-5: Right triangle

Since these two interior angles add up to 90°, the following is true of these cofunctions:

$$\sin\theta = \cos(90° - \theta) = \cos\beta$$

and

$$\cos\theta = \sin(90° - \theta) = \sin\beta$$

Example

Because they are cofunctions, the sine and cosine of 45° should have the same value.

$$\sin 45° = \cos(90° - 45°) = \cos 45°$$

and

$$\cos 45° = \sin(90° - 45°) = \sin 45°$$

Using your calculator:

$\sin 45° = 0.707106781$ and $\cos 45° = 0.707106781$

▲

Exercise 6-5

Practice using your calculator to determine the value of cofunctions for specific angles.

cos 40° =	____________	sin 50° =	____________
sin 31.5° =	____________	cos ______° =	____________
cos 18° =	____________	sin ______° =	____________
sin 8° =	____________	cos ______° =	____________
cos 79° =	____________	sin ______° =	____________

Complete the Solution

We can now complete the solution to the triangle in Figure 6-6. If you need to solve for the length of side *b*, you now have several methods available. Because you now know the value of two sides, you may use the Pythagorean Theorem. You may also use trigonometry functions, similar to the ways that you solved for side *a*.

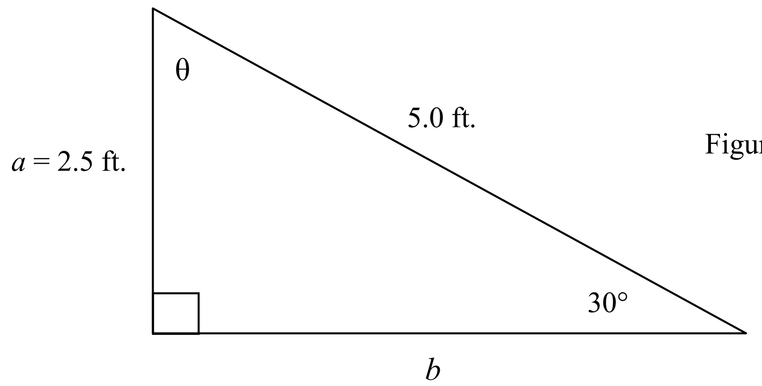


Figure 6-6: Right triangle

<u>Exercise 6-6</u>

As an exercise, solve for side *b* using five different methods.

1. Pythagorean Theorem solution to side *b*.

2. Using the 30° interior angle, and two different trigonometry functions.

3. Solve for the value of angle θ.

4. Using angle θ, and two different trigonometry functions.

Practice in Solving for Unknown Sides

Exercise 6-7

Solve for unknown side *b* in the right triangle in Figure 6-7.

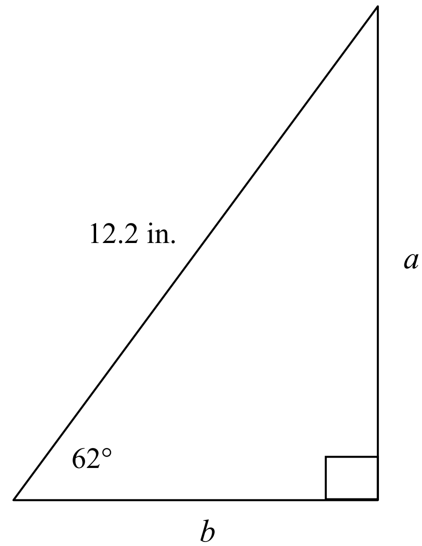


Figure 6-7: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown?	
Next, decide which interior angle you will use as a reference angle.	
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	
Create an equation using this function.	
Use algebra to solve for the unknown in the equation.	
Use your calculator to solve for the length of side *b*. Look at the triangle. Does your answer appear to be reasonable?	

Practice in Solving for Unknown Sides

Exercise 6-8

Solve for unknown side *b* in the right triangle in Figure 6-8.

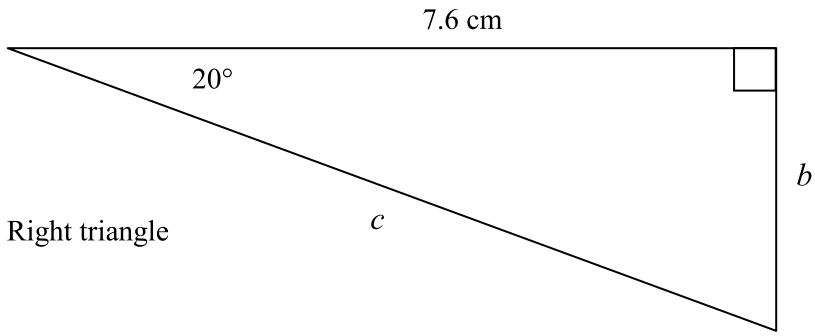


Figure 6-8: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown?	
Next, decide which interior angle you will use as a reference angle.	
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	
Create an equation using this function.	
Use algebra to solve for the unknown in the equation.	
Use your calculator to solve for the length of side *b*. Look at the triangle. Does your answer appear to be reasonable?	

Practice in Solving for Unknown Sides

Exercise 6-9

Solve for unknown side *c* in the right triangle in Figure 6-9.

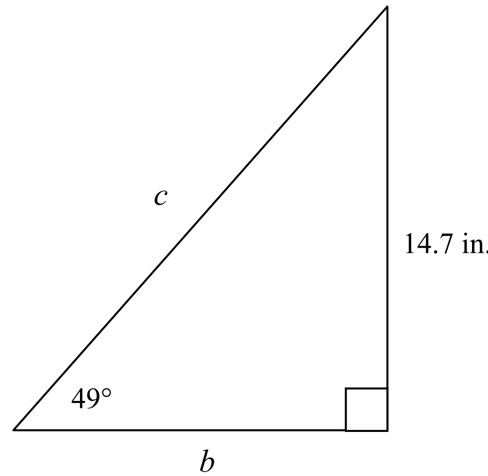


Figure 6-9: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown?	
Next, decide which interior angle you will use as a reference angle.	
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	
Create an equation using this function.	
Use algebra to solve for the unknown in the equation.	
Use your calculator to solve for the length of side *c*. Look at the triangle. Does your answer appear to be reasonable?	

Practice in Solving for Unknown Sides

Exercise 6-10

Solve for unknown side *b* in the right triangle in Figure 6-10.

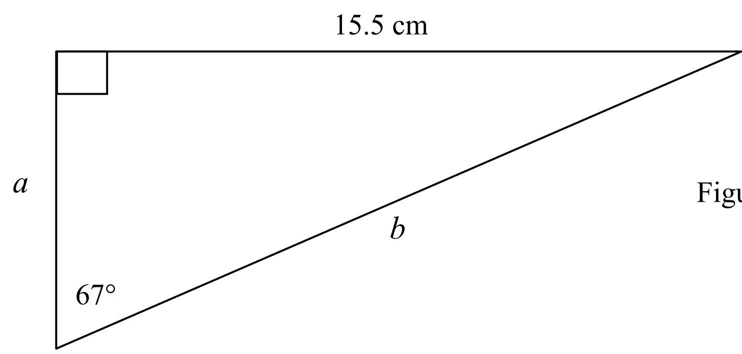


Figure 6-10: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown?	
Next, decide which interior angle you will use as a reference angle.	
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	
Create an equation using this function.	
Use algebra to solve for the unknown in the equation.	
Use your calculator to solve for the length of side *b*. Look at the triangle. Does your answer appear to be reasonable?	

Chapter 7: Solving for Unknown Angles

Introduction

Our next application of trigonometry is to solve for an unknown angle in a right triangle. To do this, you must know the length of at least two sides, and one interior angle. In this case, the known interior angle is the right angle. The other two interior angles are unknown.

See Figure 7-1 for a right triangle which has two known sides. We want to determine the value of the unknown interior angle θ. Which trigonometry function can we use to solve for the unknown angle?

The sine function relates the unknown angle θ to the two known sides as follows:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2.5\text{ ft.}}{5.2\text{ ft.}} = 0.480769231$$

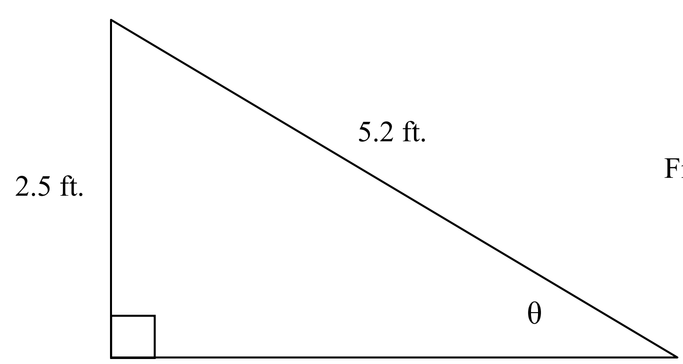


Figure 7-1: Right triangle

Let's continue solving this problem in the next example.

Solving for an Unknown Angle

Example

Here are the steps to solve this type of problem:

1. Determine which unknown you need to calculate. Do you have enough *information* to solve for this unknown?

2. In this example, in Figure 7-2, we are solving for the value of angle θ. We already know the length of two sides and one angle (the right angle), which is sufficient.

3. Next, decide which trigonometry function to use. Pick the function that relates both the two known sides and the unknown angle. As shown on the previous page, the sine function is the correct choice. Create an equation using this function.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2.5\text{ ft.}}{5.2\text{ ft.}} = 0.480769231$$

4. Use algebra to solve for the unknown in the equation.

$$\sin\theta = 0.480769231$$

$$\theta = \frac{1}{\sin} \text{ X } \frac{2.5}{5.2} = \sin^{-1} \quad (0.480769231)$$

In words, this means "θ is the angle whose sine is 0.480769231." The expression $\sin^{-1}$ represents the inverse trigonometry function for sine. In turn, $\cos^{-1}$ and $\tan^{-1}$ are the inverse functions for cosine and tangent, respectively.

5. Use your calculator to solve for angle θ. Your scientific calculator has keys for $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. These functions may be integrated with the SIN, COS, TAN buttons. Here are the key strokes for the Texas Instruments calculators:

Calculator	First operation:	Then:
TI – 30XA	2.5 divided by 5.2 = 0.480769231	$\boxed{\text{2nd}}$ $\boxed{\text{SIN}^{-1}}$ = 28.74° (rounded)
TI – 30XIIS	$\boxed{\text{2nd}}$ $\boxed{\text{SIN}^{-1}}$ (2.5/5.2) = 28.74° (rounded)	not required

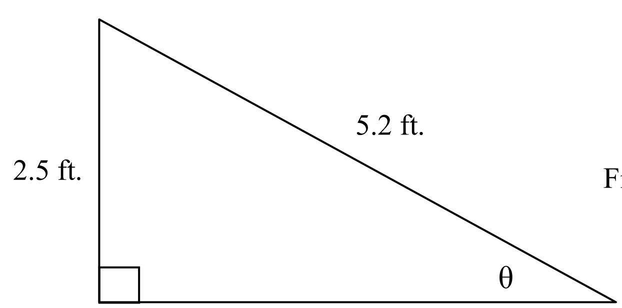


Figure 7-2: Right triangle

▲

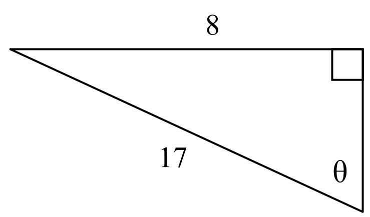
Create an Equation to Solve for an Unknown Angle

Before we continue solving this problem, we will practice the important step of creating an equation to solve for an unknown angle in a right triangle.

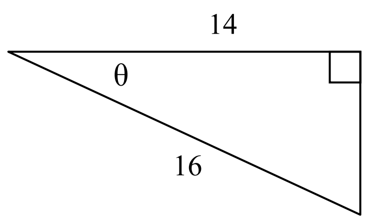
After creating an equation, use algebra to solve for the unknown.

Example

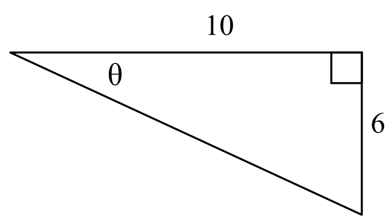
For each triangle, write an equation which relates the known information to the unknown information. Use the appropriate trigonometry function. Then, use algebra to solve for the unknown.



$$\sin\theta = \frac{8}{17} \qquad \theta = \sin^{-1}\frac{8}{17}$$



$$\cos\theta = \frac{14}{16} \qquad \theta = \cos^{-1}\frac{14}{16}$$



$$\tan\theta = \frac{6}{10} \qquad \theta = \tan^{-1}\frac{6}{10}$$

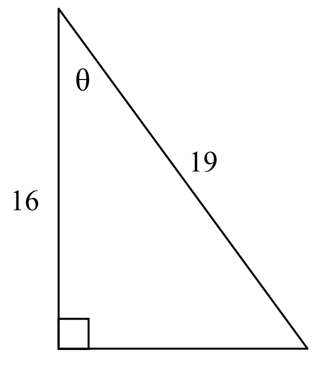
▲

Create an Equation to Solve for an Unknown Angle

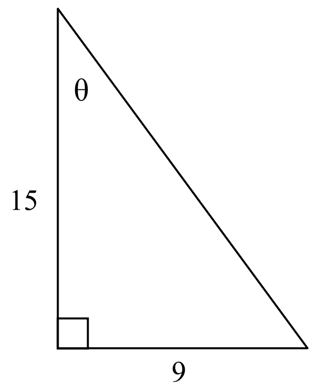
<u>Exercise 7-1</u>

For each triangle, write an equation which relates the known information to the unknown information. Use the appropriate trigonometry function. Then, use algebra to solve for the unknown, as shown in previous examples.

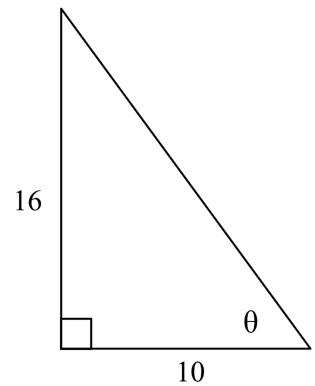
a.



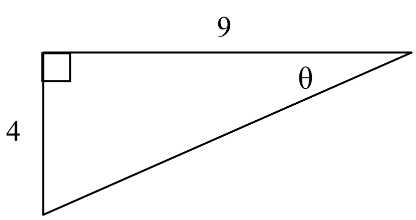
b.



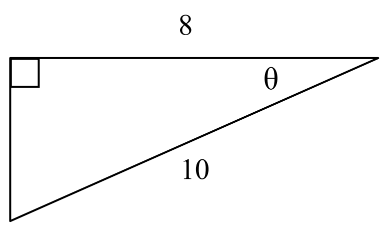
c.



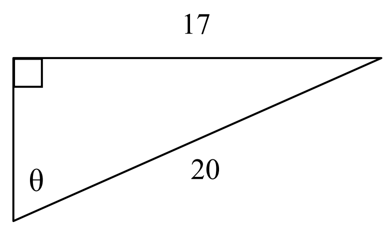
d.



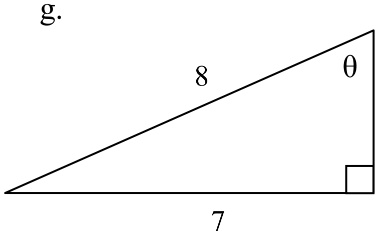
e.



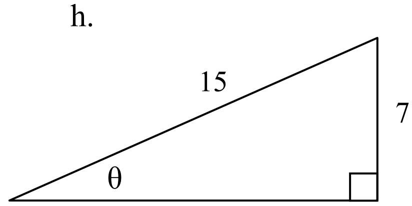
f.



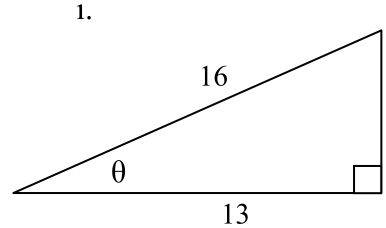
g.



h.



i.



Inverse Trigonometry Functions in Your Calculator

In the previous example, you used your calculator to find the angle whose sine is 0.480769231. You found that angle $\theta = 28.74°$. What does this mean?

This means that for *every* right triangle with a 28.74° interior angle, the *ratio* of the opposite side to the hypotenuse is 0.480769231. This is true no matter how large or small the triangle is.

Exercise 7-2

Practice using your calculator to determine an angle, using the inverse trigonometry function.

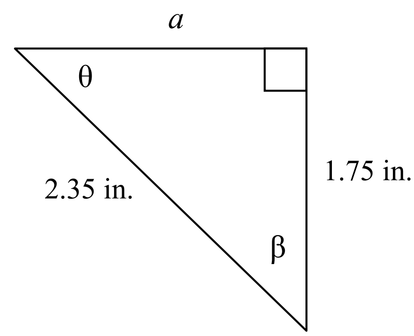
		Decimal degrees (include all decimals displayed)		Degree, minutes, seconds (to the nearest second)
$\sin^{-1} 0.15$	=	8.626926559°	=	8° 37′ 37″
$\cos^{-1} 0.85$	=		=	
$\tan^{-1} 0.35$	=		=	
$\sin^{-1} 0.75$	=		=	
$\cos^{-1} 0.23$	=		=	
$\tan^{-1} 1.12$	=		=	
$\sin^{-1} 0.95$	=		=	
$\cos^{-1} 0.55$	=		=	
$\tan^{-1} 2.25$	=		=	

Note: When solving trigonometry problems, use the value of the function provided by your calculator. Avoid writing down this value on paper, and then re-entering as a rounded value.

See the following example.

How to Avoid Rounding an Intermediate Calculation

Figure 7-3: Right triangle



Example

In Figure 7-3, determine the value of angle β.

An intermediate step is to calculate the cosine of angle β as follows:

$$\cos\beta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1.75\text{ ft.}}{2.35\text{ ft.}} = 0.744680851$$

The result of dividing 1.75 by 2.35 is an *intermediate* calculation. In other words, this result is not your final answer, but it is a calculation that you must complete before arriving at the final answer.

At this point, leave 0.744680851 displayed in your calculator. Do not write down this number and then re-enter a rounded value. Doing this will cause an inaccuracy in your final answer. The key strokes involved with the TI-30XA calculator are more susceptible to the intermediate rounding error, if you stop halfway through the series of keystrokes. The TI-30XIIS calculator includes this intermediate calculation as an integral part of the operation, making it more difficult to perform intermediate rounding.

Here are the complete series of correct keystrokes for both calculators:

Calculator	First operation:	Then:
TI – 30XA	1.75 divided by 2.35 = 0.744680851	$\boxed{\text{2nd}}$ $\boxed{\text{COS}^{-1}}$ = 41° 52′ 06″
TI – 30XIIS	$\boxed{\text{2nd}}$ $\boxed{\text{COS}^{-1}}$ (1.75/2.35) = 41° 52′ 06″	not required

▲

Errors Caused by Intermediate Rounding

In the example above, what is the effect of rounding the cosine value? Let's round the cosine value to three decimal places, 0.745

Now, use the rounded cosine value to determine the angle β.

$$\beta = \cos^{-1} 0.745 = 41.84090107° = 41° 50' 27''$$

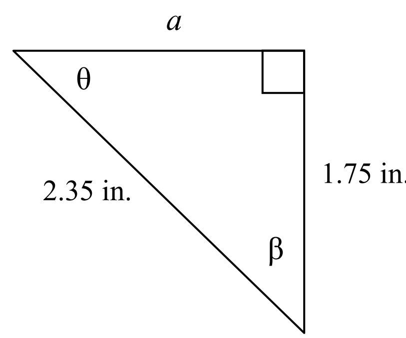
In many types of problems, this error may be unacceptable.

Complete the Solution

Exercise 7-3

From the previous example, we know that angle $\beta = 41° 52' 06''$. Now complete the solution to the triangle in Figure 7.4. The remaining unknowns are side *a* and angle θ. There are several methods to solve for each of these unknowns. Here are a few of those methods:

Figure 7-4: Right triangle



1. Solve for side *a* using the Pythagorean Theorem.

2. Solve for side *a* using the tangent of angle β.

3. Solve for angle θ using the inverse sine function.

4. Solve for angle θ using your knowledge of the sum of interior angles in a triangle.

Problem Solving Steps

Here is a summary of the steps used to solve for an unknown angle in a right triangle. The steps are illustrated using the solution for angle β in Figure 7-3.

Step	Answer
Determine which unknown you need to calculate.	angle β
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	Yes, two sides which are related to angle β are known.
Create an equation using this function.	$\cos\beta = \dfrac{1.75}{2.35}$
Solve this equation for the unknown angle using the inverse function.	$\beta = \cos^{-1}\dfrac{1.75}{2.35}$
Use your TI-30XA calculator to perform the calculations. Avoid intermediate rounding.	1.75 [÷] 2.35 [=] 0.744680851 [2nd] [COS] = 41.86830623° [2nd] [DD►DMS] = 41° 52′ 06″
Use your TI-30XIIS calculator to perform the calculations. Avoid intermediate rounding.	[2nd] [COS] (1.75 [÷] 2.35) = 41.86830623° • Next, select the units key, [° ′ ″] • Use the right [►] key until ►DMS is displayed • press [ENTER] twice • = 41° 52′ 06″

Naming Conventions for Inverse Functions

Inverse trigonometry functions are named and abbreviated in several ways. Here are names that you may find in other textbooks and technical manuals. The shaded names are the corresponding abbreviations that may also appear on your calculator.

The angle whose sine is		Arc sine	ARCSIN		Inverse sine	INVSIN		$\sin^{-1}$	$\sin^{-1}$
The angle whose cosine is		Arc cosine	ARCCOS		Inverse cosine	INVCOS		$\cos^{-1}$	$\cos^{-1}$
The angle whose tangent is		Arc tangent	ARCTAN		Inverse tangent	INVTAN		$\tan^{-1}$	$\tan^{-1}$

Practice in Solving for Unknown Angles

Exercise 7-4

Solve for unknown angle θ in the right triangle in Figure 7-5.
Express your answer in DMS format to the nearest whole second.

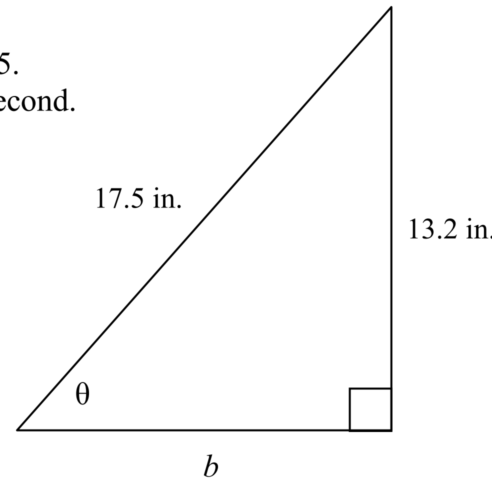


Figure 7-5: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	
Which trigonometry function relates the two known sides (using θ as the reference angle)?	
Create an equation for the unknown angle using the inverse function.	
Use your calculator to perform the calculations. Avoid intermediate rounding. Look at the triangle. Does your answer appear to be reasonable?	

Practice in Solving for Unknown Angles, continued

Exercise 7-5

Solve for unknown angle β in the right triangle in Figure 7-6.
Express your answer in DMS format to the nearest whole second.

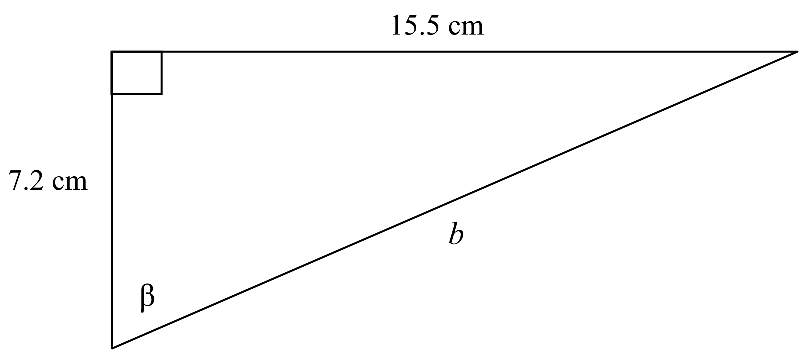


Figure 7-6: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	
Which trigonometry function relates the two known sides (using β as the reference angle)?	
Create an equation for the unknown angle using the inverse function.	
Use your calculator to perform the calculations. Avoid intermediate rounding. Look at the triangle. Does your answer appear to be reasonable?	

Practice in Solving for Unknown Angles, continued

Exercise 7-6

Solve for unknown angle θ in the right triangle in Figure 7-7.
Express your answer in DMS format to the nearest whole second.

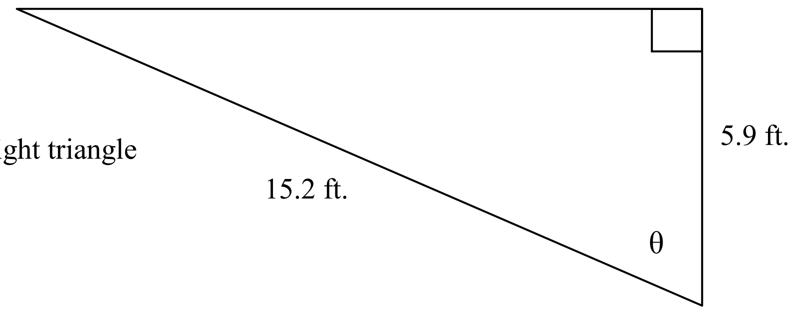


Figure 7-7: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	
Which trigonometry function relates the two known sides (using θ as the reference angle)?	
Create an equation for the unknown angle using the inverse function.	
Use your calculator to perform the calculations. Avoid intermediate rounding. Look at the triangle. Does your answer appear to be reasonable?	

Practice in Solving for Unknown Angles, continued

<u>Exercise 7-7</u>

Solve for unknown angle β in the right triangle in Figure 7-8.
Express your answer in DMS format to the nearest whole second.

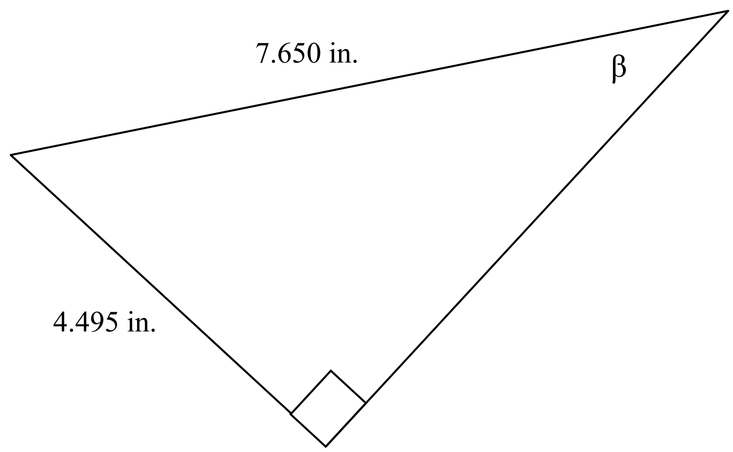
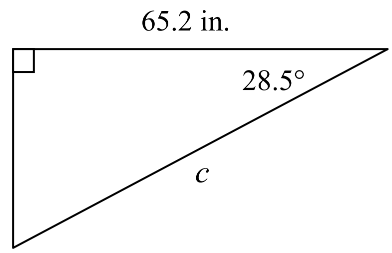


Figure 7-8: Right triangle

Step	Answer
Determine which unknown you need to calculate.	
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	
Which trigonometry function relates the two known sides (using β as the reference angle)?	
Create an equation for the unknown angle using the inverse function.	
Use your calculator to perform the calculations. Avoid intermediate rounding. Look at the triangle. Does your answer appear to be reasonable?	

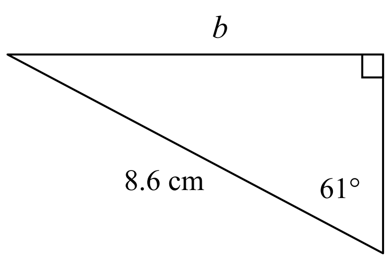
Chapters 6–7, Practice Worksheet

Include units with your answers. You may check your answers in Appendix 1.

<table>
<tr><td rowspan="2">1.
</td><td>Your estimate for the length of side c</td></tr>
<tr><td>Solve for side c using two different methods. Show your solutions.</td></tr>
</table>

Method 1

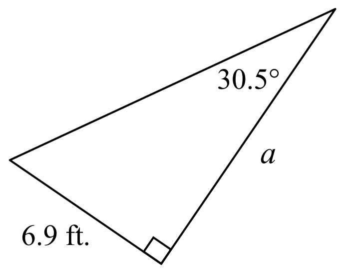
Method 2

<table>
<tr><td rowspan="2">2.
</td><td>Your estimate for the length of side b</td></tr>
<tr><td>Solve for side b using two different methods. Show your solutions.</td></tr>
</table>

Method 1

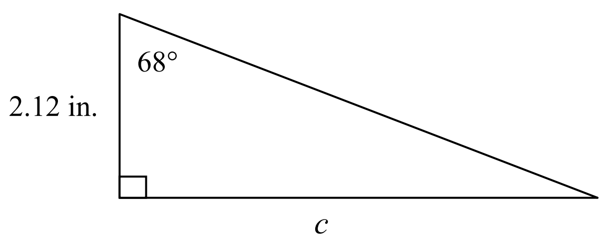
Method 2

Chapters 6–7, Practice Worksheet, continued

3. 	Your estimate for the length of side *a*
	Solve for side *a* using two different methods. Show your solutions.

Method 1

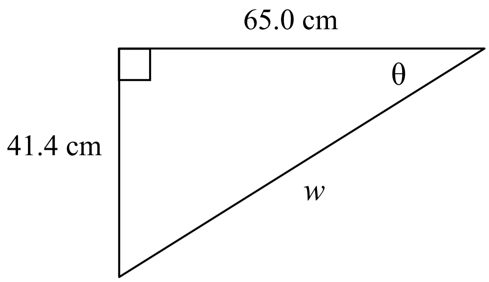
Method 2

4. 	Your estimate for the length of side *c*
	Solve for side *c* using two different methods. Show your solutions.

Method 1

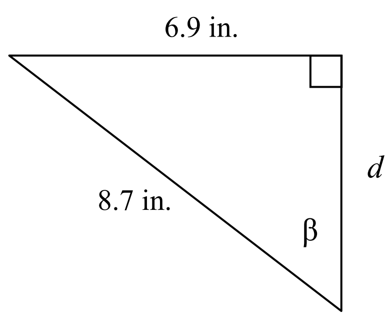
Method 2

Chapters 6–7, Practice Worksheet, continued

<table>
<tr><td rowspan="3">5.
</td><td>Your estimate for angle θ</td></tr>
<tr><td>Your estimate for side w</td></tr>
<tr><td>Solve for angle θ and side w</td></tr>
</table>

Solution for angle θ (DMS format to the nearest second).

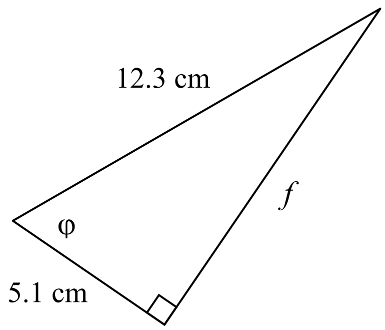
Solution for side *w* (use two different methods).

<table>
<tr><td rowspan="3">6.
</td><td>Your estimate for angle β</td></tr>
<tr><td>Your estimate for side d</td></tr>
<tr><td>Solve for angle β and side d</td></tr>
</table>

Solution for angle β (DMS format to the nearest second).

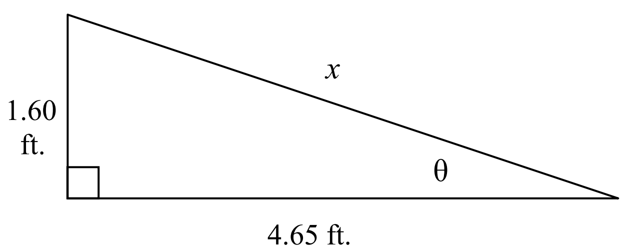
Solution for side *d* (use two different methods).

Chapters 6–7, Practice Worksheet, continued

<table>
<tr><td rowspan="3">7.
</td><td>Your estimate for angle φ</td></tr>
<tr><td>Your estimate for side f</td></tr>
<tr><td>Solve for angle φ and side f</td></tr>
</table>

Solution for angle φ (DMS format to the nearest second)

Solution for side *f* (use two different methods)

<table>
<tr><td rowspan="3">8.
</td><td>Your estimate for angle θ</td></tr>
<tr><td>Your estimate for side x</td></tr>
<tr><td>Solve for angle θ and side x</td></tr>
</table>

Solution for angle θ (DMS format to the nearest second)

Solution for side *x* (use two different methods)

Chapter 8: Mechanical Applications

Now that you have learned the basic operations in right angle trigonometry, let's apply those operations to problems that you may encounter in the fields of mechanical, civil, and architectural technology.

Mechanical Technology Applications

Here are some examples of right angle trigonometry applications in the mechanical technology field. We will discuss examples of these calculations in this chapter:

- Calculate the drill point depth to use in programming a drilling operation.
- Calculate the size of a gage block stack to use with a sine bar (measurement of an angle).
- Calculate the drill depth for an engraving operation.
- Identify X and Y locations on a part.
- Calculate the hole locations on a bolt-hole circle.
- Inspect a v-block with a gage pin.
- Inspect a dovetail slide with gage pins.
- Calculate the angle of a tapered mechanical part.

In all of these applications, you will need to sketch a right triangle as part of the solution. The right triangle that you sketch will help you to determine the answer to the problem.

Drill Point Depth

In a mechanical part, you may find a dimension for a *blind hole*. A blind hole is a hole that is not drilled all of the way through the part. In Figure 8-1, you can see that the depths of the holes are specified, but the dimensions do not include the depth of the drill point.

If this part is made on a CNC machine, the CNC program must include the drill depth to the end of the drill point.

You can use trigonometry to calculate the depth of the drill point. This depth must then be added to the drill depth that is shown on the drawing.

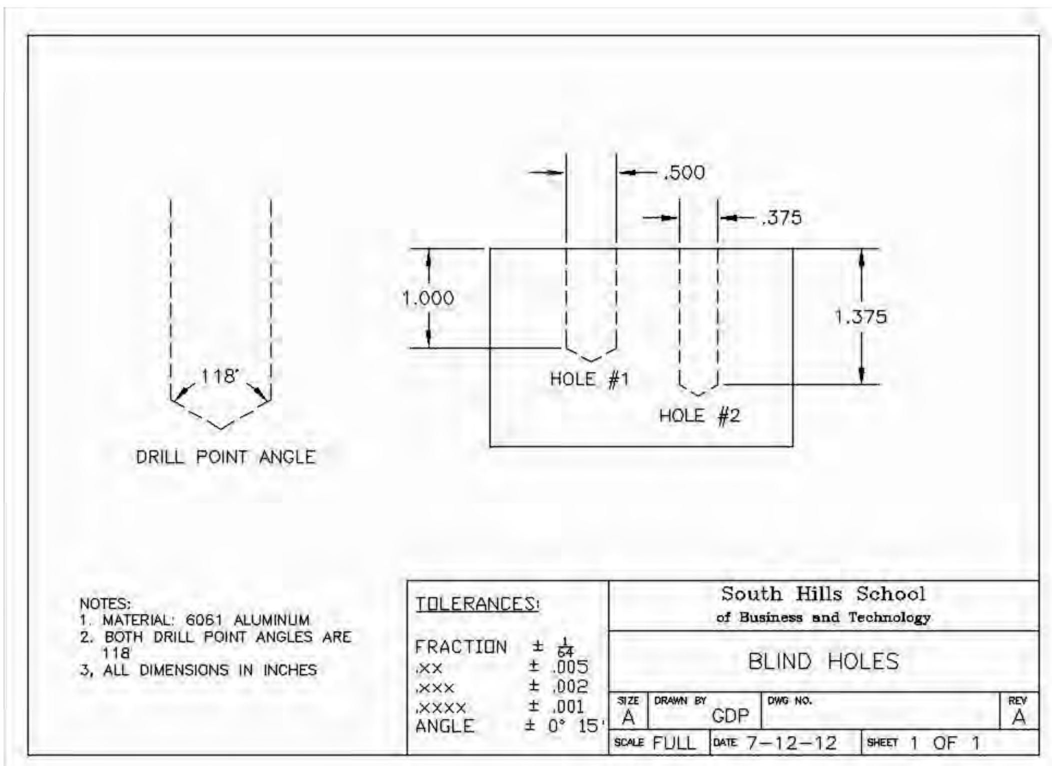


Figure 8-1: Blind holes

You must first sketch a right triangle on the drill point, as shown in Figure 8-2.

- Draw a horizontal line that separates the body of the drill from the drill point.
- Draw a line vertically from the drill point. This line, which is called a *bisector*, divides the 118° angle in half. Each right triangle then has a 59° angle.
- The side opposite the 59° angle has a length of 1/2 the drill diameter.

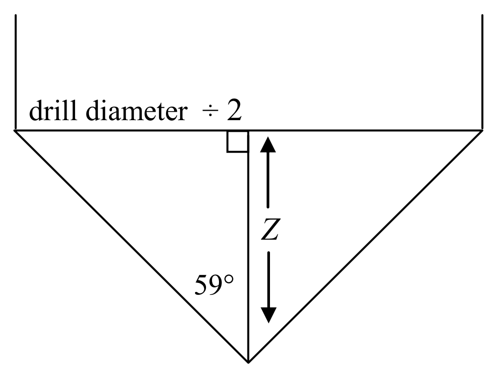


Figure 8-2: Drill point

Drill Point Depth, continued

The unknown side Z can now be calculated using the tangent function. See the following example.

Example

Solve for the drill point depth for Hole #1 in Figure 8-1.

In Figure 8-2, the side opposite the 59° angle is half of the diameter of the blind hole. From Figure 8-1, we can see that the diameter of Hole #1 is 0.500.

$$\tan 59° = \frac{\text{opposite}}{\text{adjacent}} = \frac{(0.500/2)}{Z}$$

Solve this equation for the unknown side *Z*. Round your answer to 3 decimal places. In the mechanical field, 3 decimal dimensions are common. There are some close tolerance situations that require 4-decimal accuracy.

$$Z = \frac{0.250}{\tan 59°} = 0.150$$

Conclusion: From Figure 8-1, we see that the depth of the 0.500 diameter hole should be 1.000. Therefore, in a CNC program, the drill depth will be 1.000 + 0.150 =1.150. The drill point will reach a depth of 1.150. This is necessary to make sure that the full diameter of the drill, 0.500, reaches a depth of 1.000.

▲

<u>Exercise 8-1</u>

Solve for the drill point depth for Hole #2 in Figure 8-1. Create a sketch of the drill point to help you solve the problem.

Angularity Measurement with a Sine Bar

Some mechanical parts may have a surface that is angled to the rest of the part. An example of this is shown in Figure 8-3.

There are several methods for specifying the tolerance on this angle. One method is to use the basic angle tolerance which is shown in the title block as +/– 0° 15′. A basic angle tolerance is measured directly using a bevel protractor.

If the tolerance must be much closer than a basic angle tolerance, then an angularity tolerance can be specified, as shown in Figure 8-3. Angularity is part of a tolerancing system called Geometric Dimensioning and Tolerancing (GD&T). If an angularity tolerance is specified, it takes precedence over the basic angle tolerance shown in the title block of the drawing.

In Figure 8-3, the angularity tolerance is shown as .003 in. The angularity is referenced to surface A, which is labeled as the bottom surface of the part. An angle must be referenced to another feature on a part, such as the bottom surface on this part.

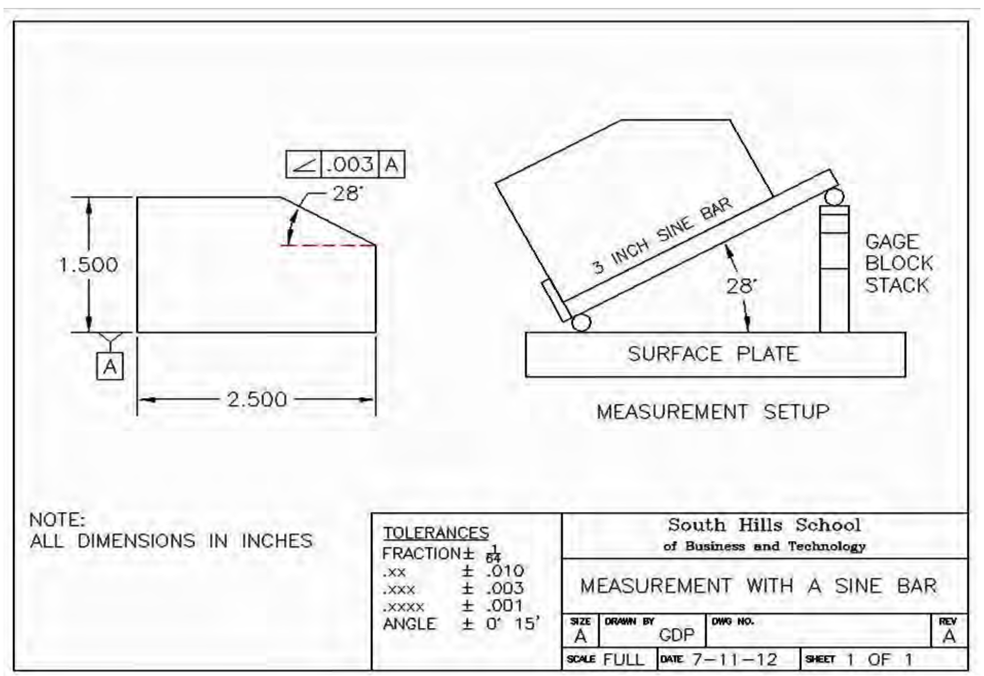


Figure 8-3: Measurement with a sine bar

What is the meaning of the .003 in. tolerance? The next page describes how to use this tolerance to measure the angled surface to verify that the part was made properly.

Angularity Measurement with a Sine Bar, continued

In order to measure this surface with an angularity tolerance, the part must be set up on a *surface plate* and a *sine bar*. The surface plate acts as a reference surface, and the sine bar is used to simulate the 28° angle on the part, as shown on the right side in Figure 8-3.

The elevated sine bar causes the angled surface to become parallel to the surface plate. The sine bar gets its name from the *sine function* which is used to solve the problem. The problem is to determine the size of the gage block stack necessary to elevate the sine bar at the desired angle.

A cylinder is attached beneath each end of a sine bar. In turn, sine bars are purchased based on the distance between these cylinders (2, 3, 5, or 10 inches), as shown in Figure 8-4.

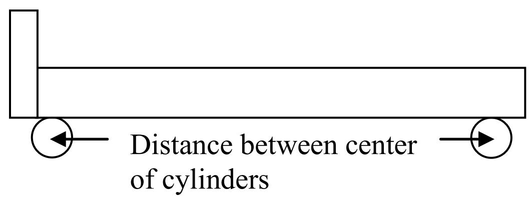


Figure 8-4: Sine bar size definition

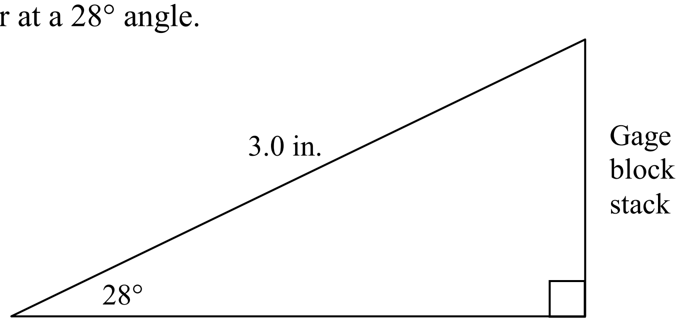
Gage blocks are precision metal blocks that are typically purchased in a set of 81 blocks. The variety of dimensions in this 81-block set allows you to create virtually any dimension by combining blocks.

The first step is to sketch the right triangle created by the sine bar and the gage blocks. This will allow you determine the desired dimension of the stack of gage blocks.

Example

This example refers to the part in Figure 8-3. Figure 8-5 is a sketch of the right triangle created by the sine bar and the gage blocks. The distance between the cylinders on the sine bar creates the hypotenuse of the right triangle. Solve this triangle to determine the dimension of the gage block stack necessary to elevate the sine bar at a 28° angle.

Figure 8-5: Right triangle created by sine bar and gage blocks



$$\sin 28° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{gage blocks}}{3.0\text{ in.}}$$

Example continued on the next page

Angularity Measurement with a Sine Bar, continued

Example, continued

Solve this equation for the gage block dimension. Round your answer to 4 decimal places. A 4-decimal answer allows you to take full advantage of the gage block set, which is capable of creating dimensions out to 4 decimal places.

$$\text{gage blocks} = (3.0 \text{ in.})(\sin 28°) = 1.4084 \text{ in.}$$

▲

Once this setup is complete, you would measure the surface. The measurement is made using a surface gage and a test indicator (similar to a dial indicator). The entire angled surface is measured with the indicator. If the surface was manufactured at exactly 28° to surface A, the pointer on the indicator dial will not move. A movement of up to .003 in. on the indicator dial is allowed.

See Figure 8-6 for a sketch of the measurement setup. Refer to a Dimensional Metrology text for a more detailed description of the measuring procedure.

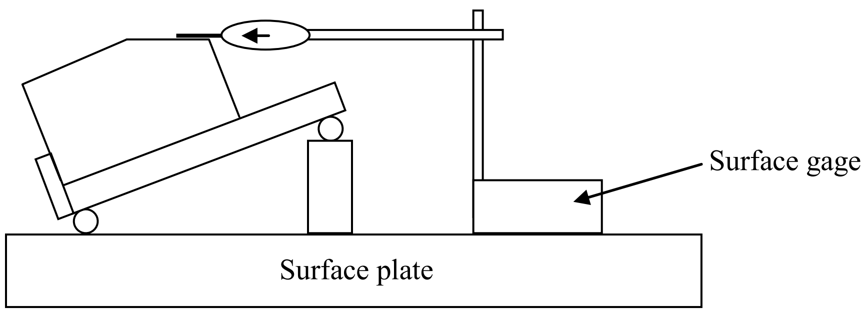
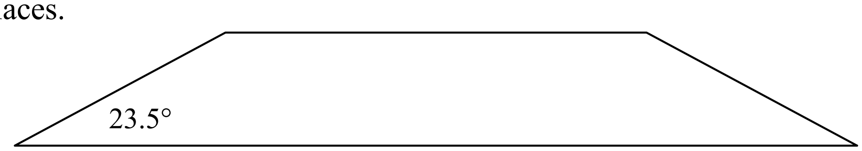


Figure 8-6: Angularity measurement setup

<u>Exercise 8-2</u>

Calculate the gage block stack necessary to measure the angled surface in the part shown in Fig. 8-7. Express your answer to 4 decimal places.

Figure 8-7: Part with an angled surface



Assume that the measurement setup will use a 5-inch sine bar (distance between the cylinders).

Drill Depth for Engraving

Another operation that requires a trigonometry calculation is engraving. A mechanical part may require engraved text as part of the manufacturing process. See the example in Figure 8-8.

The engraving in this example is performed on a milling machine with an engraving tool. The engraving tool has a 60° point and a 1/8" diameter. The width of the engraved letters is determined by how deep the tool is plunged into the top of the part.

Section A-A in the drawing is a side view to show the shape of the engraving created by the engraving tool. Detail B is a close-up view of section A-A. Detail B shows that the desired width of the lettering is 0.080 inch.

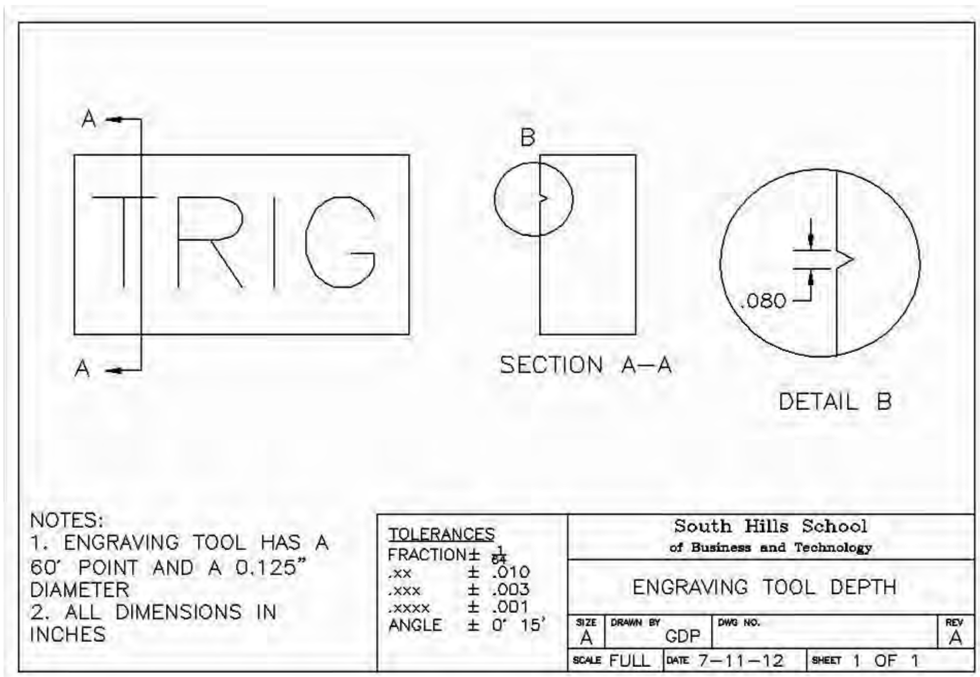


Figure 8-8: Engraving tool depth

Example

The first step is to sketch a right triangle on the engraving tool as shown in Figure 8-9. This right triangle must include the tool depth to be used in the engraving operation.

Use *Z* for the unknown tool depth. *Z* is typically used in milling operations for the depth dimension.

Example continued on the next page

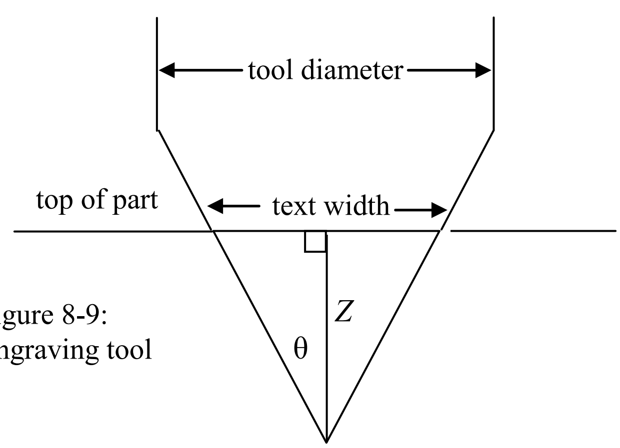


Figure 8-9:
Engraving tool

Drill Depth for Engraving, continued

Example, continued

Now you can use this right triangle to solve for the tool depth Z. Recall that the tool point angle is 60° and the text width is .080 inch. Note that the right triangle *bisects* the tool point angle. The right triangle also bisects the desired text width.

$$\tan(60/2)^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{(0.080/2)\text{ in.}}{Z}$$

Solve this equation for the tool depth Z. Round your answer to 3 decimal places.

$$Z = \frac{(0.080/2)\text{ in.}}{\tan(60/2)^\circ} = \frac{0.040\text{ in.}}{\tan 30^\circ} = 0.069\text{ in.}$$

The tool depth that will achieve the desired text width is 0.069 in.

▲

Note that the diameter of the engraving tool (in this case 1/8 inch) is not important to the problem. As long as the tool diameter is larger than the desired text width, then the tool depth is what determines the width of the text

Exercise 8-3

Solve for the engraving tool depth, given the following information:

- Tool point angle: 60°
- Tool diameter: 3/16 inch
- Desired text width: 0.125 inch

As part of the solution, sketch and label the tool and the right triangle that you need to solve the problem.

Identify X and Y Locations on a Part

Computer Numerical Control (CNC) machines can machine a precision part by taking instructions from a computer program. The CNC program uses the Cartesian coordinate system to position the machine tool on the part. The coordinates used are *X*, *Y*, and *Z*, with *Z* being the depth coordinate.

If you needed to write a CNC program, you first need to know the *X* and *Y* coordinates of key locations on the part. See Figure 8-10 for a part that will be machined with a CNC program.

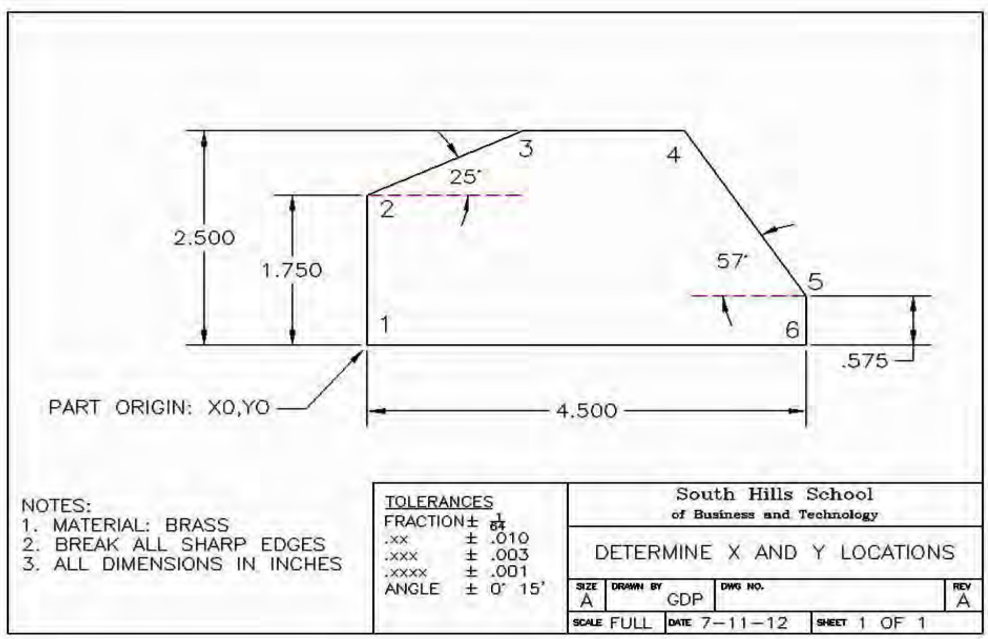


Figure 8-10: Part to be manufactured with a CNC program

See Figure 8-11 for a review of the Cartesian coordinate system. The part is typically placed in Quadrant I to ensure that all locations on the part have positive *X* and *Y* coordinates.

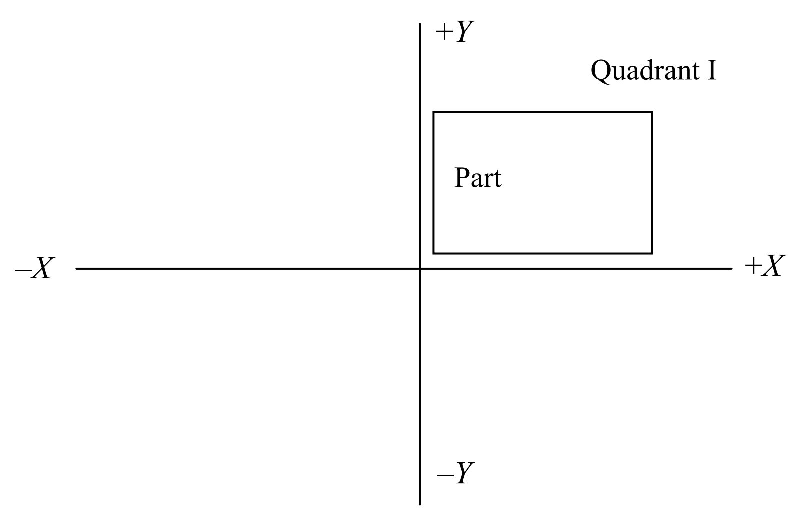


Figure 8-11: Cartesian coordinates

Identify *X* and *Y* Locations on a Part, continued

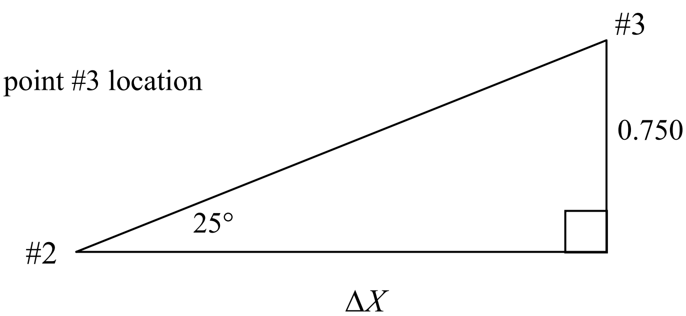
To place the part in Quadrant I, the lower left corner of the part is set at a position of X 0.000 and Y 0.000.

Many of the X and Y locations of the labeled points can be determined by reading and interpreting the dimensions on the drawing. Some of the locations must be determined using trigonometry. Let's calculate the X location of point #3 in Figure 8-10.

Example

The first step is to sketch the right triangle needed to solve the problem. Sketch a triangle whose hypotenuse is the line segment between points #2 and #3. We will use ΔX to stand for the unknown side of this triangle. The ΔX value is equal to the X location of point #3.

Figure 8-12: Sketch to solve for point #3 location



Solve for ΔX in Figure 8-12.

$$\tan 25° = \frac{\text{opposite}}{\text{adjacent}} = \frac{0.750 \text{ in.}}{\Delta X}$$

Solve this equation for the unknown side ΔX. Round your answer to 3 decimal places.

$$\Delta X = \frac{0.750 \text{ in.}}{\tan 25°} = 1.608 \text{ in.}$$

The X location of point #3 is 1.608 in.
▲

Exercise 8-4

Determine the X location of point #4 in Figure 8-10. As part of your solution, sketch and label the right triangle that you need to solve the problem.

Hole Locations on a Bolt-Hole Circle

Another mechanical application is the determination of the *X* and *Y* locations for holes in a plate. A common design is called a *bolt-hole circle*. A number of holes are located on this circle, which has a known radius.

An example of a bolt-hole circle on a plate is shown in Figure 8-13. The six holes are equally spaced around the circle. The lower left corner of the plate is positioned at X0.000 and Y 0.000 in the Cartesian coordinate system.

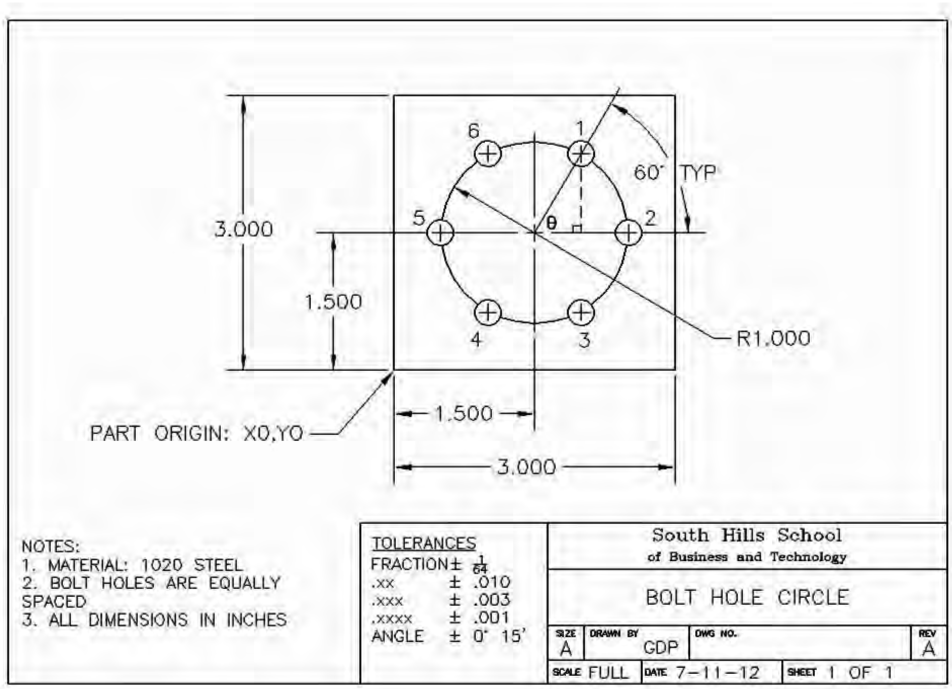


Figure 8-13: Bolt-hole circle

Start by determining the *X* and *Y* coordinates of Hole #1. As with the previous problems, you will start by sketching a right triangle which will help to solve the problem. See how the sketch in Figure 8-14 relates to the drawing in Figure 8-13.

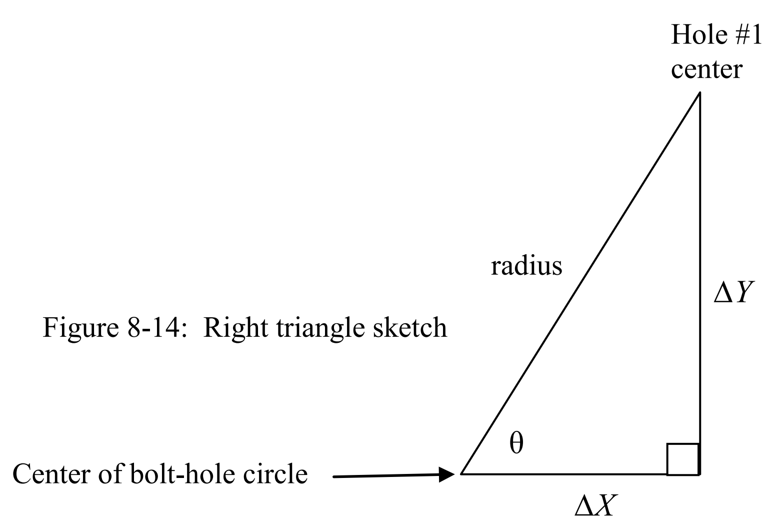


Figure 8-14: Right triangle sketch

Hole Locations on a Bolt-Hole Circle, continued

Here is an explanation for some of the notations used on the drawing in Figure 8-13:
The dimension marked with R indicates the radius of the bolt-hole circle.
The TYP notation is an abbreviation for typical and indicates that all of the angles between the six holes are the same.

Example

First, solve for solve for ΔX in Figure 8-13. The angle θ and the radius of the circle are known.

$$\cos 60° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\Delta X}{1.000}$$

Solve this equation for the unknown side ΔX. Round your answer to 3 decimal places.

$$\Delta X = (1.000)(\cos 60°) = 0.500 \text{ in.}$$

This calculation locates Hole #1 in the X direction from the center of the circle. Now finish the calculation by determining the X location of Hole #1 from the lower left corner of the part. For the purposes of locating the hole, the lower left corner is the *part origin*.

X value of hole #1 = $\Delta X + X$ at the center of the circle = 0.500 + 1.500 = 2.000 in.

Next, solve for ΔY in Figure 8-13.

$$\sin 60° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\Delta \mathrm{Y}}{1.000}$$

Solve this equation for the unknown side ΔY. Round your answer to 3 decimal places.

$$\Delta Y = (1.000)(\sin 60°) = 0.866 \text{ in.}$$

Y value of hole #1 = $\Delta Y + Y$ at the center of the circle = 0.866 + 1.500 = 2.366 in.

Exercise 8-5

Solve for the X location of Hole #4 in Figure 8-13. Include a labeled sketch of the right triangle that you need to help solve the problem. Use the next page if you need more room.

Hole Locations on a Bolt-Hole Circle, continued

Exercise 8-6

Solve for the *Y* location of Hole #4 in Figure 8-13. Include a labeled sketch of the right triangle that you need to help solve the problem.

Exercise 8-7

Complete the solution for the *X* and *Y* locations of all holes in Figure 8-13. Express your answers to three decimals.

Hole	X	Y
1		
2		
3		
4		
5		
6		

V-Block Inspection

A v-block is a mechanical device that is used to hold round parts. A round part can be held during a machining process, or the part can be held during an inspection process.

This example is a discussion of how the v-block itself is inspected, to verify that it was manufactured to specifications.

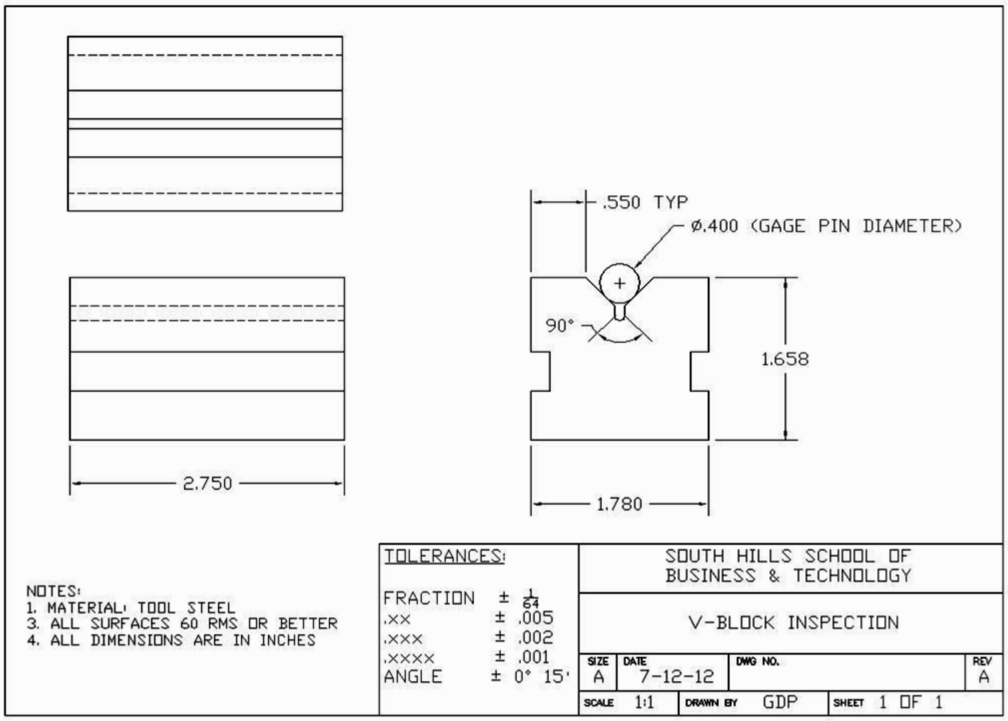


Figure 8-15: V-block inspection

A v-block is inspected using a precision gage pin. A gage pin is manufactured to very close tolerances, making it useful in the inspection of mechanical parts.

The gage pin is placed in the v-block as shown in Figure 8-15. This gage pin has a diameter of 0.400 inch. The symbol Ø is used in drawings to indicate a diameter dimension.

As we saw in an earlier drawing, the TYP notation is used in this drawing. The .550 TYP (typical) notation means that the horizontal surface to the right of the “vee” is the same dimension as the horizontal surface to the left of the “vee”.

V-Block Inspection, continued

We will assume that the angle of the "vee", shown as 90° on the drawing, has already been inspected and is within the allowed tolerance.

The next inspection is to check the depth of the "vee" into the block. A gage pin of known diameter, in this case 0.400 inch, is placed in the "vee" of the v-block. Figure 8-16 shows the details of the "vee". Dimension H is the expected dimension, which confirms that the depth of the "vee" was manufactured correctly. Dimension H represents how far the top of the gage pin extends beyond the top of the v-block. For example, if the "vee" was machined too deeply into the block, the H dimension found during inspection will be too low.

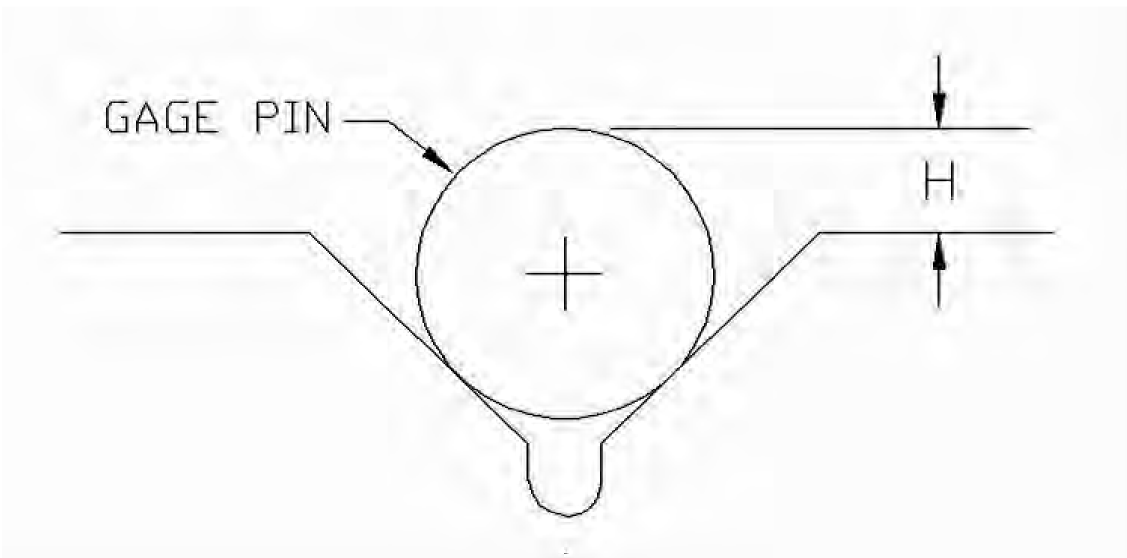


Figure 8-16: Calculation of the inspection dimension, H

Exercise 8-8

Sketch two right triangles on Figure 8-16. These triangles must be helpful in solving for the unknown dimension H.

Label the vertices on these two triangles so that equations can be created to help solve for H.

Hint: A radius drawn to a tangent line is perpendicular to the tangent line.

V-Block Inspection, continued

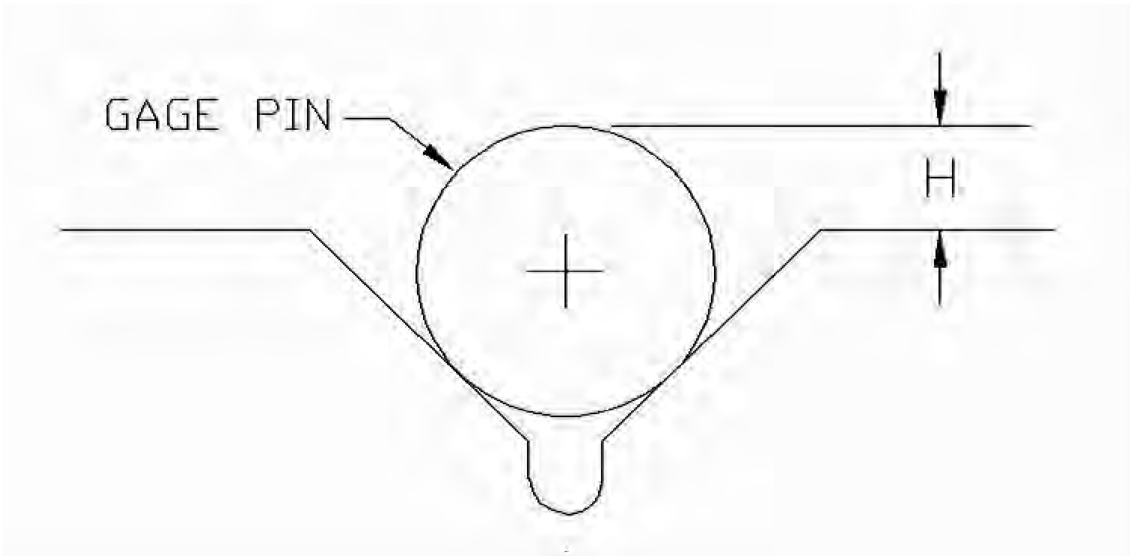


Figure 8-17: Calculation of the inspection dimension, H

Exercise 8-9

Create equations that can be used to solve for H.

Exercise 8-10

Calculate dimension H. See Figure 8-15 for the necessary dimensions. Express your answer to three decimals. All dimensions are in inches.

Dovetail Slide Inspection

A dovetail slide is a mechanical device that is found on many machines. For example, you may find a dovetail slide on a milling machine and on a lathe. The slide is used to facilitate precision linear movement.

A dovetail slide is inspected using precision gage pins, as shown in Figure 8-18. As shown on the drawing, the gage pin diameter is 0.220 inch. The inspection dimension, *X*, is the distance across the outside of the two gage pins. Assume that the two angles of the vees were already inspected with a bevel protractor and confirmed to be 60°.

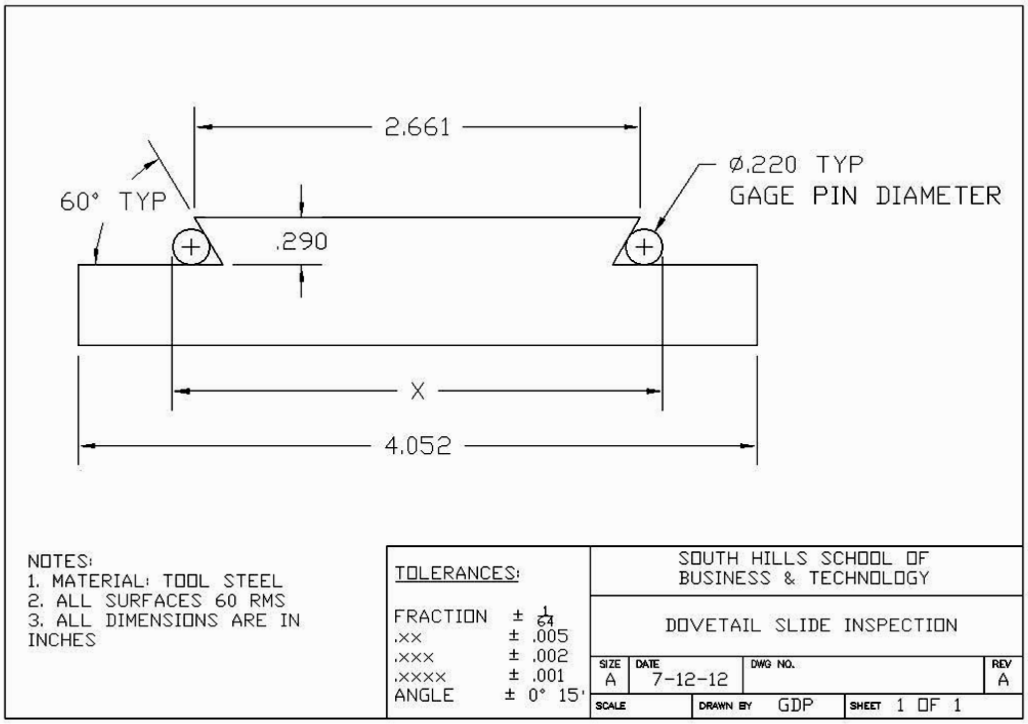


Figure 8-18: Dovetail slide inspection

<u>Exercise 8-11</u>

Sketch two right triangles on Figure 8-19. These two triangles must be useful in the calculation of the inspection dimension, *X*.

Label the vertices of the triangles to help with the solution.

Hint: A radius drawn to a tangent line is perpendicular to the tangent line.

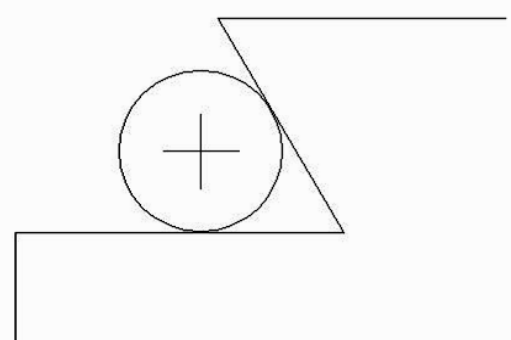


Figure 8-19: Slide detail

Dovetail Slide Inspection, continued

Exercise 8-12

Calculate the dimension *X*. This is the expected dimension, which confirms that the dovetail was manufactured correctly.

Express your answer to three decimals. All dimensions are in inches.

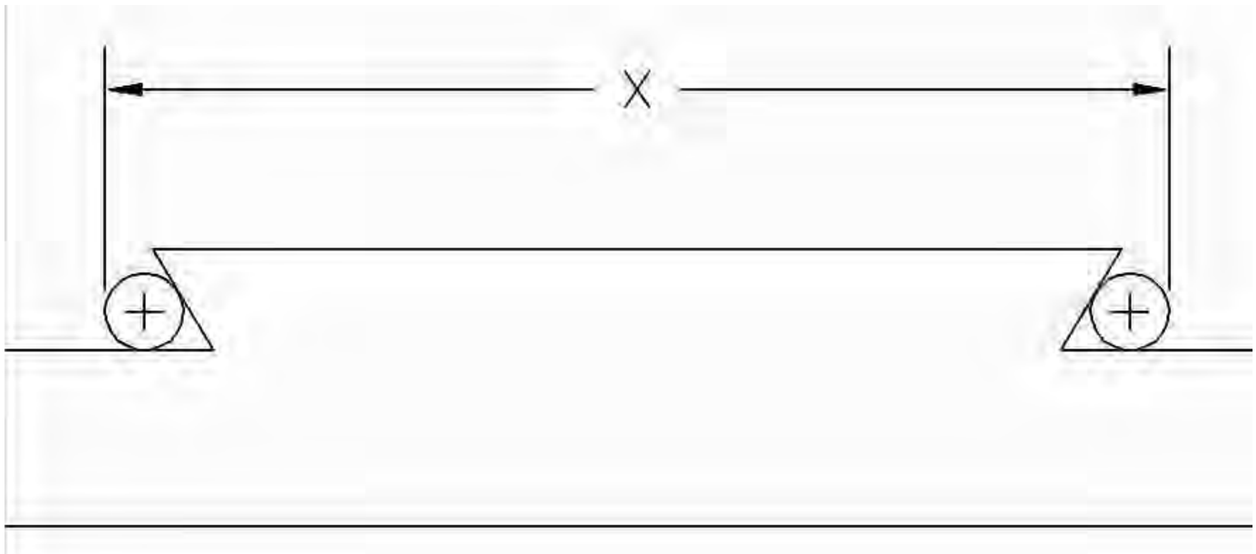


Figure 8-20: Dovetail slide inspection dimension

Tapers

Tapers are a common mechanical device in machines. For example, tapers are used to mount tools into milling machines and lathes. The taper provides a secure holding method, such as the tapered shank on the drill in Figure 8-21.

Figure 8-21: Tapered shank drill

An example of a tapered part is shown in Figure 8-22 below. The two diameters on the side view represent the largest and smallest diameters on the part.

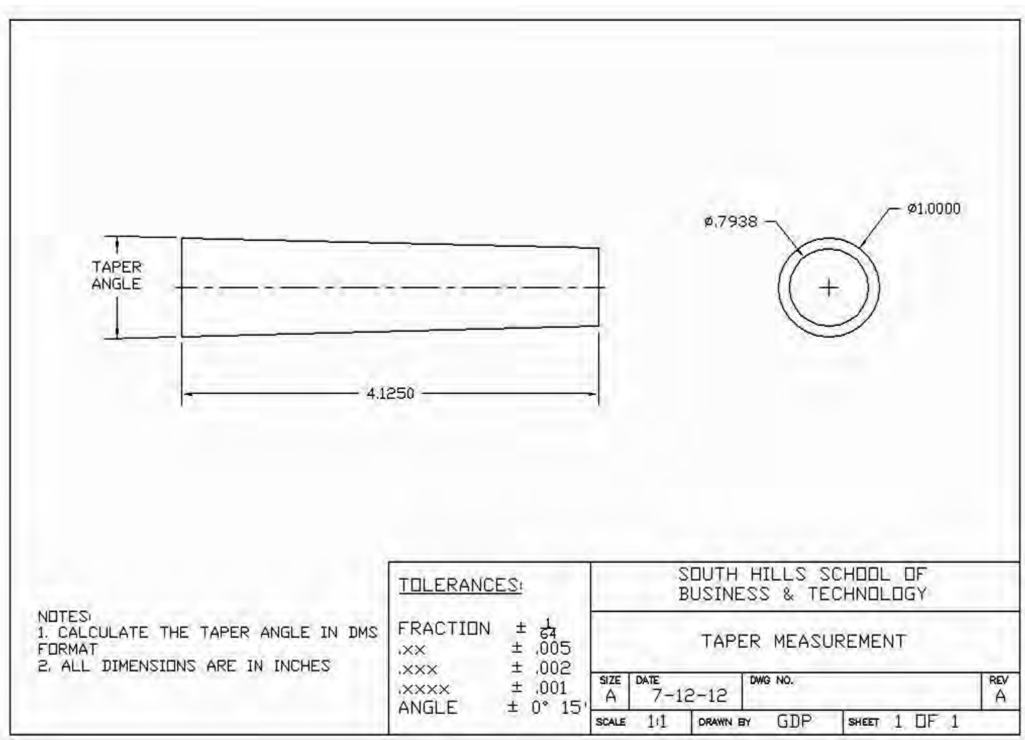


Figure 8-22: Taper

The angle of the taper can be calculated using trigonometry. This is the angle created on the taper as the diameter increases from one end of the taper to the other end.

Tapers, continued

Exercise 8-13

On Figure 8-23, sketch two right triangles that will help to calculate the angle of the taper, α.

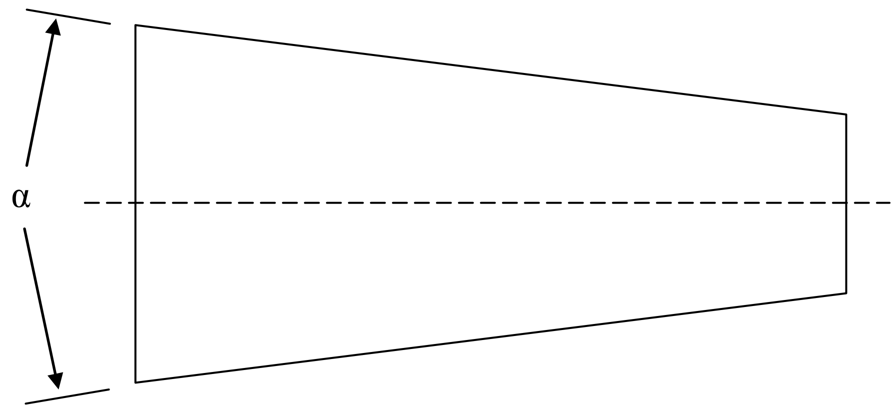


Figure 8-23: Taper sketch

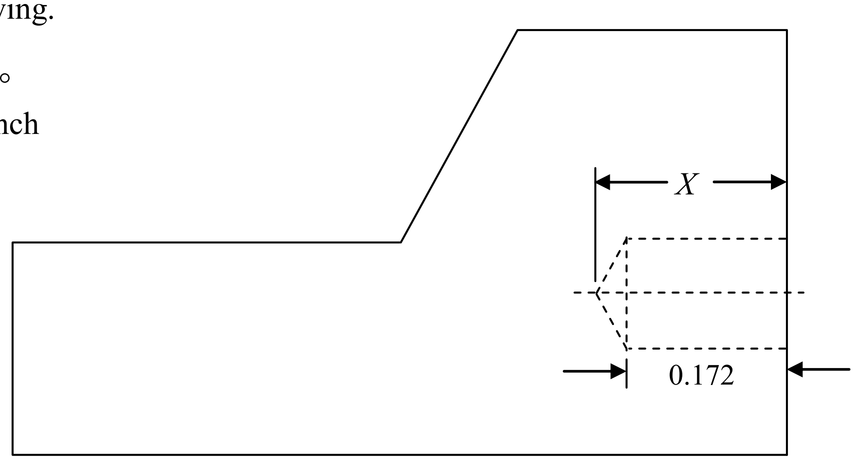
Exercise 8-14

Calculate the taper angle, α, for the taper shown in Figure 8-22. Express your answer in the DMS format to the nearest second.

Practice Worksheet

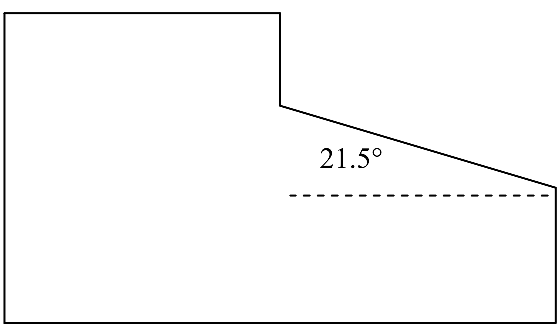
1. Solve for the drilling depth *X*. First, solve for the height of the drill point, and then add that value to the depth dimension on the drawing.

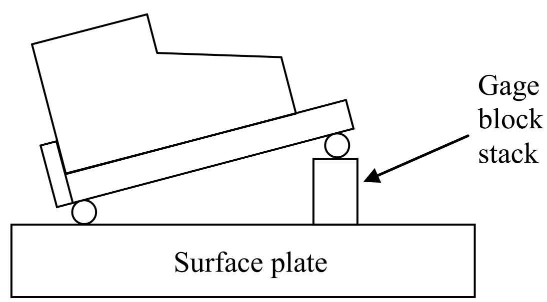
- The drill point angle is 118°
- The drill diameter is 5/32 inch
- Answer to 3 decimals



2. Calculate the size of the gage block stack necessary to measure the angled surface in the part shown below. Express your answer to 4 decimal places.

- Assume that the measurement setup will use a 5-inch sine bar (distance between the cylinders).

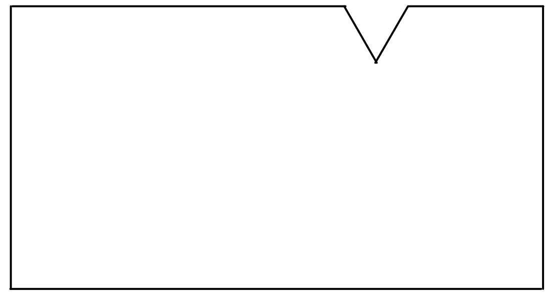




Practice Worksheet, continued

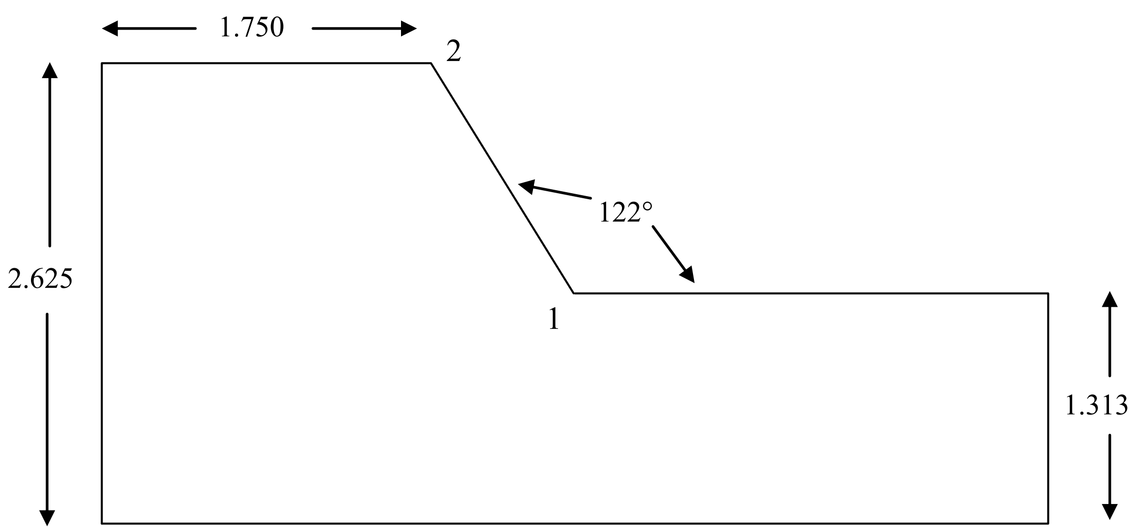
3. Solve for the engraving tool depth, given the following information:

- Tool point angle: 60°
- Tool diameter: 3/16 inch
- Desired text width: 0.008 inch



As part of the solution, sketch and label the tool and the right triangle that you need to solve the problem. Round your answer to 3 decimal places.

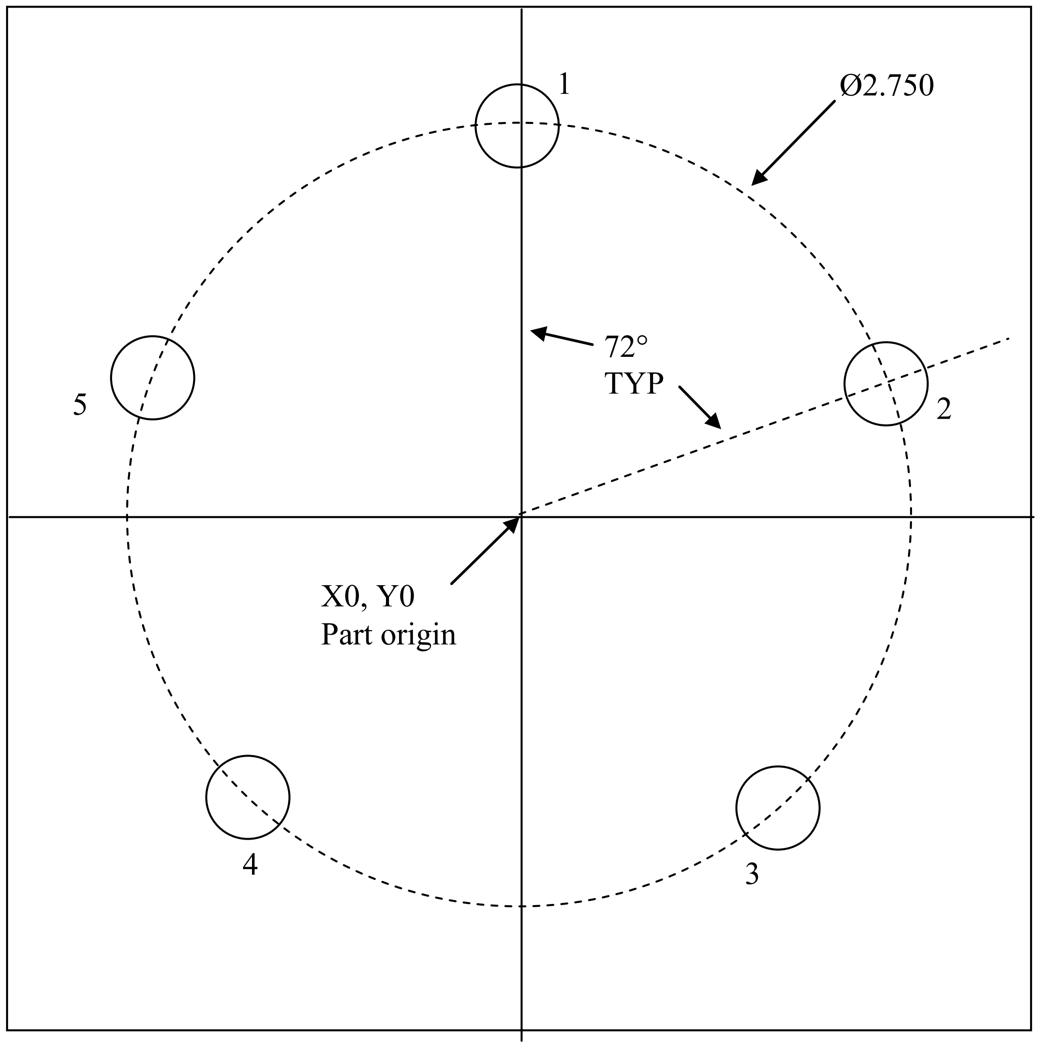
4. Determine the *X* location of point #1 on this part. As part of your solution, sketch and label the right triangle that you need to solve the problem. Answer to 3 decimals.



Practice Worksheet, continued

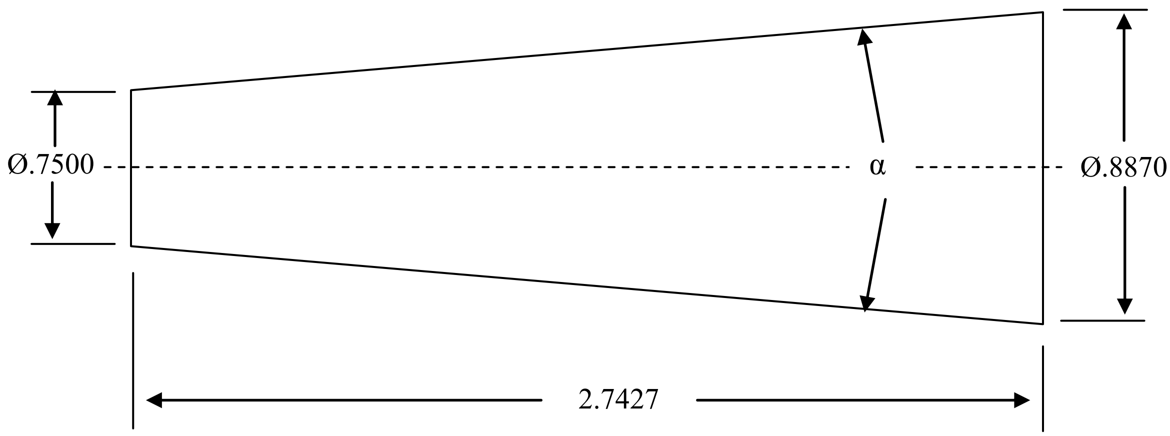
5. Solve for the *X* and *Y* locations of holes #2 and #4 in this part.

- The center of the part has *X* and *Y* coordinates of X0, Y0 (part origin).



Practice Worksheet, continued

6. Calculate the taper angle, α, in this taper. Express your answer in DMS format to the nearest second.



Chapter 9: Civil Applications

Now let us look at some applications of trigonometry in the civil technology field.

Civil Technology Applications

Here are some examples of right angle trigonometry applications in the civil technology field. We will discuss examples of these calculations in this chapter:

- Calculation of horizontal distance and slope angle
- Calculation of the height of a structure
- Calculation of distance across an obstruction
- Calculation of the Northing and Easting coordinates of a survey point
- Bearing format for directions
- Slope angle and grade

As in the previous chapter, you will need to sketch a right triangle as part of the solution. The right triangle that you sketch must help you to determine the answer to the problem.

Horizontal Distance and Slope Angle

Every field of study has a unique way of measuring and displaying precision and units. In civil engineering, distances are expressed in decimal feet to two decimals. These units are used when measuring and displaying both horizontal and vertical distances, or *elevations*.

A surveying operation is shown in Figure 9-1. A *transit* and tripod are set up on a sloped terrain, at point A. Point A is a *benchmark*, or a reference point, for other locations on this property. The transit scope is leveled and aimed towards Point B.

At Point B, a surveying assistant holds a *level rod* vertically with the help of a level. The level rod is graduated in decimal feet to two decimals.

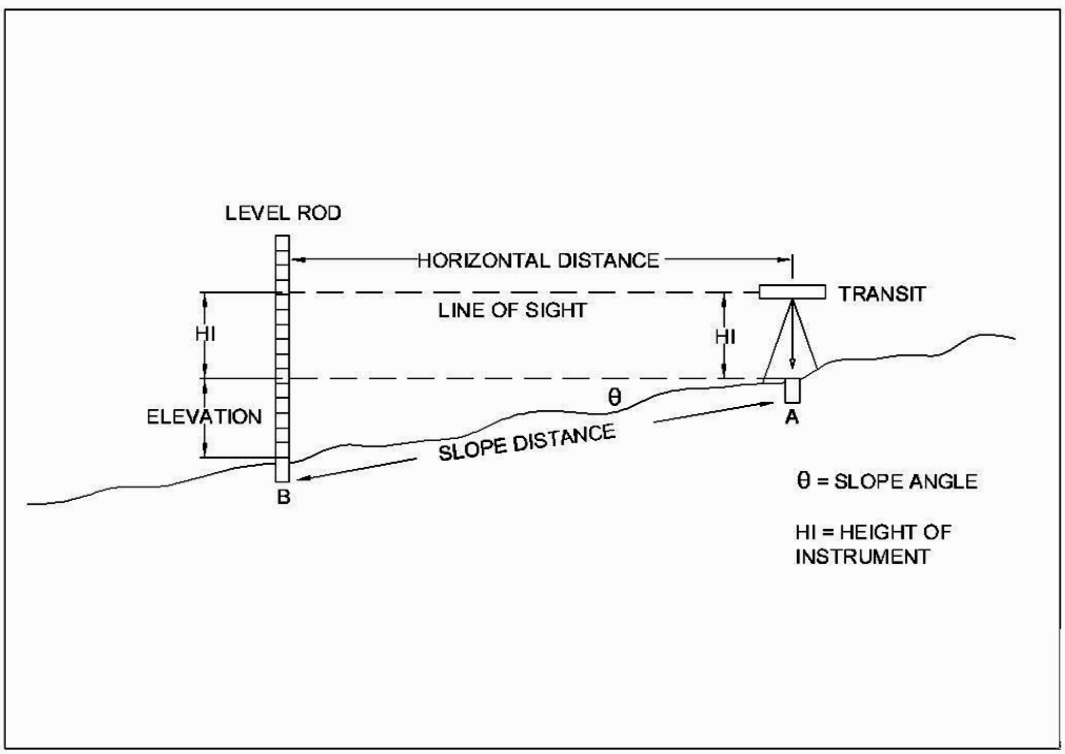


Figure 9-1: Surveying operation

For the purposes of mapping, the distance between points A and B must be shown as a *horizontal distance*. This distance can be measured directly. In this problem, we will calculate horizontal distance using other measurements. The surveyors will provide us with a distance measured along the slope, the elevation measurement, and the height of instrument above point A.

We will also calculate the slope angle, θ, from the given information.

Horizontal Distance and Slope Angle, continued

Example

Calculate the horizontal distance and slope angle in Figure 9-1 given the following information:

- Slope distance = 34.20 feet
- Reading on the level rod = 11.18 feet
- Height of instrument (HI) = 5.82 feet

First, draw a sketch of the right triangle that will help you to solve the problem. See Figure 9-2.

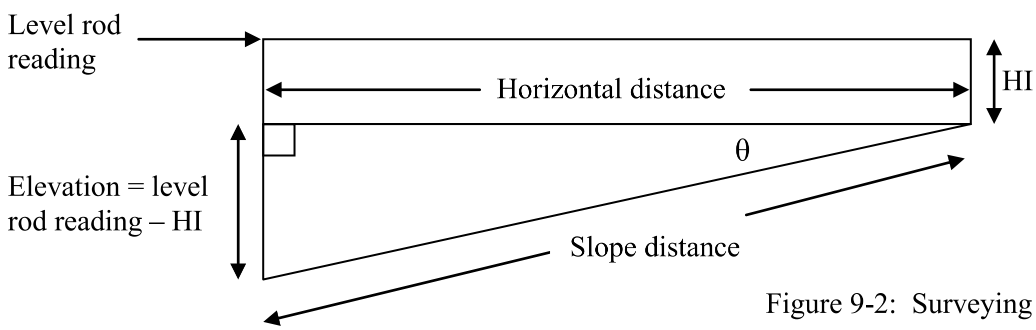


Figure 9-2: Surveying sketch

Let's start by calculating the elevation, which is the reading on the level rod minus HI, the height of the instrument. Thus, 11.18 ft – 5.82 ft. = 5.36. Now we can calculate the slope angle. We can then use the slope angle to calculate the horizontal distance. Express your answer in decimal degrees and degrees/minutes/seconds.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{elevation}}{\text{slope distance}} = \frac{5.36 \text{ ft.}}{34.20 \text{ ft.}}$$

Use the inverse sine function to complete the calculation.

$$\theta = \sin^{-1}\frac{5.36 \text{ ft.}}{34.20 \text{ ft.}} = 9.016862801° = 9° \, 01' \, 01''$$

Now calculate the horizontal distance using the slope angle and the tangent function.

$$\tan 9° \, 01' \, 01'' = \frac{\text{opposite}}{\text{adjacent}} = \frac{5.36 \text{ ft.}}{\text{horizontal distance}}$$

Solve this equation for the horizontal distance. Round your answer to 2 decimal places.

$$\text{horizontal distance} = \frac{5.36 \text{ ft.}}{\tan 9° \, 01' \, 01''} = 33.78 \text{ feet}$$

We will continue this problem on the next page.

▲

Horizontal Distance and Slope Angle, continued

Exercise 9-1

Referring to Figure 9-2, use the Pythagorean Theorem to calculate the horizontal distance. Compare your result to our calculation on the previous page.

Exercise 9-2

In the example on the previous page, we used the tangent function to calculate horizontal distance. Try using the cosine function to calculate the horizontal distance. Compare your result to the previous calculations.

The Height of a Structure

A transit can also be used to measure a vertical angle. A *vertical angle* is the angle between the horizontal line of sight and some object that is elevated above or below the transit elevation. See Figure 9-3 for an example of measuring the vertical angle to the top of a building.

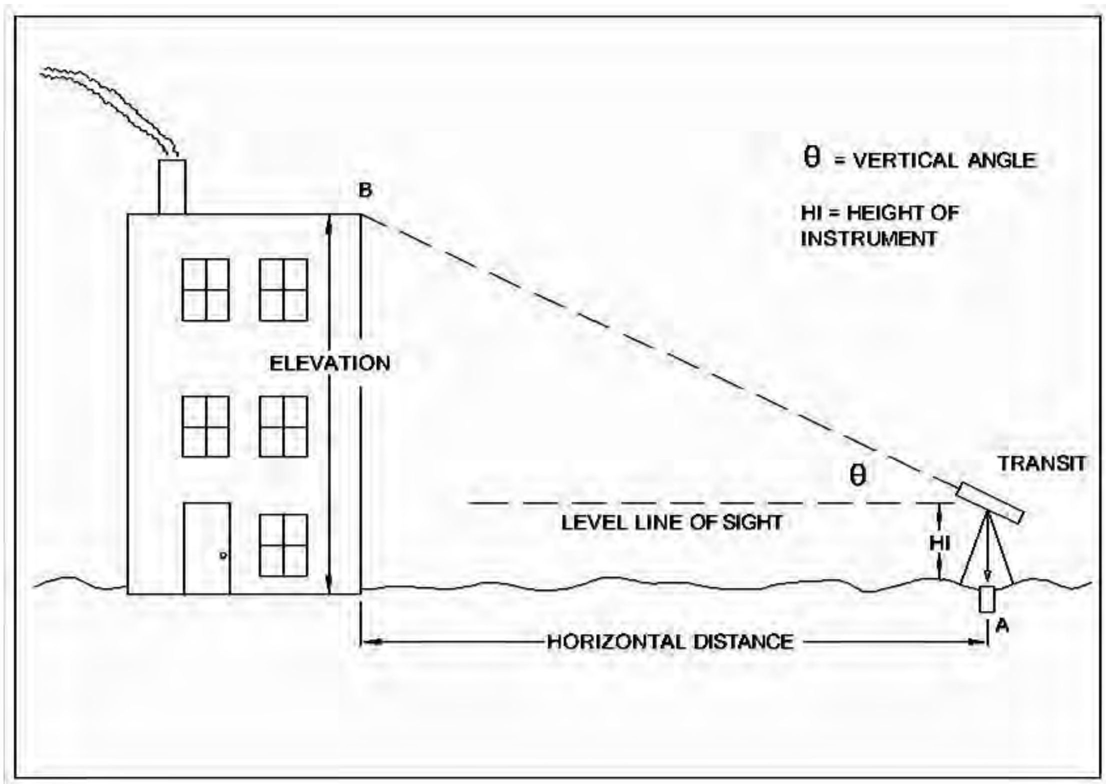


Figure 9-3: Building elevation

The vertical angle measurement can be used to help calculate the height of the building. The other required information includes the horizontal distance to the building and the HI (height of instrument).

Let's calculate the height of this building, given the following information:

- The horizontal distance from the building to the transit = 40.40 ft.
- The HI = 5.50 ft.
- The vertical angle is measured as 25° 04′.

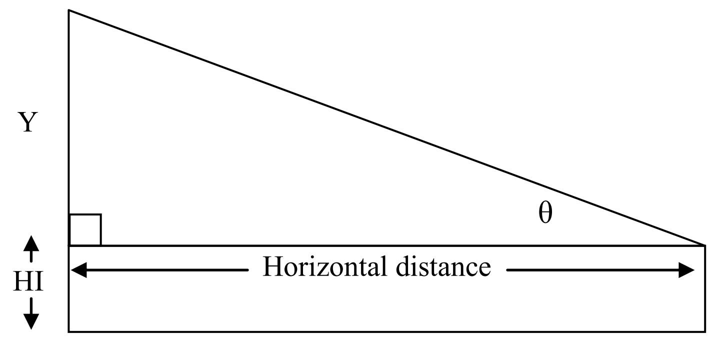
The first step is to sketch the right triangle that will help to solve the problem. See the sketch on the next page.

The Height of a Structure, continued

Example

Here is a sketch of the right triangle we will use to solve this problem.

Figure 9-4: Building height solution



First, we will solve for the side Y of the triangle. Then to find the overall height of the building, we will add the HI, height of instrument, to Y.

$$\tan 25° \, 04' = \frac{\text{opposite}}{\text{adjacent}} = \frac{Y}{\text{horizontal distance}} = \frac{Y}{40.40 \text{ ft.}}$$

$$Y = (40.40 \text{ ft.}) (\tan 25° \, 04') = 18.90 \text{ ft.}$$

Building height = Y + HI = 18.90 ft. + 5.50 ft. = 24.40 ft.

▲

Exercise 9-3

Repeat this problem for a different building, using the following measurements:

- The horizontal distance from the building to the transit = 120.00 ft.
- The HI = 5.75 ft.
- The vertical angle is measured as 32° 10′.

Include a labeled sketch of the right triangle that you need to help solve the problem.

Distance across an Obstruction

Surveyors may need to measure a distance between two points that are separated by an obstruction. The obstruction may limit direct access to the terrain between the two points. Examples of obstructions are rivers, brooks, ponds, and canyons. Current distance-measuring technology includes electronic distance measurement, using a laser and reflector. This technology can eliminate the need for direct measurement using a tape. However, electronic distance measurement may require the placement of a laser reflector at the distant point. This may not be practical due to an obstruction.

The sketch in Figure 9-5 shows an example of measuring between two points which are separated by a river.

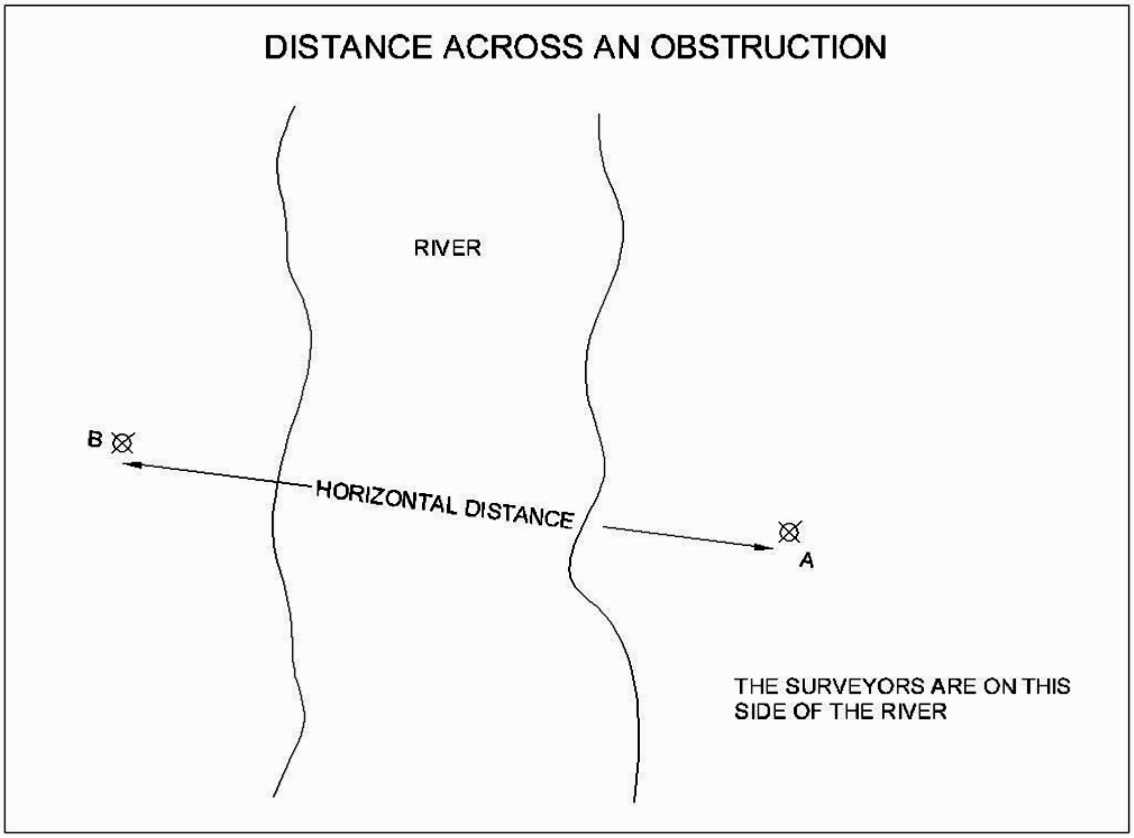


Figure 9-5: Distance across an obstruction

Without the aid of electronic distance measurement, another method must be used to measure between the points *A* and *B*. One method is to lay out a third point *C* in a way that creates a right triangle. This new point and the right triangle are shown in Figure 9-6 on the next page.

Distance across an Obstruction, continued

Example

As shown in Figure 9-6, the new point *C* is located at a distance of 150.00 feet from point *A*. A transit is able not only to read an angle between two given points, but also to create an angle. The transit operator at point *A* first sights on point *B*, and then turns the transit clockwise 90°. The new point *C* is then placed on this new direction, at a distance of 150.00 feet from point *A*. Thus, point *C* is located at 90° from the line between *A* and *B*.

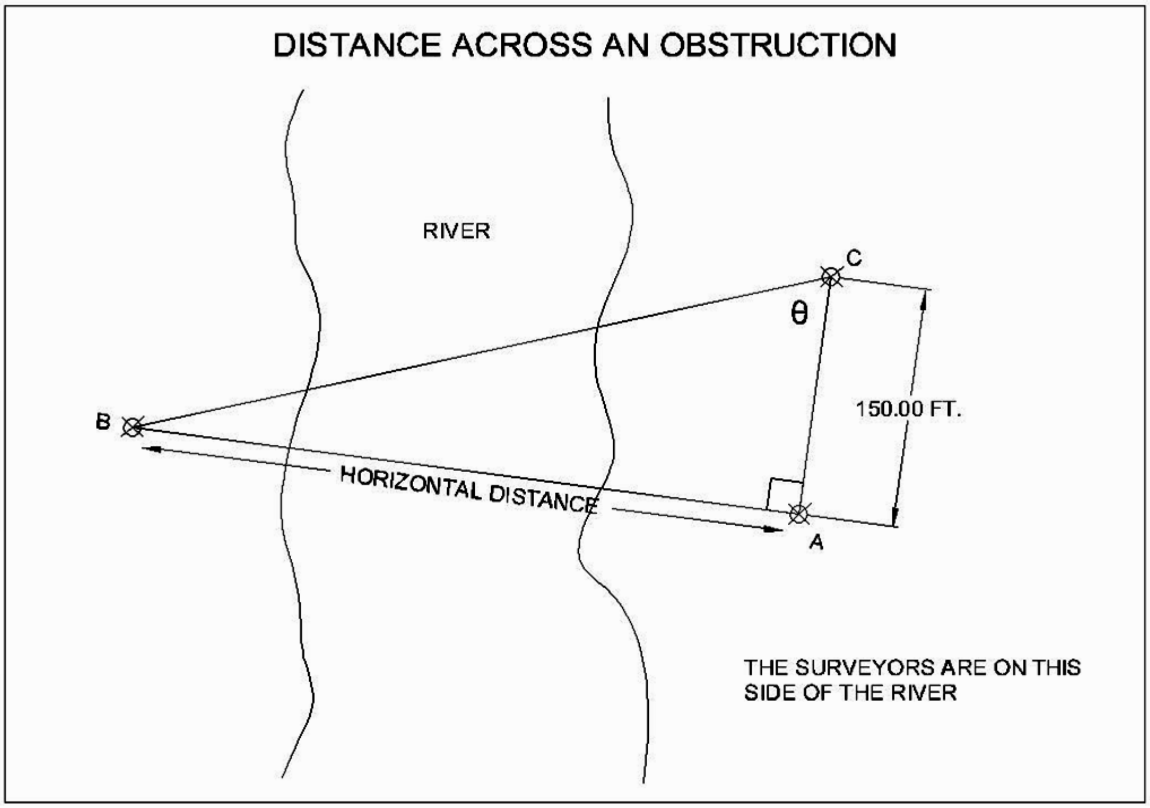


Figure 9-6: Measuring across an obstruction

The surveyor now relocates the transit from point *A* to the new point *C*. At point *C*, the surveyor sights on point *A*, and then turns the transit to point *B* and measures the angle. This angle measures 70° 02′. Now you have enough information to calculate the distance between points *A* and *B*.

$$\tan 70° \, 02' = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{horizontal distance}}{150.00 \text{ ft.}}$$

$$\text{horizontal distance} = (150.00 \text{ ft.})(\tan 70° \, 02') = 412.87 \text{ ft.}$$

▲

Distance across an Obstruction, continued

Exercise 9-4

Refer to the sketch in Figure 9-7. Calculate the distance between points *A* and *B*. Use the following additional information:

- A third point, C, is located at a distance of 90.00 feet from point *A*.
- Point *C* is located in a way that creates a right triangle with points *A*, *B*, and *C*.
- The angle measured by the surveyor from point *C* is 67° 37′ 14″ (sight on point *A* and turn towards point *B*).

As part of your solution, sketch the new point *C* and the resulting right triangle on Figure 9-7.

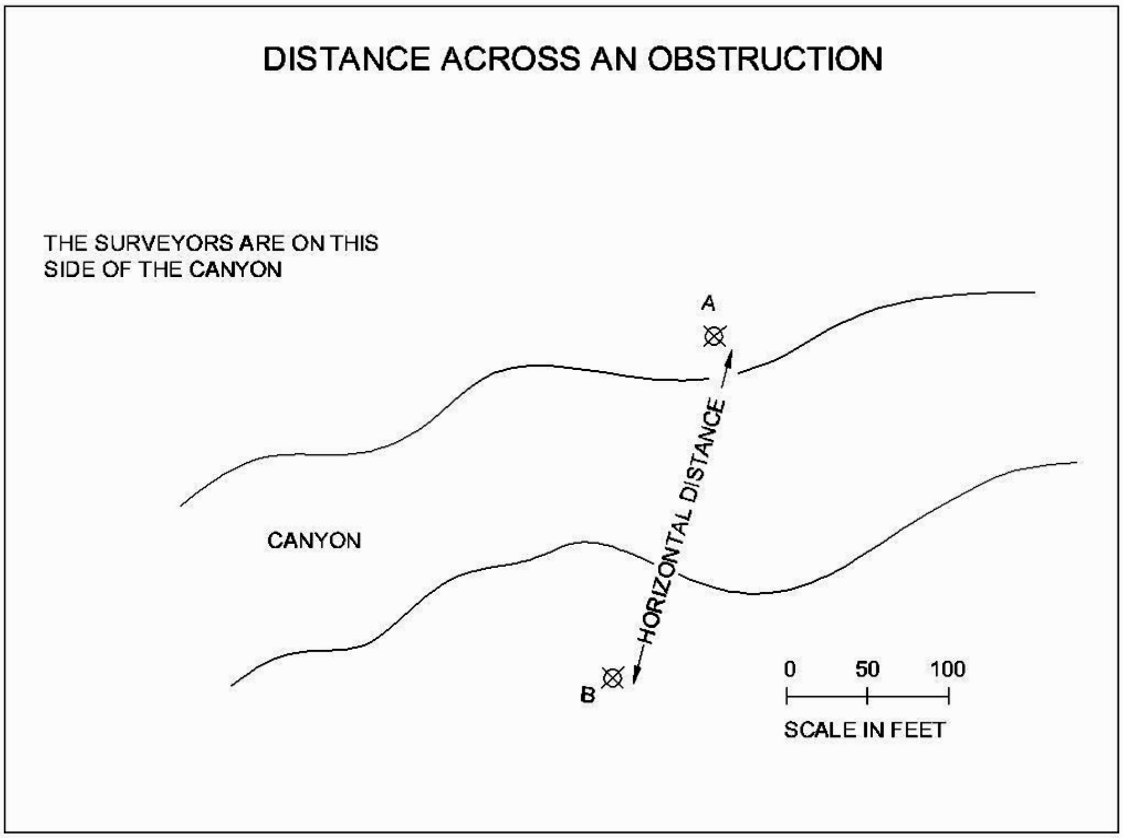


Figure 9-7: Distance across a canyon

Northing and Easting of a Survey Point

Survey Points

Survey points are typically imported into a CAD program. These points are then used to create a site plan for a property. The survey points can be features such as buildings, sidewalks, utility poles, and stormwater drains. The points may also be ground points used to characterize the topography.

When points are inserted into a CAD program, they are in a coordinate format. The coordinate system is the same as the *Cartesian coordinate system* that you already know, with a few differences.

The survey points have a *Z* coordinate which represents the elevation of the point. This elevation may be the distance above sea level or it may be the distance above or below a local benchmark point that is on or near the property.

Instead of using *X* and *Y* for the horizontal coordinates, survey points use the term *Northing* and *Easting*. The Northing coordinate is in the *Y* direction, and the Easting coordinate is in the *X* direction, as shown in Figure 9-8.

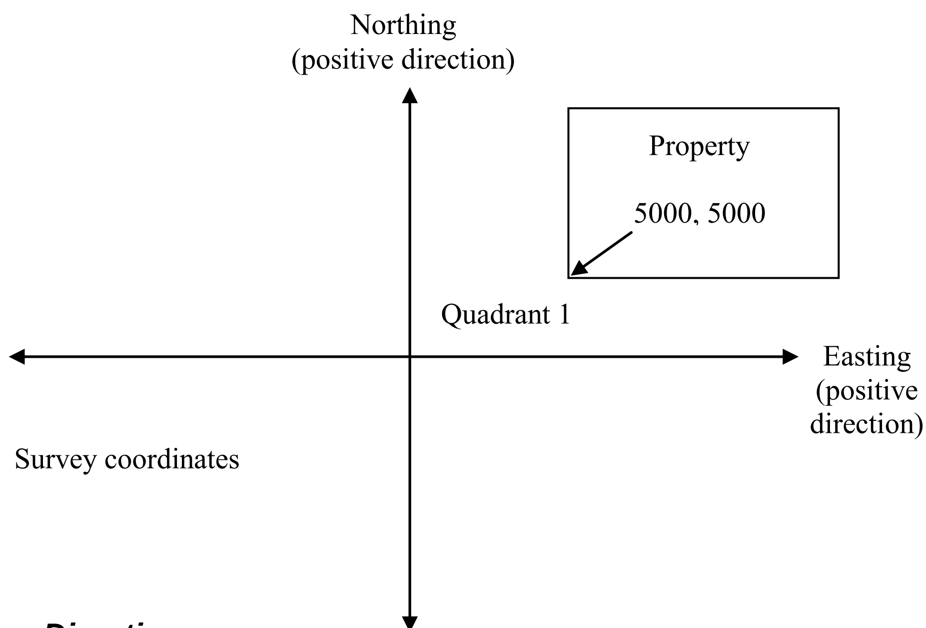


Figure 9-8: Survey coordinates

Positive and Negative Directions

Northing and Easting use the same convention as *X* and *Y*. To the right (or East) is the positive direction for Easting. To the left (or West) is the negative direction for Easting. The North direction is the positive direction for Northing. The South direction is the negative direction for Northing.

Placement of Property on Coordinates

The property in Figure 9-8 is placed in Quadrant I. Using Quadrant I ensures that all points on or near the property will have only positive values for Northing and Easting. The southwestern corner of the property is given a Northing and Easting location of 5000, 5000 to further ensure that no points have a negative value.

Northing and Easting of a Survey Point, continued

Example

Let's calculate the Northing and Easting of a survey point shown in Figure 9-9. This figure shows a property with a house. The surveyors have set up the transit at point *A*, which is the southwestern corner of the property. This point *A* is a temporary benchmark from which other points are measured. Point A has a Northing and Easting coordinate of 5000, 5000.

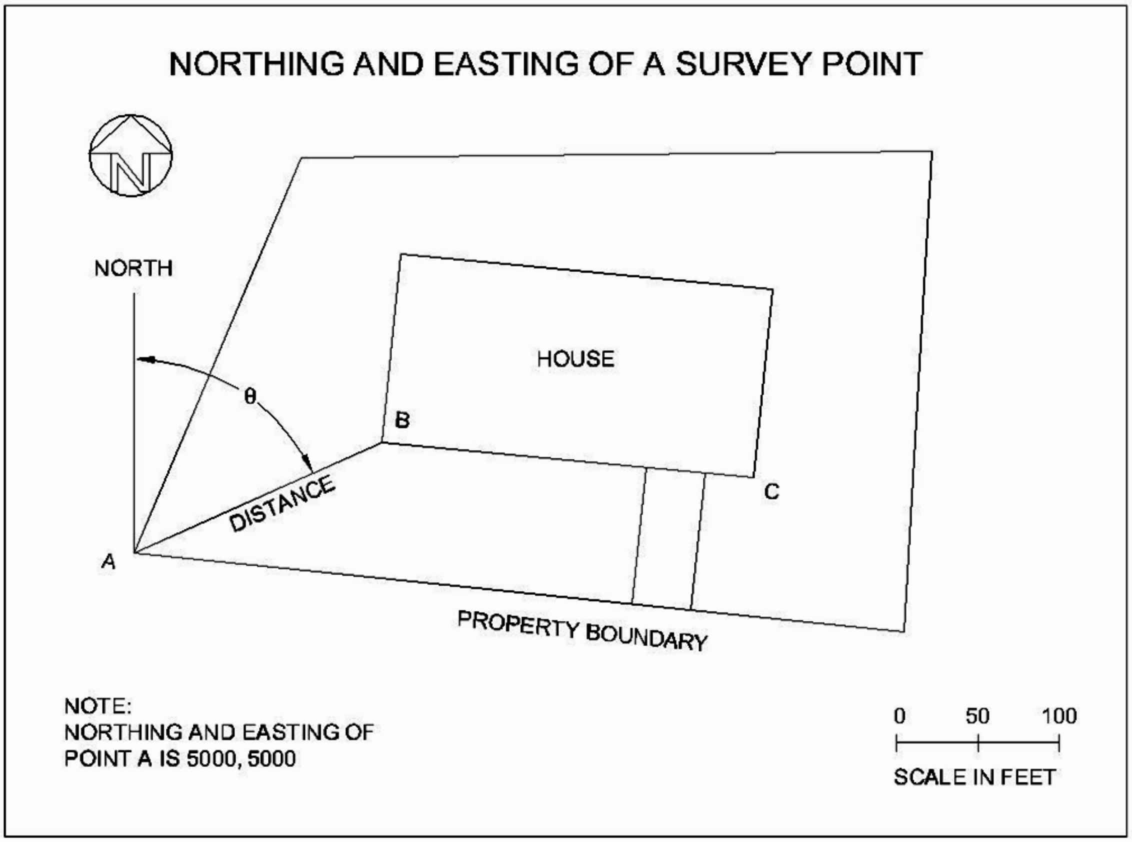


Figure 9-9: Property with a house

Distance from *A* to *B*

The surveyors measure the distance from the benchmark point *A* to the corner of the house at point *B*. The distance is measured as 167.44 feet. This distance is the hypotenuse of our right triangle.

Azimuth

The angle from point A to point B is measured by sighting the transit on the North direction and then turning clockwise to point B. This angle is called an *azimuth*. The azimuth convention is defined as an angle turned clockwise from North. The North direction is defined as an azimuth of 0°. An azimuth can be anywhere between 0° and 360°. In this example, the surveyors measure the azimuth as 65° 47′ 42″.

Northing and Easting of a Survey Point, continued

Sketch of the Solution

The first step in the solution is to sketch the right triangle that helps us to solve the problem. We are trying to determine the Northing and Easting of point B, given the following information:

- Point A has a Northing and Easting of 5000, 5000
- The distance from point A to point B = 167.44 feet
- The azimuth from point A to point B = angle $\theta = 65° 47' 42''$

Figure 9-10 is a sketch of the right triangle.

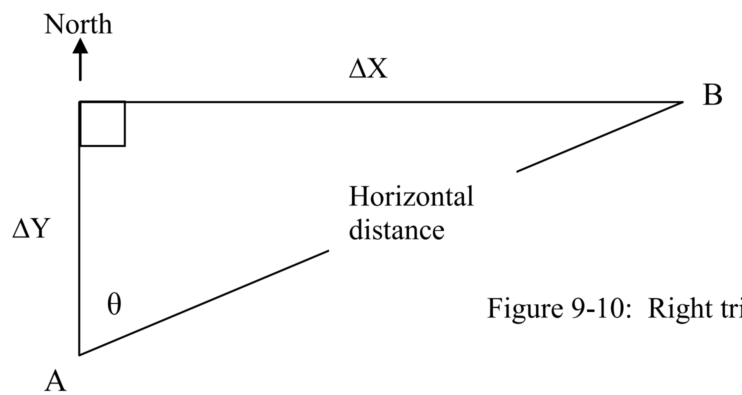


Figure 9-10: Right triangle

Solve for Northing

$$\cos 65° 47' 42'' = \frac{\Delta Y}{\text{horizontal distance}} = \frac{\Delta Y}{167.44 \text{ ft.}}$$

$$\Delta Y = (167.44 \text{ ft.})(\cos 65° 47' 42'') = 68.65 \text{ ft.}$$

Northing of point B = Northing of point A + ΔY = 5000 + 68.65 = 5068.65 ft.

Solve for Easting

$$\sin 65° 47' 42'' = \frac{\Delta X}{\text{horizontal distance}} = \frac{\Delta X}{167.44 \text{ ft.}}$$

$$\Delta X = (167.44 \text{ ft.})(\sin 65° 47' 42'') = 152.72 \text{ ft.}$$

Easting of point B = Easting of point A + ΔX = 5000 + 152.72 = 5152.72 ft.

▲

Northing and Easting of a Survey Point, continued

Exercise 9-5

Use Figure 9-11 below to calculate the Northing and Easting coordinates of point *C*, which is the southeastern corner of the house. Use the following information in your calculations:

- Point *A* has a Northing and Easing of 5000, 5000
- The distance from Point *A* to Point *C* = 384.29 feet
- The azimuth from Point *A* to Point *C* = angle θ = 82° 59′ 30″

As part of your solution, sketch the necessary right triangle in Figure 9-11.

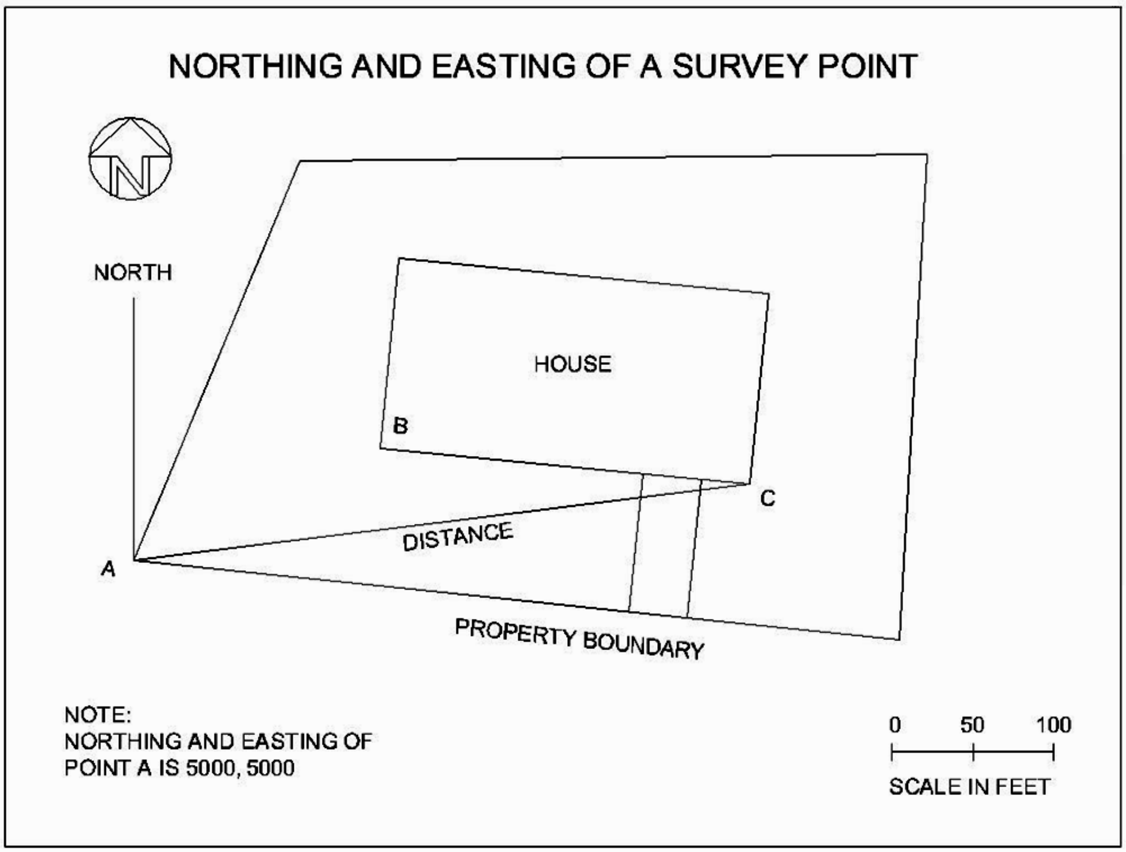


Figure 9-11: Coordinates of point C

Solve for Northing

Solve for Easting

Bearing Format for Directions

In the civil technology field, a direction is often displayed as a *bearing* rather than as an *azimuth*.

Earlier, the direction from point *A* to point *C* was given as an azimuth. The azimuth is referenced to the North direction, which is 0°.

A bearing can be referenced to either North or South. The direction is expressed as an angle towards the East or West directions. The numerical value of the angle is always less than or equal to 90°.

The earlier azimuth can be expressed in bearing format as N82° 59′ 30″E. This means that an observer can look directly North, then turn towards the east a total of 82° 59′ 30″.

The direction of boundary lines on a site plan are often expressed as bearings.

Example

The following four azimuths are shown on the bearing axes in Figure 9-12, in the bearing format. You can see how bearings are shown in each of the four quadrants.

Line A: azimuth of 32°
Line B: azimuth of 127°
Line C: azimuth of 248°
Line D: azimuth of 334°

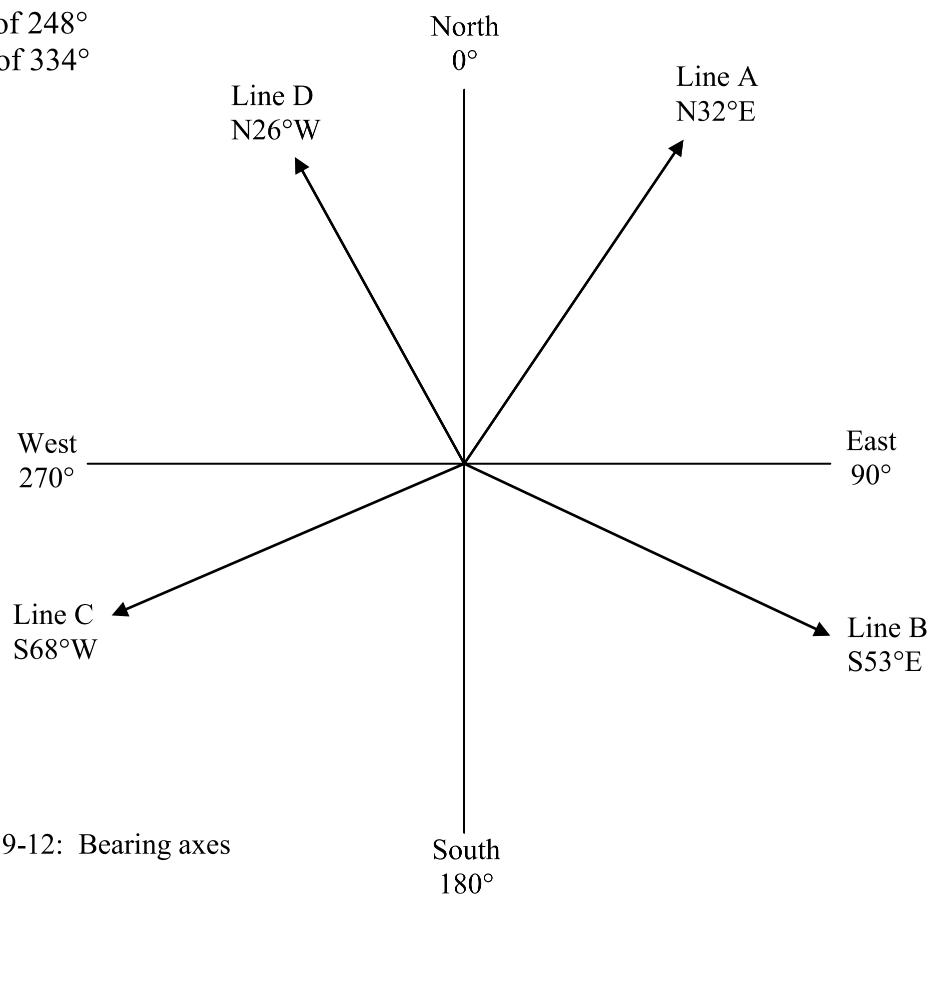


Figure 9-12: Bearing axes

▲

Bearing Format for Directions, continued

Exercise 9-6

Express the following azimuths as bearings, and the following bearings as azimuths.

Azimuth	Bearing
282° 45'	
	S17° 20′W
153° 35′	
	N29° 15′E

Exercise 9-7

Sketch the four bearings from the table above. Use Figure 9-13 below to sketch a line and label each line with the bearing.

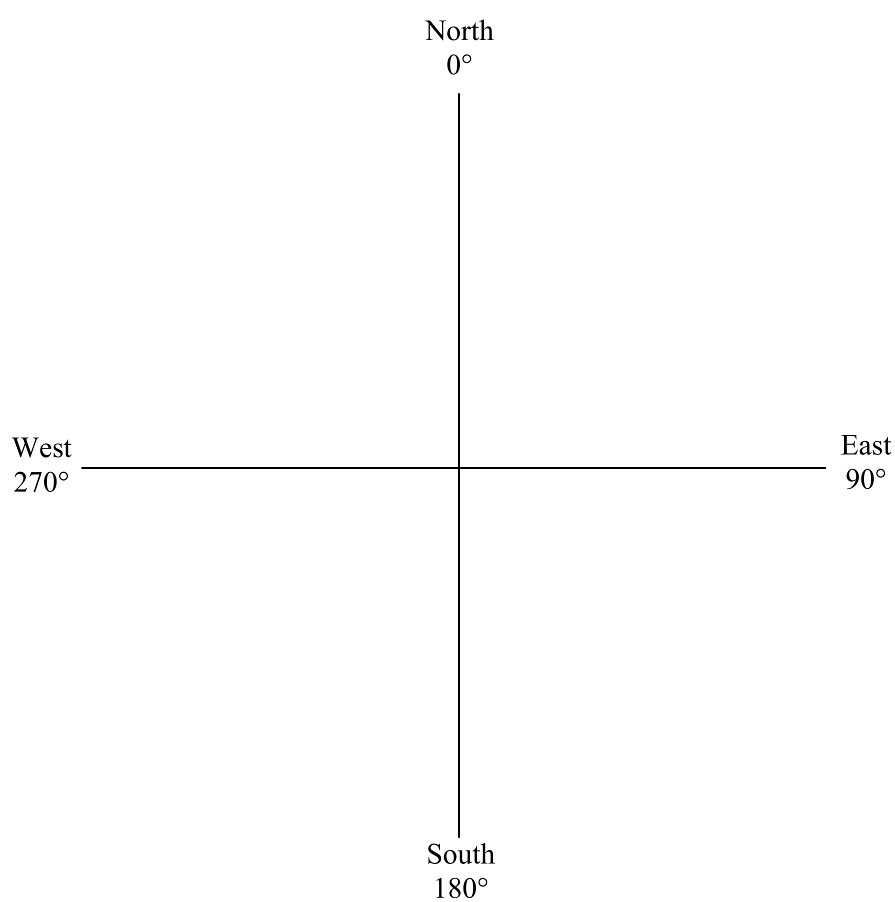


Figure 9-13: Bearing axes

Slope Angle and Grade

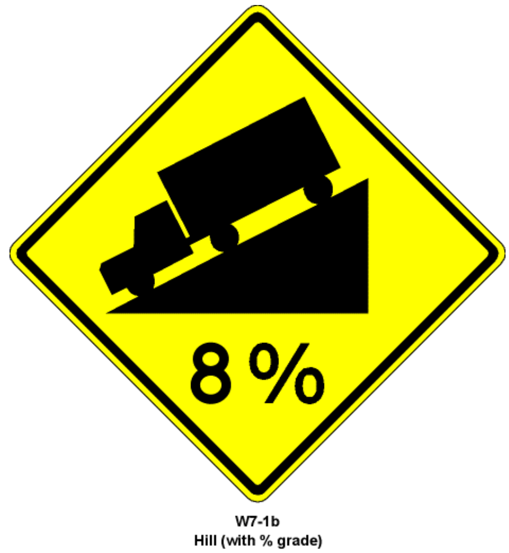
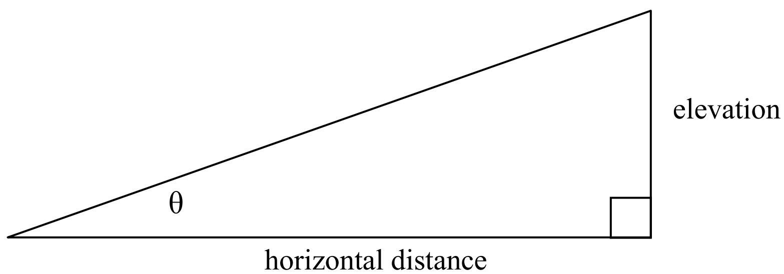


Fig. 9-14: Grade warning sign
Source: Manual of Traffic Signs at
http://www.trafficsign.us/index.html

Slope Angle

Slope angle is the angle at which the ground is rising or falling between two points. An assumption is that the elevations changes linearly between these two points. The ground does not normally slope in a linear fashion.

Example



$$\text{slope} \;=\; \frac{\text{rise}}{\text{run}} \;=\; \frac{\text{elevation}}{\text{horizontal distance}}$$

$$\text{slope angle} \;=\; \theta \;=\; \tan^{-1} \frac{\text{elevation}}{\text{horizontal distance}}$$

Highway and Road Slope and Grade Requirements

State and local governments publish slope and grade requirements for roadway and highway design. Some of the specifications are maximum limits and some are minimum limits.

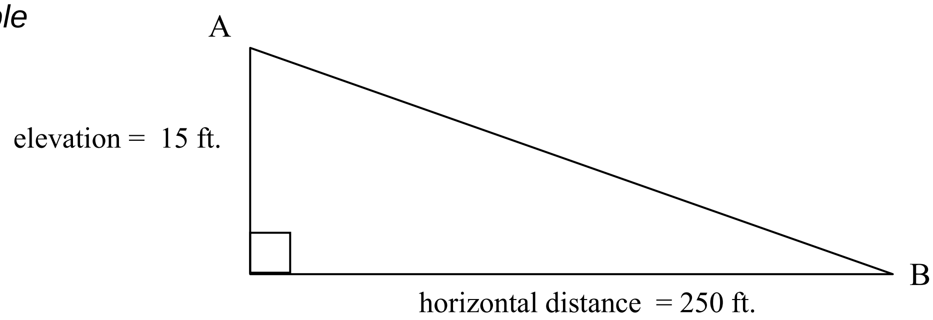
One excellent example is the California Highway Design Manual, section 204. This manual is available at http://www.dot.ca.gov/hq/oppd/hdm/hdmtoc.htm

Slope Angle and Grade, continued

Grade

Grade, or percent grade, is the slope expressed as a percent rather than as an angle. Grade may also be thought of as the elevation change for every 100 feet of horizontal travel.

Example



$$\text{Grade} = \frac{\text{elevation}}{\text{horizontal distance}} \times 100\%$$

$$\text{Grade} = \frac{15\text{ ft.}}{250\text{ ft.}} \times 100\% = 6\%\text{ grade}$$

▲

Contour Lines

Topographical maps may be used to determine the slope and grade of terrain, either in rural or urban areas. The U.S. Geological Survey publishes topographical maps, called 7.5-minute quadrangle maps.

These maps can be viewed and ordered at http://nationalmap.gov/ustopo/index.html

A topographical map uses *contour lines* to show elevations at particular locations on the map. An example of contour lines on a map is shown in Figure 9-14.

A contour line represents all points that have the same *elevation*. For example, one of the contour lines in Figure 9-14 is labeled as 120 feet. This means that all points on that contour line are at an elevation of 120 feet.

The elevation is the *vertical distance* above sea level, which is 0 feet. The elevation may also be referenced to a local benchmark elevation, rather than to sea level.

Slope Angle and Grade, continued

Contour Lines, continued

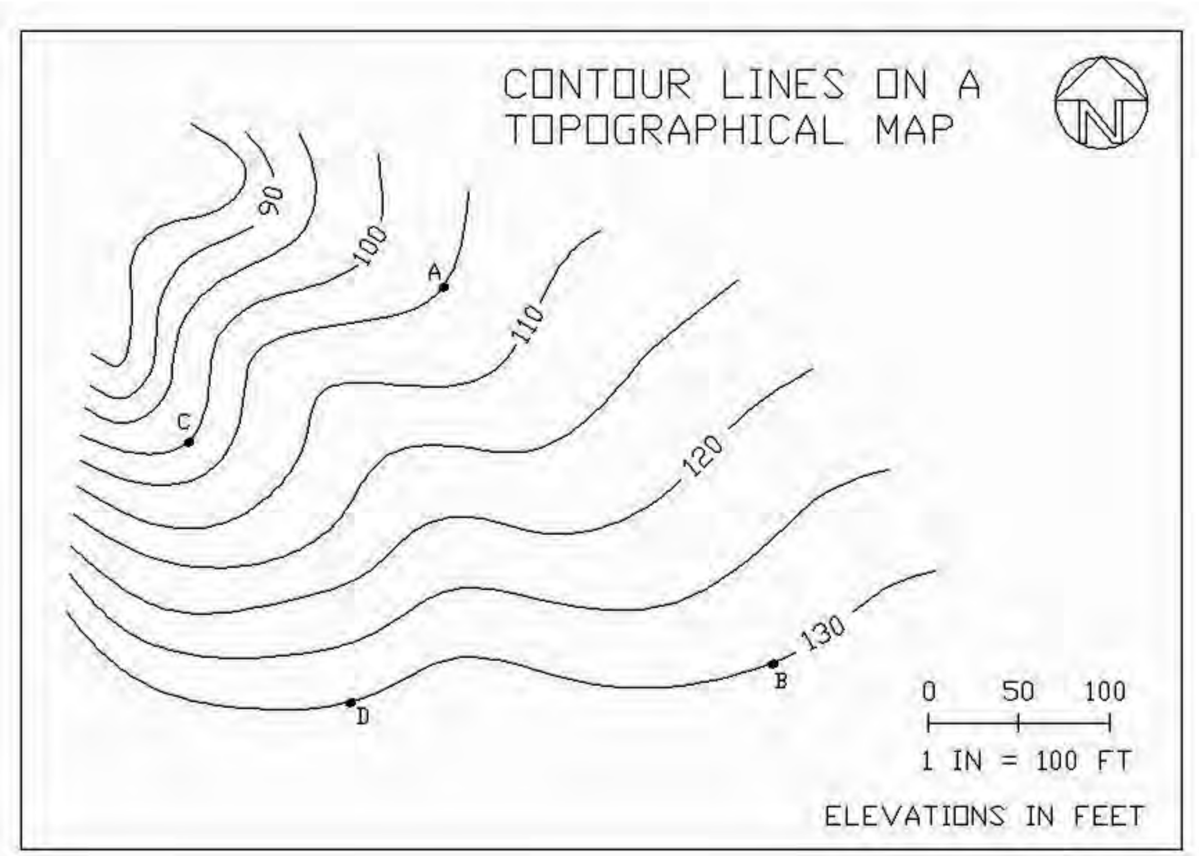


Figure 9-14: Contour lines

Example

You can use the contour lines on a map to calculate both the slope and grade between two points. In this exercise, we will calculate the slope angle between points A and B in Figure 9-14.

The elevation rise between points A and B is read from the contour lines. The rise between A and B is 130 feet – 105 feet = 25 feet.

The horizontal distance between points A and B must be measured and then converted into a real distance using the scale, shown in the lower right corner of the map.

$$\text{horizontal distance} = \text{measured distance} \times \text{scale} = 2\ 5/8 \text{ in.} \times \frac{100 \text{ ft.}}{1 \text{ inch}} = 263 \text{ ft.}$$

This example continues on the next page

Slope Angle and Grade, continued

Contour Lines, continued

Example, continued

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{elevation difference between A and B}}{\text{horizontal distance between A and B}} = \frac{25 \text{ ft.}}{263 \text{ ft.}}$$

$$\text{slope angle} = \theta = \tan^{-1} \frac{25 \text{ ft.}}{263 \text{ ft.}} = 5.430051073° = 5° \, 25' \, 48''$$

▲

Exercise 9-8

Now calculate the *grade* between points *A* and *B* in Figure 9-14. Express your answer in percent to one decimal.

Exercise 9-9

Calculate the slope angle between points *C* and *D* in Figure 9-14. Express your answer in DMS format to the nearest second.

Elevation difference between points *C* and *D* =

Horizontal distance between points *C* and *D* =

Exercise 9-10

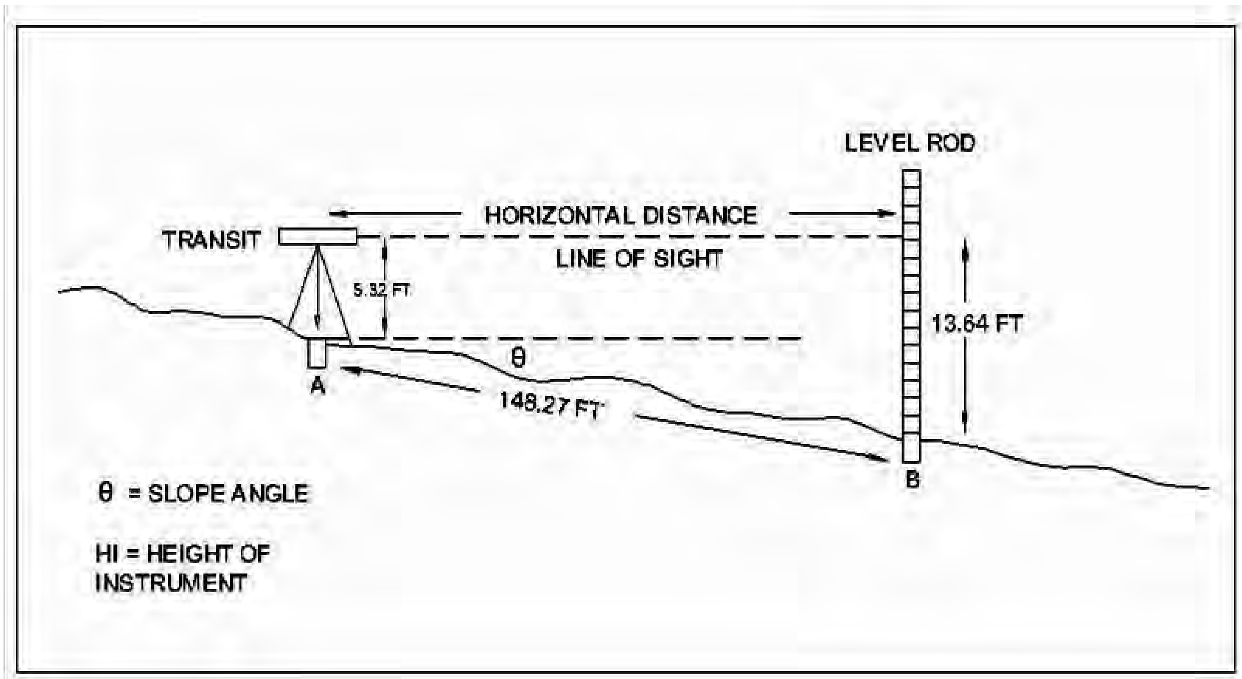
Calculate the grade between points *C* and *D* in Figure 9-14. Express your answer in percent to one decimal.

Practice Worksheet

1. Calculate the horizontal distance and slope angle.

First, draw a sketch of the right triangle that will help you to solve the problem.

Note: This drawing is not to scale.



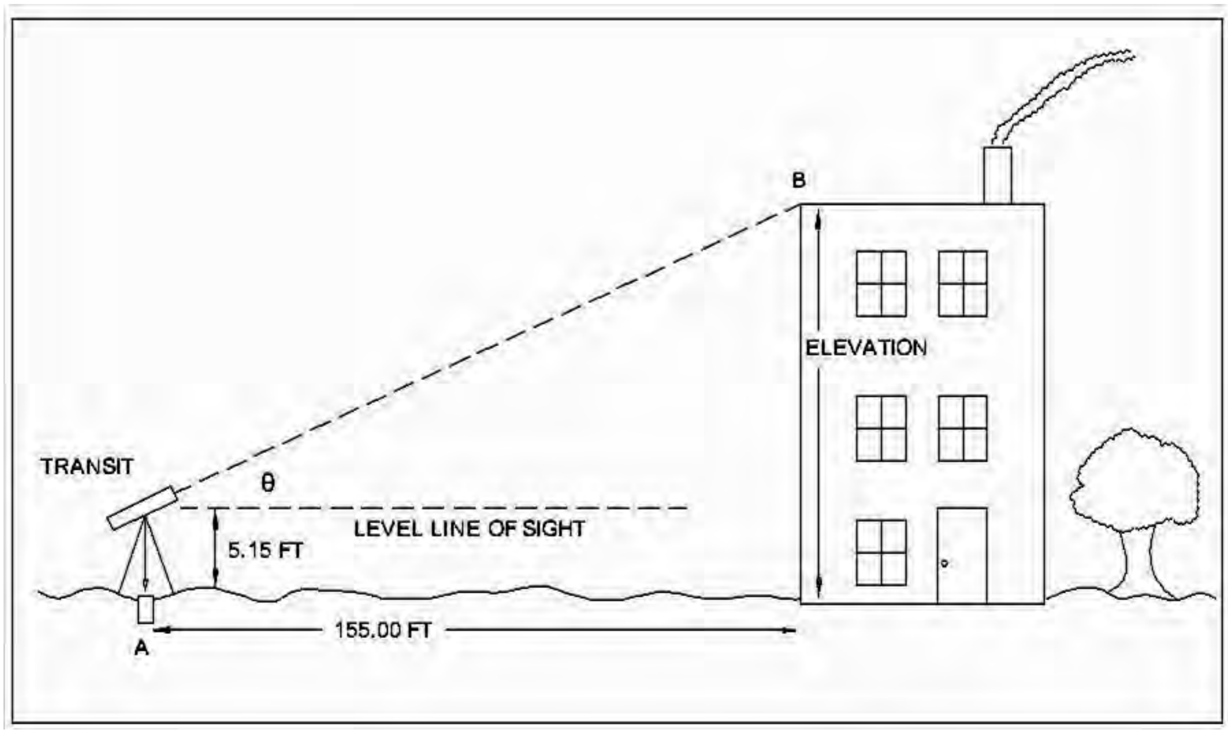
Practice Worksheet, continued

2. Calculate the height of this building, using the following measurements:

- The vertical angle θ is measured as 31°40′.

Include a labeled sketch of the right triangle that you need to help solve the problem.

Note: The drawing is not to scale.

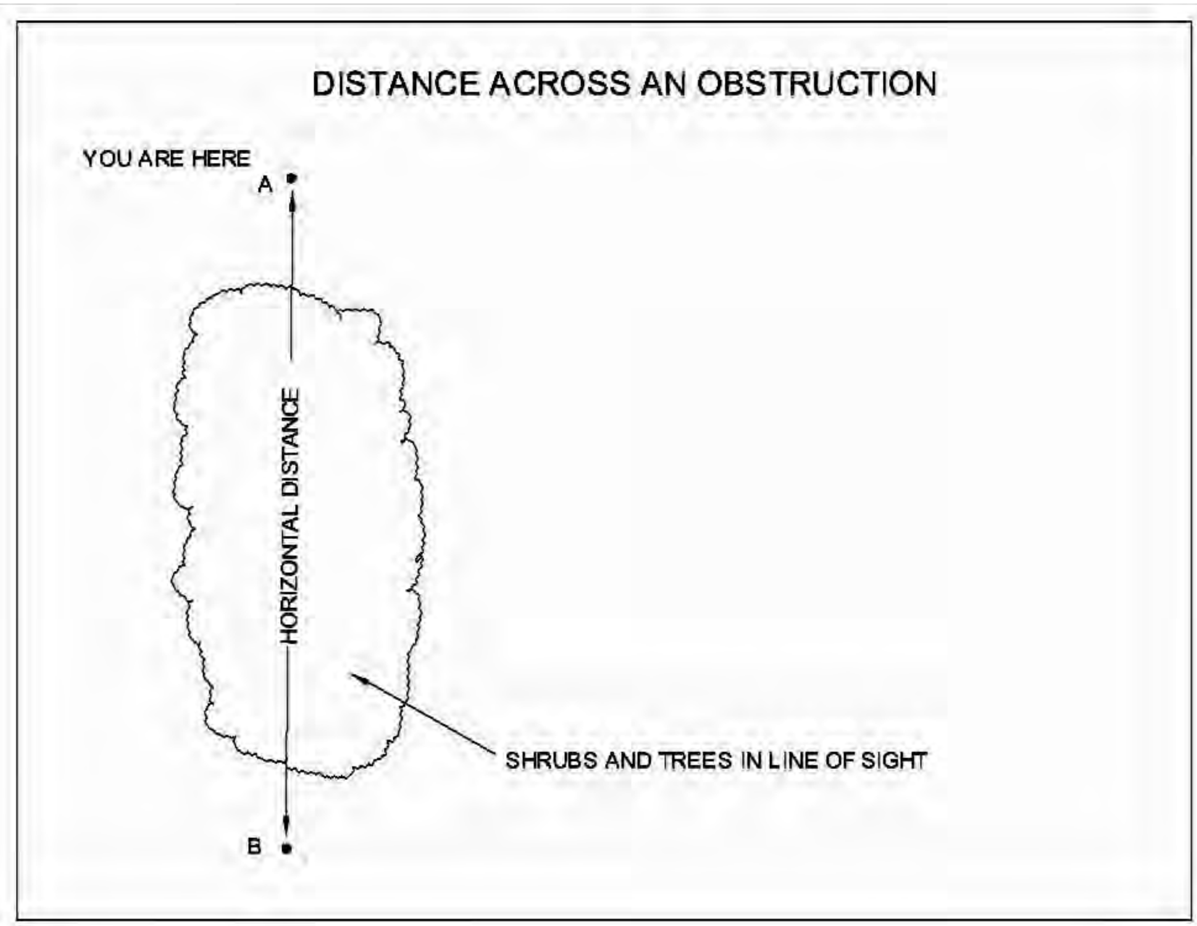


Practice Worksheet, continued

3. Calculate the distance between points *A* and *B*. Use the following additional information:

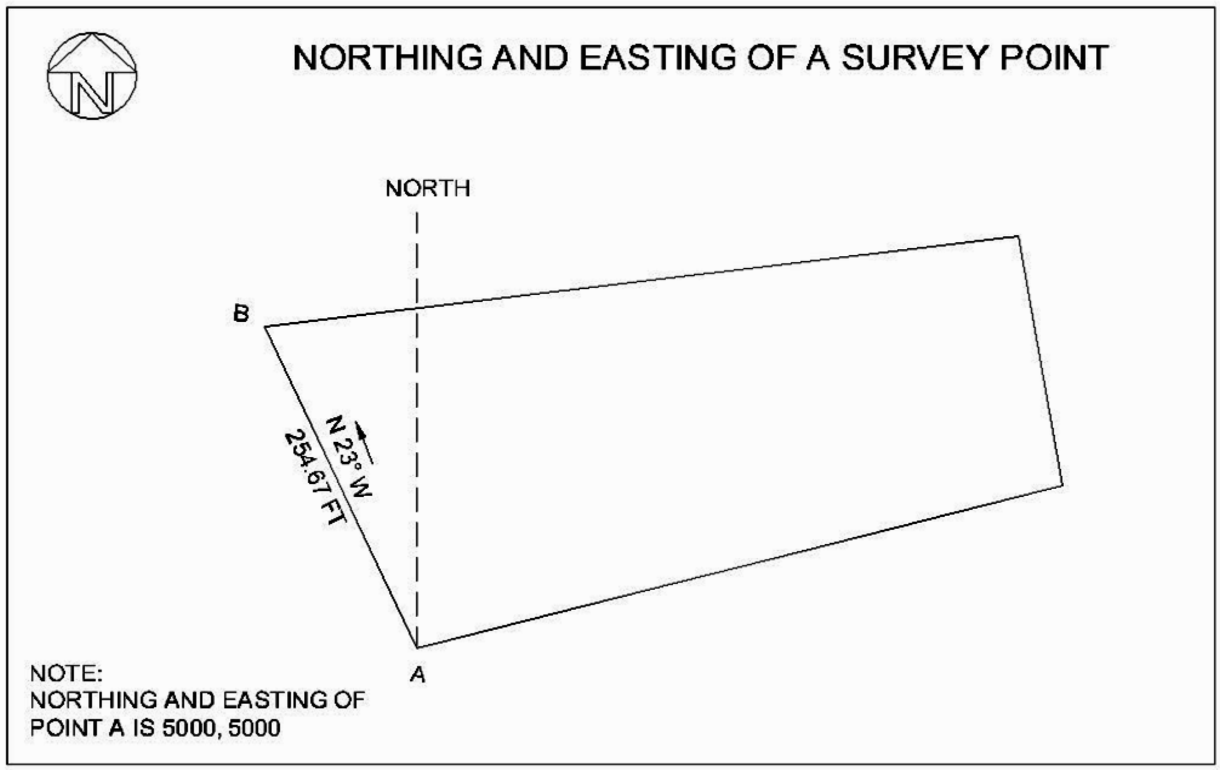
- A third point *C* is located at a distance of 75.00 feet from point *A*.
- Point *C* is located in a way that creates a right triangle with points *A*, *B*, and *C*.
- The angle measured by the surveyor from point *C* is 51°22′ (sight on point *B* and turn towards point *A*).

As part of your solution, sketch the new point *C* and the resulting right triangle.



Practice Worksheet, continued

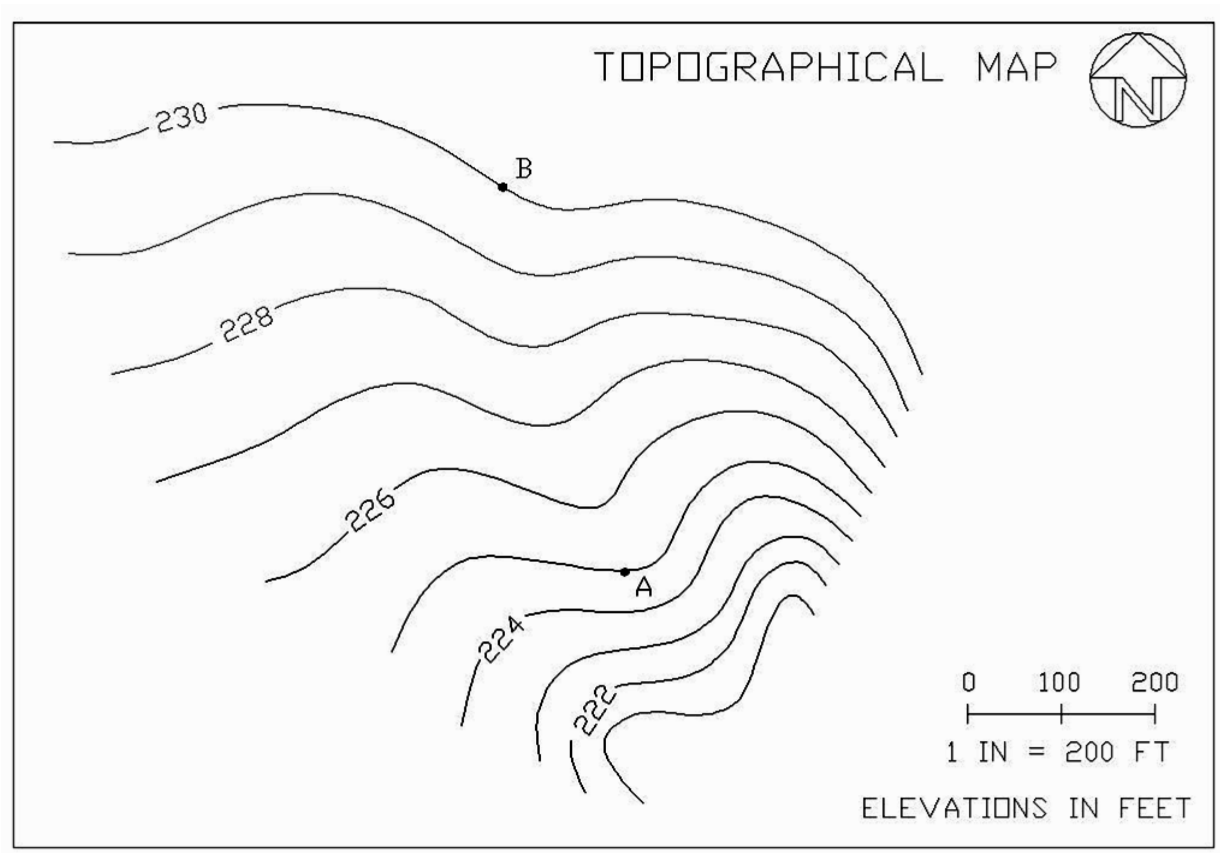
4. Calculate the Northing and Easting coordinates for point *B* in the following sketch. Express the answer in feet to two decimals.



Practice Worksheet, continued

5. Calculate the slope angle and grade between points *A* and *B* on this topographical map.

Express slope angle in DMS format. Express grade in percent, to two decimals.



Chapter 10: Architectural and Construction Applications

Now let's look at some applications of trigonometry in the architectural and construction fields.

Architectural and Construction Applications

Here are some examples of right angle trigonometry applications in the architectural technology field. We will discuss examples of these calculations in this chapter:

- Calculation and use of roof pitch
- Calculation of roof truss dimensions
- Use of the 3-4-5 triangle
- Calculation of stair dimensions

As in the previous chapter, you will need to sketch a right triangle as part of the solution. The right triangle that you sketch must help you to determine the answer to the problem.

Roof Pitch

What is Roof Pitch?

Roof pitch is a ratio that expresses the slope of the roof. This ratio expresses the *rise* and *run* of the slope. Figure 10-1 shows an example of roof pitch.

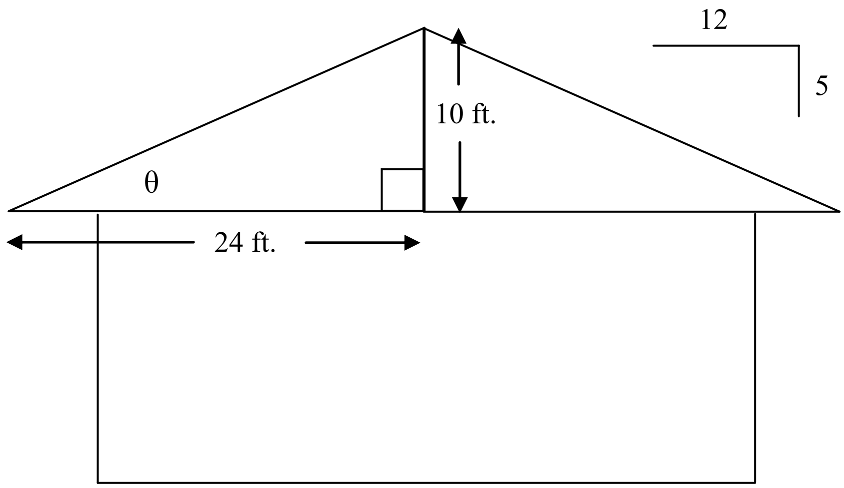


Figure 10-1: Roof pitch

Roof Pitch Notation

Roof pitch can be expressed in several ways. In addition to the notation in Figure 10-1, you may see this roof pitch expressed as 5/12 or 5:12. The roof pitch is expressed as the simplified form of the rise/run ratio. The denominator is typically 12, which represents 12 inches in a foot. This makes it easy for anyone to measure a roof pitch by measuring the rise and run in inches. In Figure 10-1, the rise is 10 feet and the run is 24 feet. This ratio reduces to:

$$\frac{\text{rise}}{\text{run}} = \frac{10 \text{ feet}}{24 \text{ feet}} = \frac{5}{12} = 5/12 = 5{:}12$$

Example

There is enough information in Figure 10-1 to make two additional calculations. Calculate the roof angle θ and also the *rafter* length. The rafter is the hypotenuse of the right triangle.

Roof Angle (decimal degrees to two decimals):

$$\tan\theta = \frac{10 \text{ ft.}}{24 \text{ ft.}} = \frac{5}{12} \qquad \theta = \tan^{-1}\frac{5}{12} = 22.62°$$

Rafter Length (feet and inches to the nearest inch):

$$\cos 22.62° = \frac{24 \text{ ft.}}{\text{rafter}} \qquad \text{rafter} = \frac{24 \text{ ft.}}{\cos 22.62°} = 26.00 \text{ ft.} = 26 \text{ ft. } 0 \text{ in.}$$

▲

Roof Pitch, continued

Exercise 10-1

Calculate the roof pitch for the roof shown in Figure 10-2. Assume that the roof is symmetrical. Also, calculate the roof angle, θ, in decimal degrees.

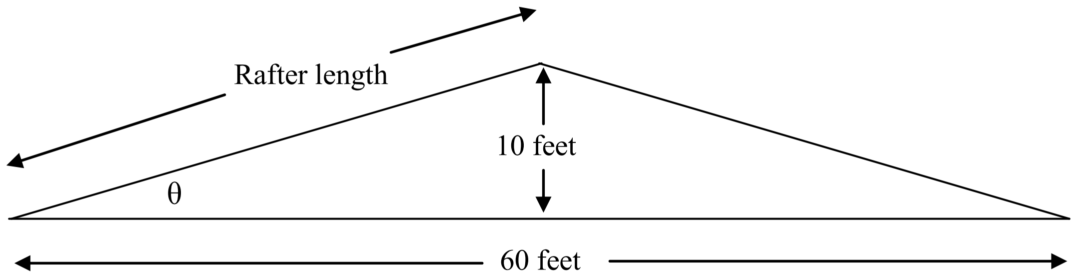


Figure 10-2: Roof pitch calculation

Exercise 10-2

In Figure 10-3, the roof angle is given, but the roof pitch is not known. Using the given angle, calculate the roof pitch. Express the roof pitch as a ratio with a denominator of 12 (x:12).

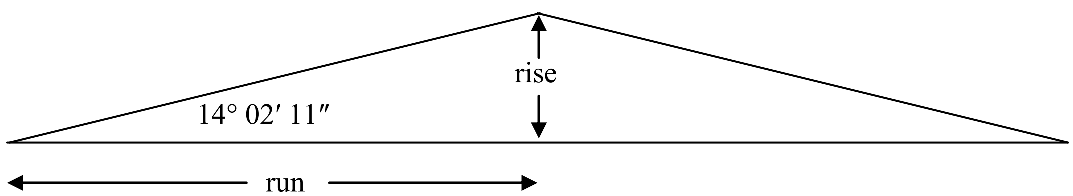


Figure 10-3: Roof pitch calculation

Roof Truss Dimensions

A roof *truss* is a *structure* made up of straight boards connected at joints. A truss is shown in Figure 10-4. We will refer to the individual boards as *webs*. For example, web DE is the board whose endpoints are the joints D and E. Notice that the webs in this truss form a number of triangles. Triangular shapes are used in a truss because triangular structures are very strong and stable.

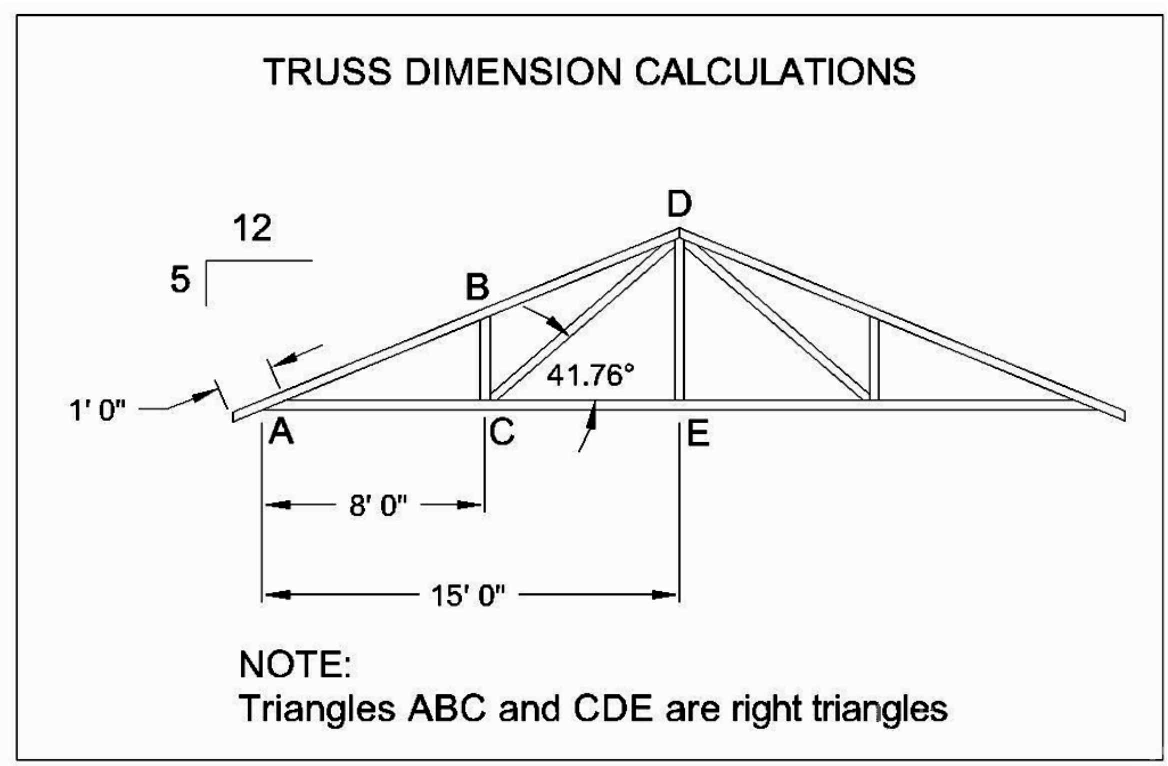


Figure 10-4: Roof truss dimensions

Roof truss dimensions can be calculated using right angle trigonometry. We will solve for unknown dimensions in the roof truss shown in Figure 10-4 above.

We will calculate the length of web BC, web DE, and rafter ABD. First, we will calculate angle BAC from the roof pitch, which is 5:12.

Example

Use the inverse tangent function to calculate the roof angle BAC.

$$\text{angle BAC} = \tan^{-1} \frac{5}{12} = 22.62° = 22° \, 37' \, 12''$$

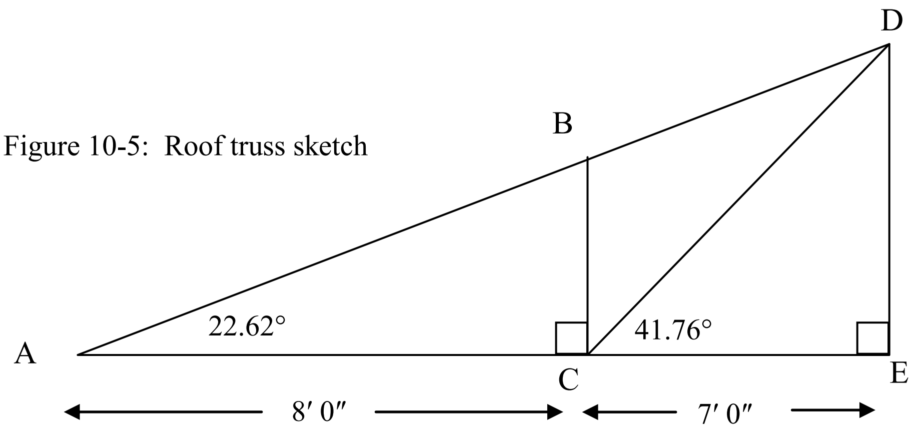
▲

Roof Truss Dimensions, continued

Example

Calculate the length of web BC. Use the value of the roof angle that we calculated and the triangle which is sketched in Figure 10-5. Express your answer in feet and inches.

Figure 10-5: Roof truss sketch



$$\tan 22.62° = \frac{BC}{8 \text{ ft.}}$$

$$BC = (8.0 \text{ ft.})(\tan 22.62°) = 3.33 \text{ ft.} = 3 \text{ ft. } 4 \text{ in.}$$

▲

Exercise 10-3

Calculate the length of web DE. Use the value of the roof angle that we calculated and the triangle that is sketched in Figure 10-5.

Exercise 10-4

Calculate the length of rafter ABD. Use the value of the roof angle that we calculated and the triangle that is sketched in Figure 10-5. Remember to add the 1-foot overhang shown in Figure 10-4.

Roof Truss Dimensions, continued

Exercises 10-5 through 10-8 are related to the truss shown in Figure 10-6 below.

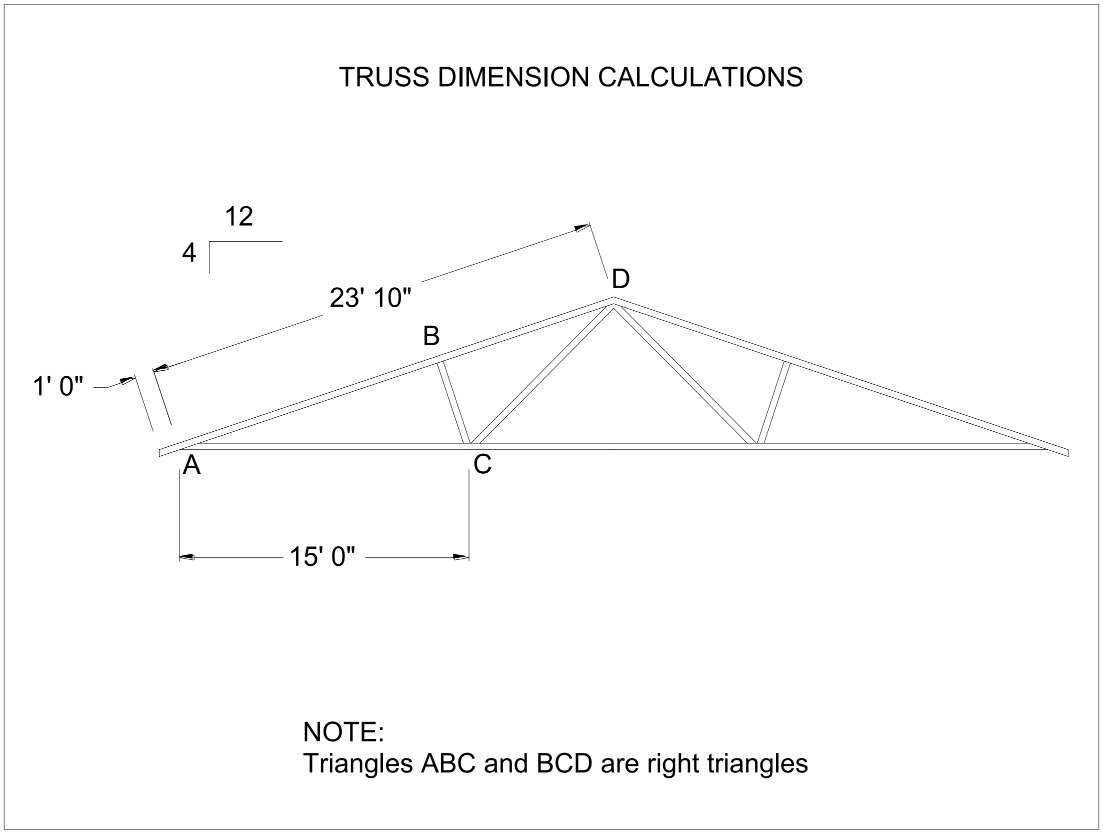


Figure 10-6: Roof truss

Exercise 10-5

Calculate angle BAC from the roof pitch, which is 4:12. You will need this angle to help calculate the unknown lengths in the truss. Express your answer in decimal degrees to two decimals.

angle BAC =

Roof Truss Dimensions, continued

Refer to Figure 10-6 for the exercises on this page. Express lengths in feet and inches, to the nearest whole inch.

Exercise 10-6

Calculate the length of web BC and the length of AB. Use the value of the roof angle that you calculated in Exercise 10-5. As part of your solution, sketch the right triangle that you need to solve for the unknown length.

Exercise 10-7

Calculate the length of web CD. As part of your solution, sketch the right triangle that you need to solve for the unknown length. Use the Pythagorean Theorem.

Exercise 10-8

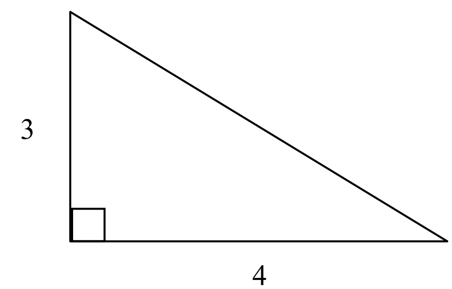
Calculate the value of angles BCD and BDC. Express your answer in DMS format to the nearest minute. As part of your solution, sketch the right triangle that you need to solve for the unknown angles.

3-4-5 Triangles

There are several practical uses for a “3-4-5” triangle. This right triangle has a hypotenuse with a length of 5. The two legs have lengths of 3 and 4. Because the three sides are in ratios of whole numbers, this triangle has practical applications, as shown in this section.

Exercise 10-9

Use the Pythagorean Theorem to confirm that a right triangle with leg lengths of 3 and 4 has a hypotenuse with a length of 5.

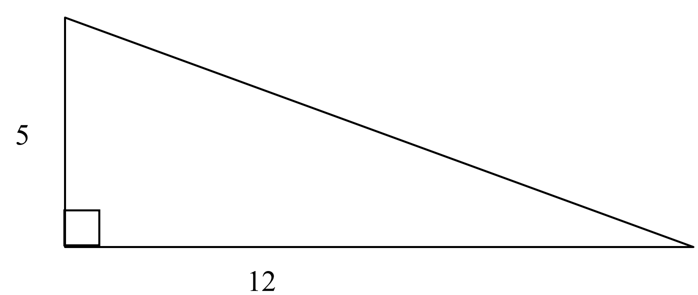


5-12-13 Triangle

Another special triangle is the 5-12-13 triangle. Its sides also exist in a ratio of whole numbers.

Exercise 10-10

Use the Pythagorean Theorem to confirm that a right triangle with leg lengths of 5 and 12 has a hypotenuse with a length of 13.



3-4-5 Triangles, continued

Using the 3-4-5 Triangle to Check a 90° Angle

This special triangle can be used to check that an existing angle is 90° as intended. An application is to check that two walls are at 90° as intended. See Figure 10-7 for an illustration of this application.

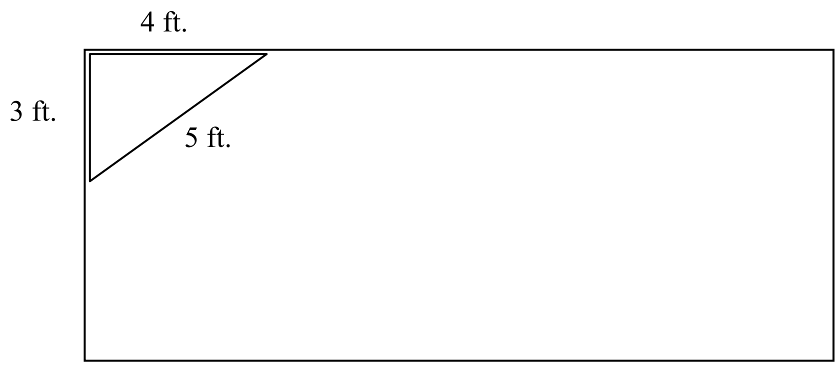


Figure 10-7: Checking a 90° angle in a corner

Application in the classroom

To simulate this triangle, a flexible tape measure is used. One leg is simulated by the 0 to 3 foot section of the tape, the other leg is simulated with the 3 to 7 foot section of the tape, and the hypotenuse is simulated with the 7 to 12 foot section of the tape. See Figure 10-8 below for this application.

This application may require three people to hold the tape.

If the two walls are at a 90° angle, then the 12-foot mark on the tape will match up with the 0-foot mark.

If the 12-foot mark on the tape does not match with the 0-foot mark, the two walls are not "square" (either more or less than 90°).

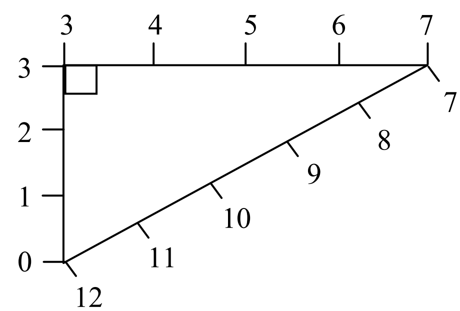


Figure 10-8: Using a tape to check a 90° angle

3-4-5 Triangles, continued

Using the 3-4-5 Triangle to Lay Out a 90° Angle

Another application of the 3-4-5 triangle is to lay out a 90° angle in preparation for construction. Before we look at an example of that application, let's take a closer look at the 3-4-5 triangle.

The 3-4-5 dimensions of this right triangle represent a *ratio* of the lengths of the sides. There are many versions of this triangle. These other versions are merely multiples of the 3-4-5 ratio.

<u>Exercise 10-11</u>

In Figure 10-9, label the 3-4-5 triangles with multiples of this ratio. One of them is done as an example.

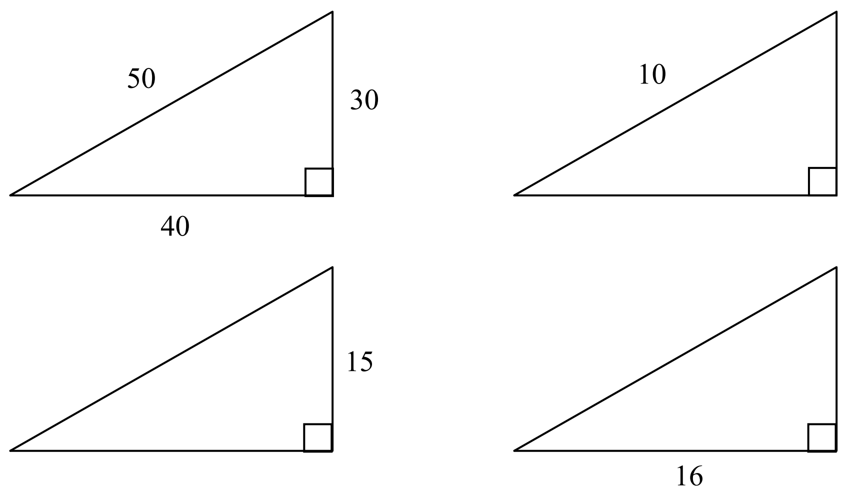


Figure 10-9: Variations of the 3-4-5 triangle

The 3-4-5 triangle can be used to lay out a 90° angle at a construction site. Locations of a foundation can be "staked out" using a long tape to simulate the 90° angle and the walls. See Figure 10-10 for an example.

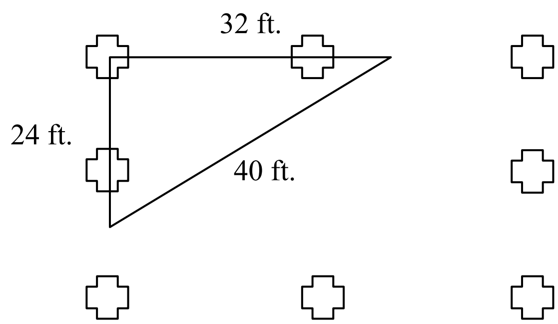


Figure 10-10: Foundation locations at a building site

Stair Dimensions

Trigonometry can also be used to calculate various dimensions of stairs in a residential or commercial building. In many cases, the staircase must fit within a certain dimension on the floor plan. These dimensions will then dictate the dimensions of the *risers* and *treads*, as well as how many risers and treads are required.

A simple staircase sketch is shown in Figure 10-11. A tread is the horizontal board that supports the weight of a person using the stairs. The riser is the vertical board that bridges the gap between the treads.

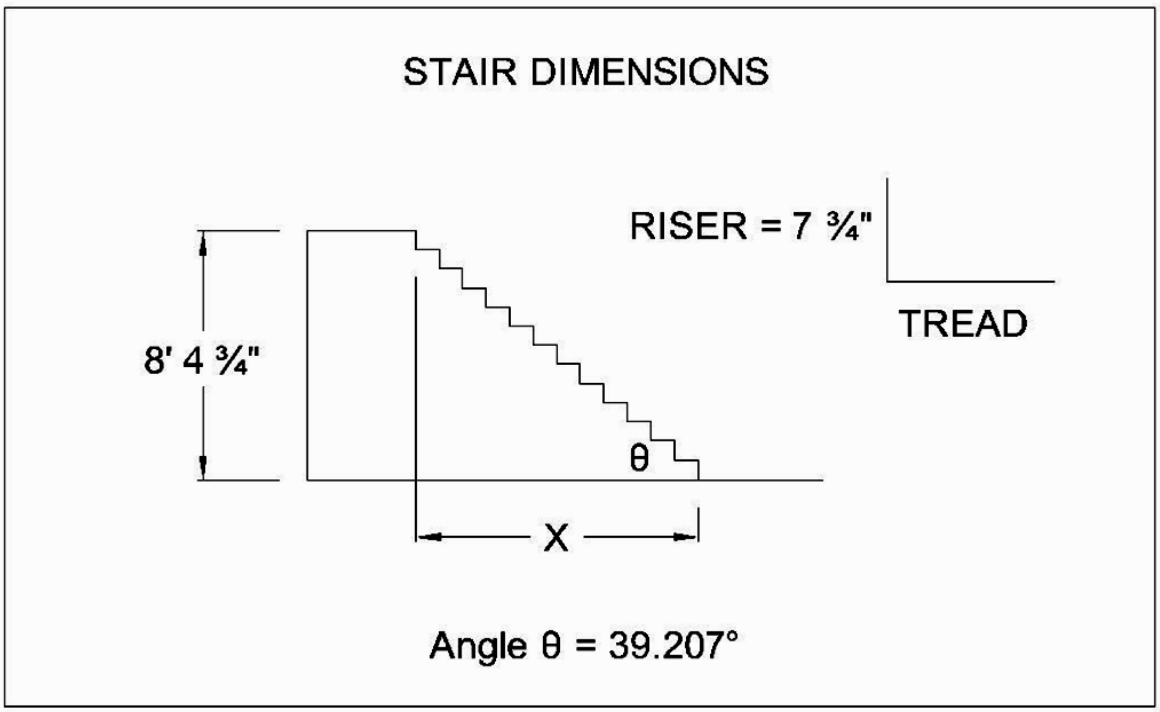


Figure 10-11: Staircase dimensions

Reference

The residential and commercial building codes include restrictions on the tread and riser dimensions.

<u>2012 ICC International Building Code® (IBC)</u>
The scope of this code covers all buildings except detached one and two family dwellings and townhouses not more than 3 stories in height.

<u>2012 ICC International Residential Code® (IRC) — One and Two-Family Dwellings</u>
This comprehensive code compiles all building, plumbing, mechanical, fuel, gas, and electrical requirements for one- and two-family dwellings.

Stair Dimensions, continued

Exercise 10-12

Using the angle θ and the riser height given in Figure 10-11, calculate the tread depth (horizontal dimension). Express your answer to the nearest 1/4 inch

Exercise 10-13

Using the given angle θ in Figure 10-11, calculate the dimension X. This is the dimension from the floor plan that restricts the horizontal size of the staircase.

Note: The triangle formed by the X dimension does not include all of the risers shown in the drawing. See the sketched right triangle in Figure 10-12.

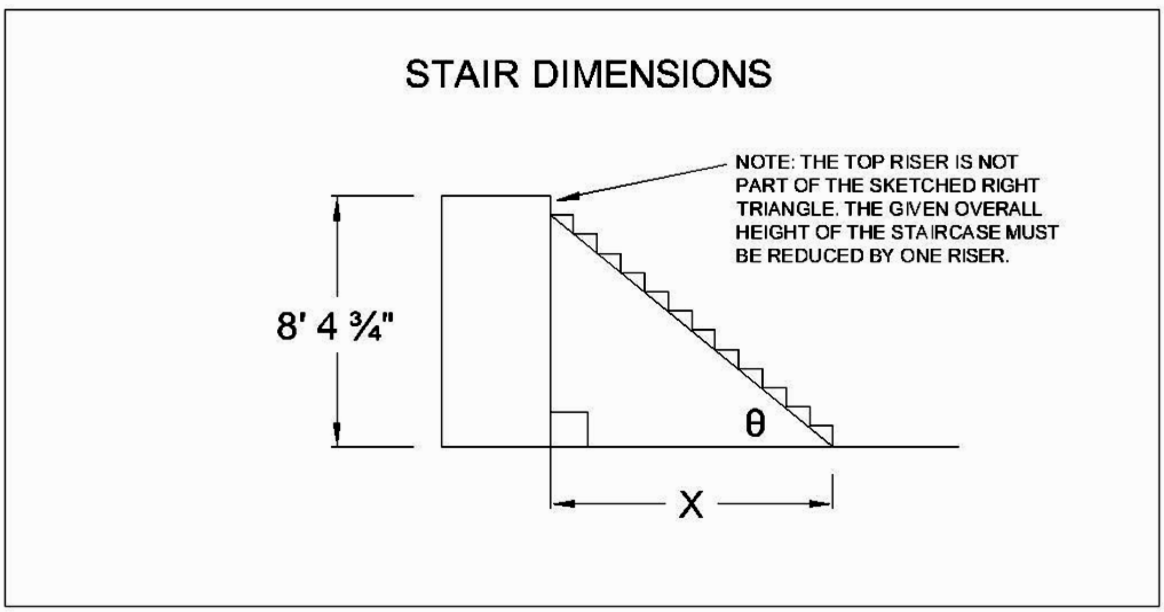


Figure 10-12: Sketched right triangle

Stair Dimensions, continued

Use the staircase sketch in Figure 10-13 to solve Exercises 10-14 and 10-15.

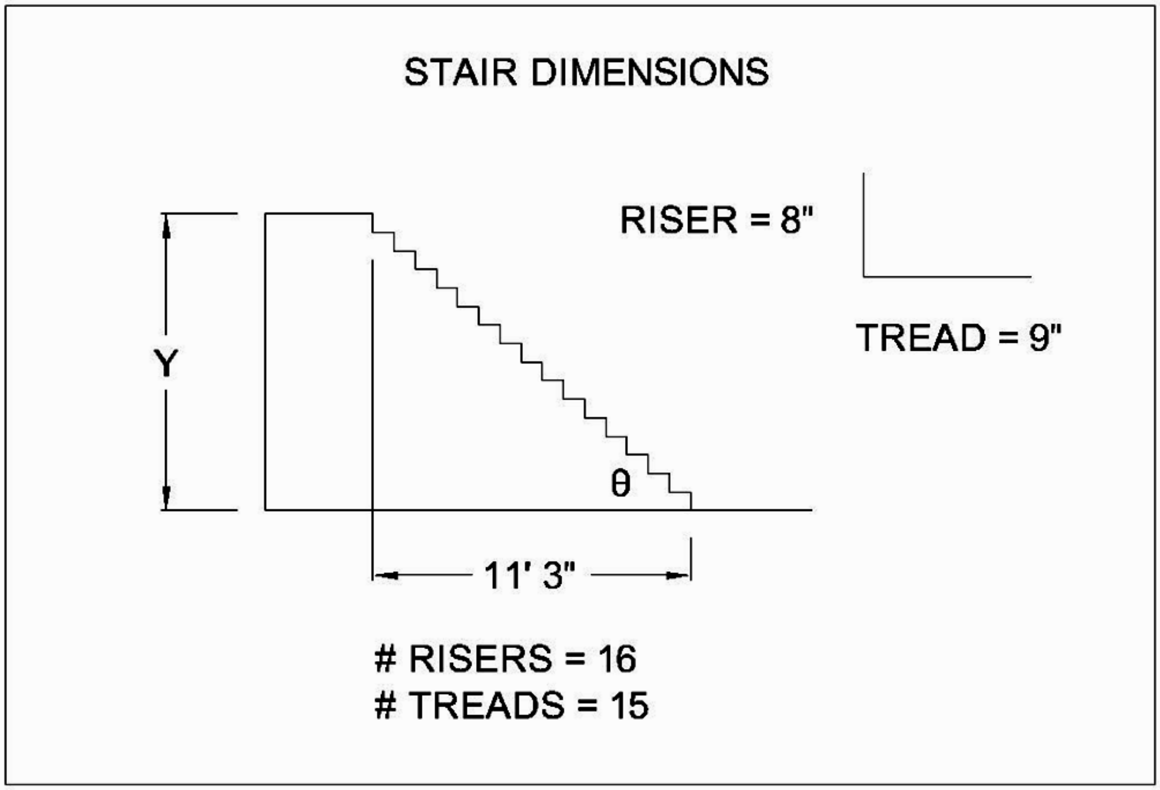


Figure 10-13: Staircase dimensions

Exercise 10-14

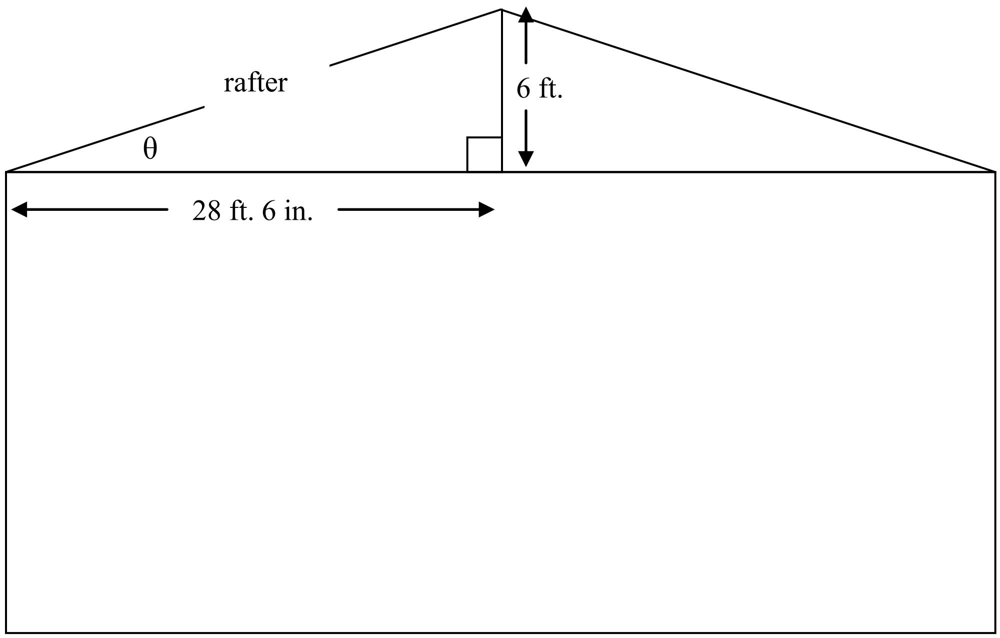
Using the riser and tread dimensions, calculate the stair angle θ.

Exercise 10-15

Calculate dimension Y, the vertical distance from the first floor to the second floor. Express your answer in feet and inches to the nearest inch. Use the stair angle θ to perform the calculation. Check your answer using the riser dimension.

Practice Worksheet

1. Calculate the roof angle, θ, and also the rafter length. Express the angle in DMS format to the nearest second. Express the rafter length in feet and inches to the nearest 1/4".



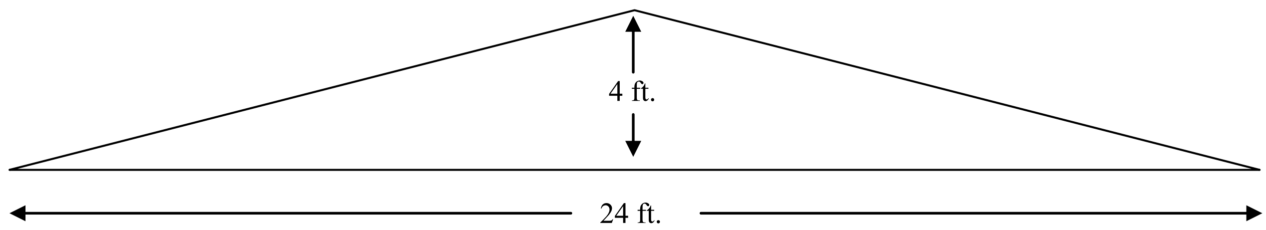
Roof angle

Rafter length

Practice Worksheet, continued

2. Calculate the roof pitch for the roof shown below. Assume that the roof is symmetrical.

   Also, calculate the roof angle, θ, in decimal degrees, to two decimals.

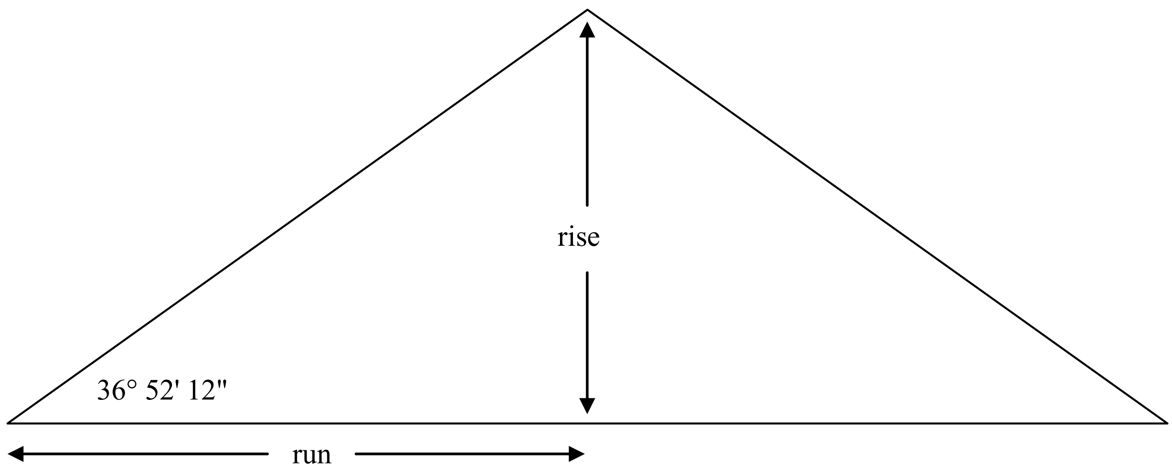


Roof pitch

Roof angle

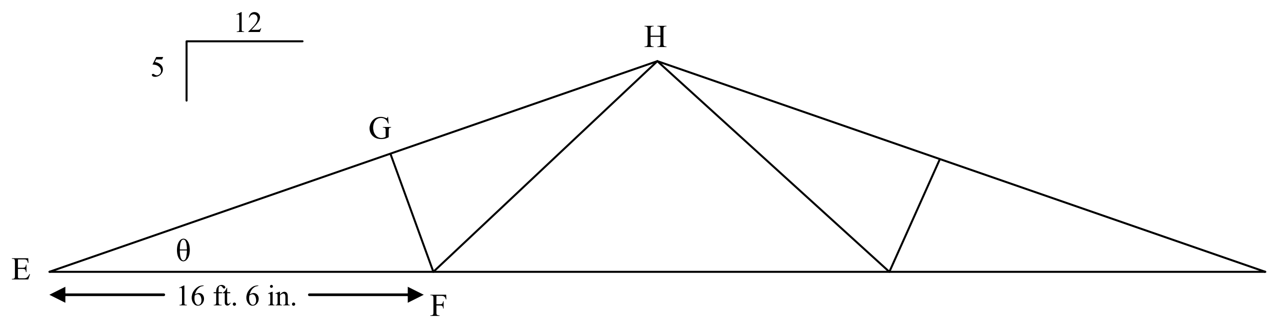
Practice Worksheet, continued

3. In the figure below, the roof angle is given but the roof pitch is not known. Using the given angle, calculate the roof pitch. Express the roof pitch as a ratio with a denominator of 12 (x:12).



Practice Worksheet, continued

4. Calculate the length of web FG. Express the web length in feet and inches to the nearest 1/2 inch.
   Note: Triangles EFG and FGH are right triangles.

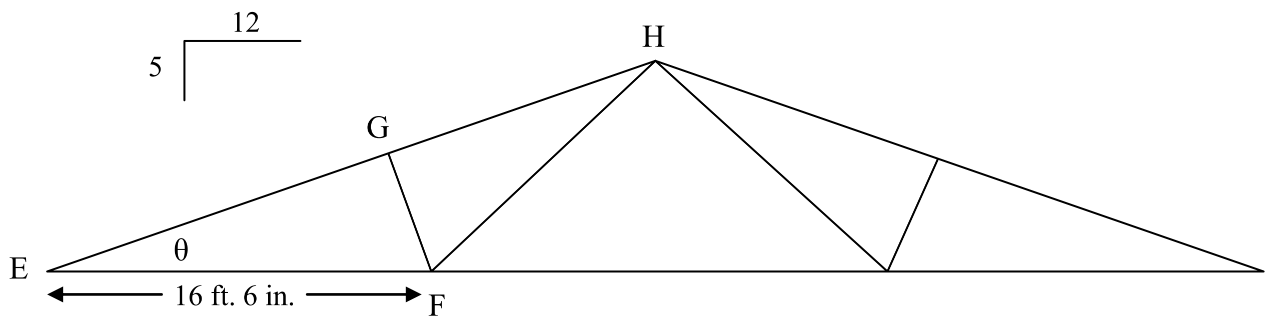


First, calculate the roof angle, θ.

Then, calculate the length of web FG.

Practice Worksheet, continued

5. Calculate the length of rafter EGH as shown on the previous problem. Express the rafter length in feet and inches to the nearest $1/2$ inch.



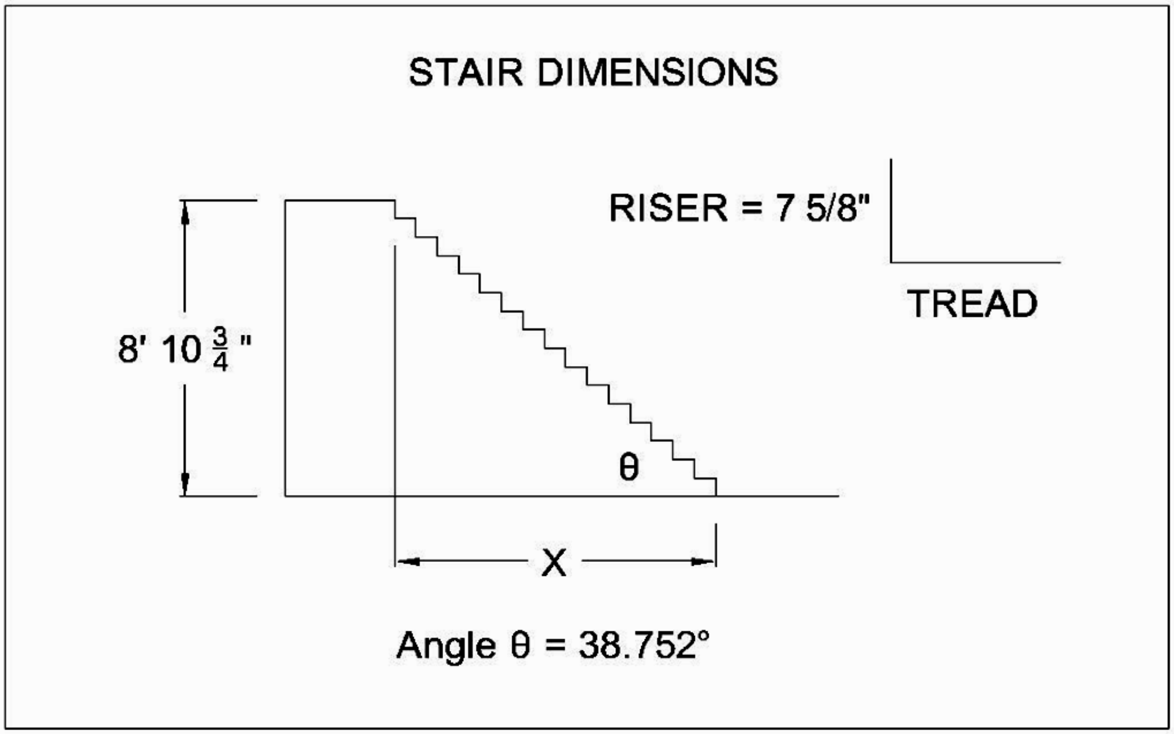
Given: $\angle GHF = 24.5°$

First, calculate the length of web EG.

Then, calculate the length of web GH.

Practice Worksheet, continued

6. Use the staircase sketch shown below to solve the following exercises.



a. Calculate the tread depth (horizontal dimension), to the nearest 1/8 inch.

b. How many risers are required to reach the second floor (nearest whole number)?

c. Calculate the dimension X, the dimension that the stairs take up on the floor plan.

d. Calculate how many treads are required in this staircase (nearest whole number).

Chapter 11: Graphing Trigonometry Functions

The trigonometry functions can be graphed. These graphs are useful in studying and describing mechanical and electrical phenomena.

Here are some examples of mechanical, acoustic, and electrical phenomena that can be described visually with a graph:

- Ocean waves
- Oscillating linear motion or rotational motion
- Sound waves
- Alternating current electricity

Mechanical Waves

Ocean Waves

One example of a mechanical wave is an ocean wave. This type of wave takes the shape of a sine wave. We will first define some terminology that is used for all types of mechanical waves.

Medium: For a mechanical wave, the medium is the material through which the wave is travelling. For example, an ocean wave travels through water. A sound wave travels through air.

Equilibrium, or rest position: This is the position of the medium when a wave is not present. For example if the ocean is calm, without any waves, the level of the ocean represents the equilibrium, or rest, position.

Displacement: This is the distance that the wave moves the medium from its equilibrium position. Sometimes the displacement is positive, when the wave raises the ocean level. The displacement may also be negative, when the wave causes the ocean level to fall below the equilibrium position.

Amplitude: Amplitude is the maximum displacement caused by a wave. The maximum displacement occurs two times during one full wave.

Cycle: One cycle represents one full wave. A full wave “rotates” through 360° (see Figure 11-1).

Example

Let’s first look at the shape of a sine wave. The basic equation for this wave is $y = A \sin \theta$, where A is the *amplitude*.

To illustrate, we need some points to graph on the Cartesian coordinates. If amplitude, A, is 1, our equation is $y = \sin \theta$. We will plot the wave in increments of 45°, for one full wave, or one cycle. The points are calculated for you in Table 11-1.

Table 11-1: Sine wave points

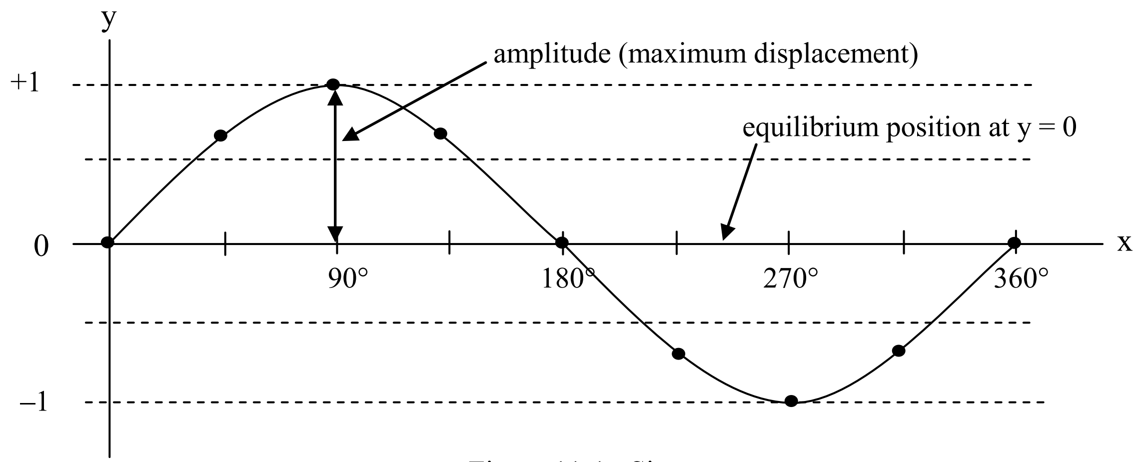
x-axis (angle θ)	y = sin θ
0°	0
45°	0.707
90°	1
135°	0.707
180°	0
225°	-.707
270°	-1
315°	-.707
360°	0

This example continues on the next page.

Ocean Waves, continued

Now we will plot the points in Figure 11-1. The amplitude in this example is 1. Notice that the points are not connected with straight lines. The sine function creates a curved shape, similar to a natural phenomenon like an ocean wave. This graph displays one complete wave, or cycle.

One full wave contains both positive displacement from the equilibrium position, and negative displacement from the equilibrium position.



▲ Figure 11-1: Sine wave

Amplitude

The wave in the previous example has an amplitude of 1. Mechanical waves, including ocean waves, may have an amplitude that is more or less that 1. Amplitude will also have units. In the next exercise, we will assign units of feet to the amplitude.

Exercise 11-1

Using the basic equation of $y = A \sin \theta$, let's graph a wave with the equation $y = 2 \sin \theta$. The first step is to calculate the points that you will plot on the graph. Complete the following Table 11-2.

Table 11-2: Sine wave points

x-axis (angle θ)	y = 2 sin θ
0°	0
45°	1.414
90°	2
135°	
180°	
225°	
270°	
315°	
360°	

This exercise continues on the next page.

Ocean Waves, continued

Exercise 11-1, continued

Now plot the points from Table 11-2 on the graph in Figure 11-2. The wave from Figure 11-1 is also shown on this graph so that you can compare the amplitude of the two waves.

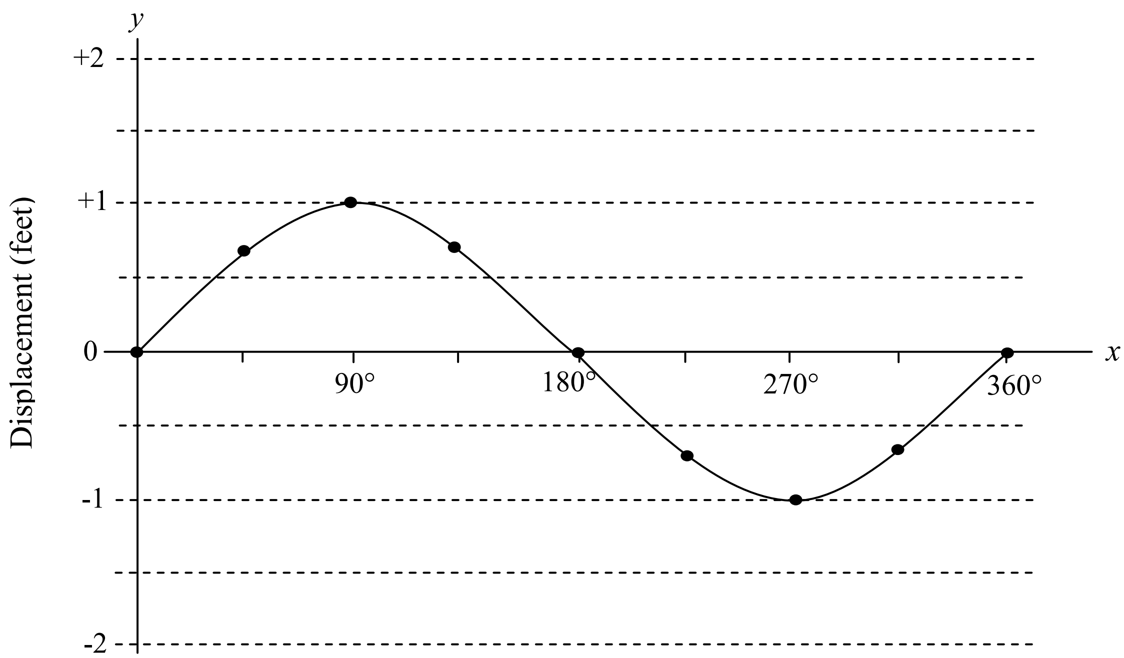


Figure 11-2: Sine wave

Exercise 11-2

Here is one more wave to plot on the Figure 11-2 graph. Use the equation y = 0.5 sin θ. Calculate the points for this graph in Table 11-3 below.

Once you have the points calculated, plot the points on the graph above, and observe how the amplitude of this wave compares with the other two waves.

Table 11-3: Sine wave points

x-axis (angle θ)	y = 0.5 sin θ
0°	
45°	
90°	
135°	
180°	
225°	
270°	
315°	
360°	

Mechanical Waves, continued

Oscillating Motion of a Spring

Another example of a mechanical wave is the oscillating motion of a spring. In this example, we have a spring with a weight W hanging on the end. The spring is set in motion and oscillates up and down. We can then graph the position of the end of the spring as it oscillates. The equilibrium, or rest, position is established before the spring is set in motion.

Example

We will declare the rest position to be at the end of the spring, as shown in Figure 11-3. This position will be 0 on the *y*-axis of the graph.

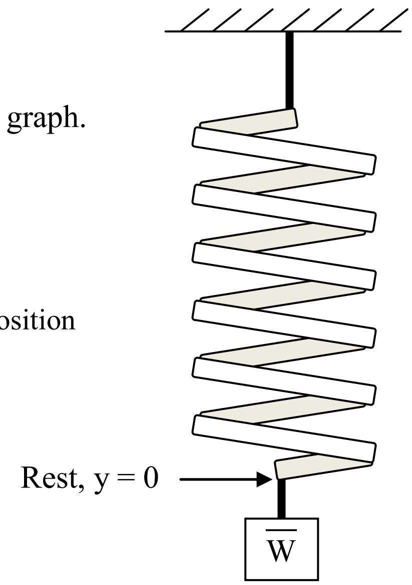


Figure 11-3: Spring at rest position

Figure 11-4 below shows positions of the spring that can be graphed. The amplitude of the oscillation will be the *y*-axis of our graph. The time of the oscillation will be on the *x*-axis.

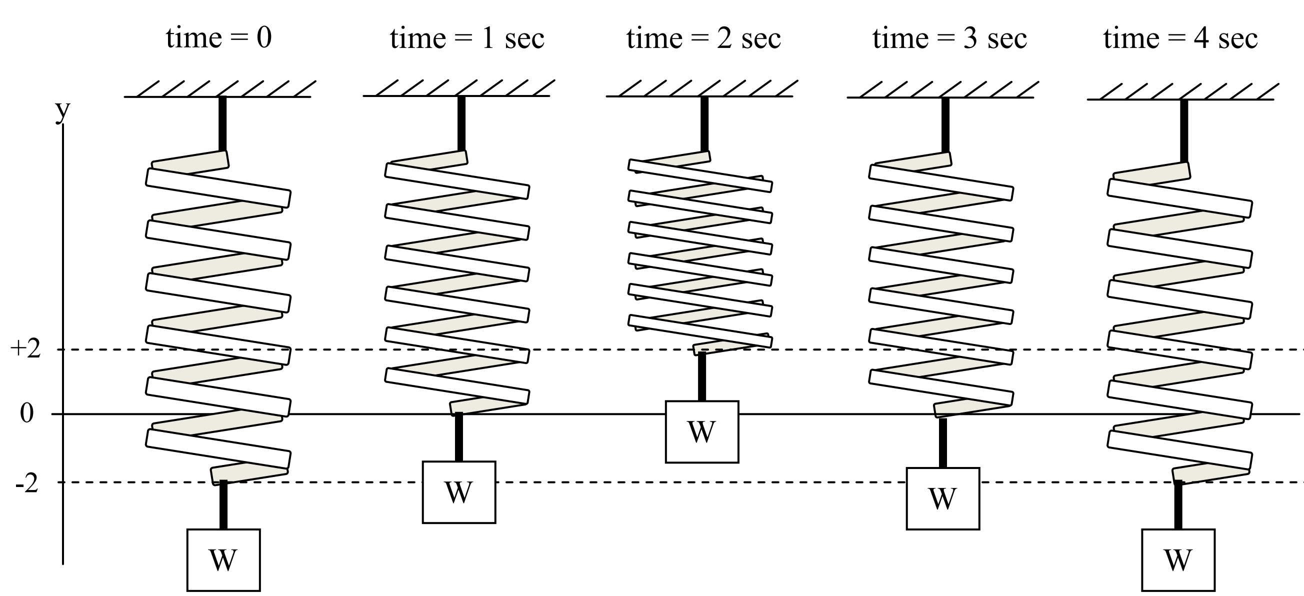


Figure 11-4: Positions of oscillating spring

Mechanical Waves, continued

Example, continued

Now let's finish this example by graphing the spring position as it oscillates, in Figure 11-5.

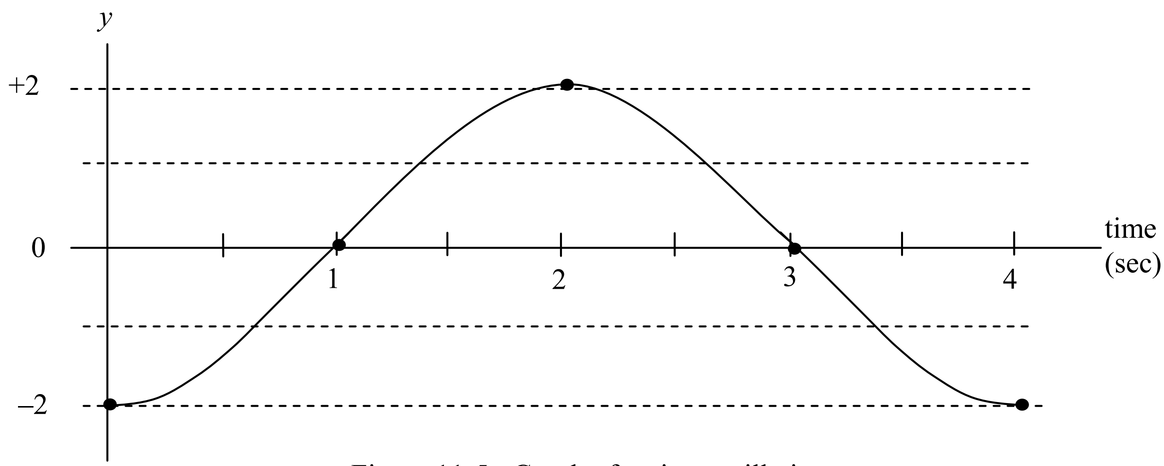


Figure 11-5: Graph of spring oscillation

This graph has the shape of a sine wave. It looks different from the sine wave in Figure 11-1 because the spring starts its oscillation at a position of -2, with the spring pulled downwards. Once the spring returns to the rest position at t = 1 second, the graph resumes a traditional sine wave shape.

We did not need to use the sine function to graph the spring motion because we had data from Figure 11-4. The spring oscillation, however, could be described with the sine function.
▲

Example

We can create an equation that describes the oscillation of this spring.

If we use the equation $y = \sin\theta$, we will get a sine wave with an amplitude of 1. This spring has an amplitude of 2, so we will change the equation to $y = 2 \sin\theta$.

Next, we need to adapt the equation to incorporate the time data on the *x*-axis.

One complete cycle of the sine wave occurs over 360°. This complete cycle takes a total of 4 seconds. The time for a complete cycle is called the *wave period* and is designated by the uppercase letter T.

The angle θ, at any point in the wave cycle, can be described by the time at that point in the cycle as follows:

Angle θ at any time $t = (360° (t/T))$, where T = wave period

This example continues on the next page.

Mechanical Waves, continued

Example, continued

The equation then becomes:

$y = 2 \sin(360^\circ (t/4))$, where the wave period T = 4 seconds

Finally, we must account for the fact that the wave starts at a displacement of y = –2 instead of starting at y = 0. The graph is shifted to the right by one-fourth of a cycle. We may also say that the peak of the wave is delayed by one-fourth of a cycle. This shift is sometimes called a *phase shift*. See Figure 11-6 below for a graphical description of the phase shift.

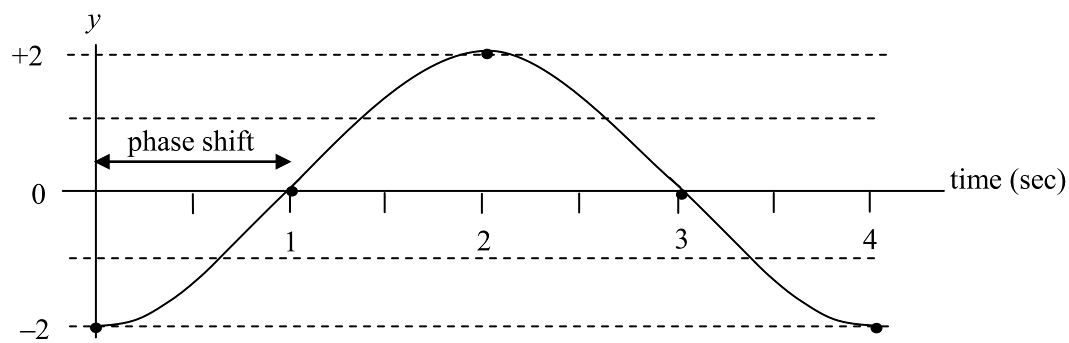


Figure 11-6: Phase shift

We can account for the phase shift by including the one-fourth of a cycle as follows:

$y = 2 \sin(360^\circ (t/4) - 90^\circ)$

phase shift

Complete the following table of calculations for the y value, or position, of the spring.

Table 11-4: Spring displacement calculations

time, t (seconds)	calculation	y
0	$y = 2 \sin(-90^\circ)$	–2
1	$y = 2 \sin(360^\circ (1/4) - 90^\circ)$	0
2	$y = 2 \sin(360^\circ (2/4) - 90^\circ)$	2
3	$y = 2 \sin(360^\circ (3/4) - 90^\circ)$	0
4	$y = 2 \sin(360^\circ (4/4) - 90^\circ)$	–2

You can see that these calculations match the data provided for the spring in Figure 11-4.
▲

Mechanical Waves, continued

<u>Exercise 11-3</u>

Create an equation for a sine wave, similar to the previous example. Use the following information about the wave:

- Amplitude of the wave = 2.5 centimeters
- The wave period, T = 1.5 seconds
- The wave has a phase shift of 45°, shifted to the left. In other words, the peak of the wave occurs 45° earlier than a wave that does not have a phase shift.

<u>Exercise 11-4</u>

Create an equation for the wave shown in the Figure 11-7 below. Use the figure to determine the amplitude, wave period, and phase shift.

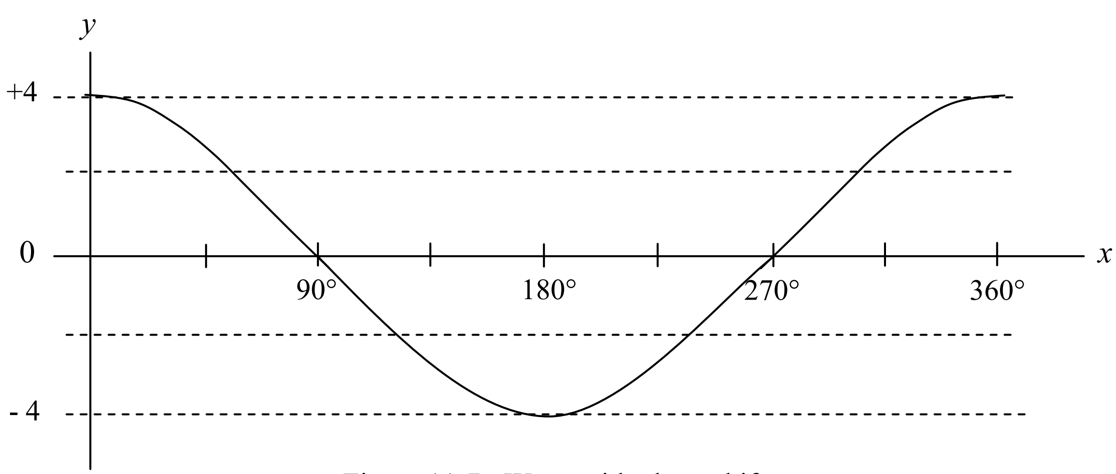


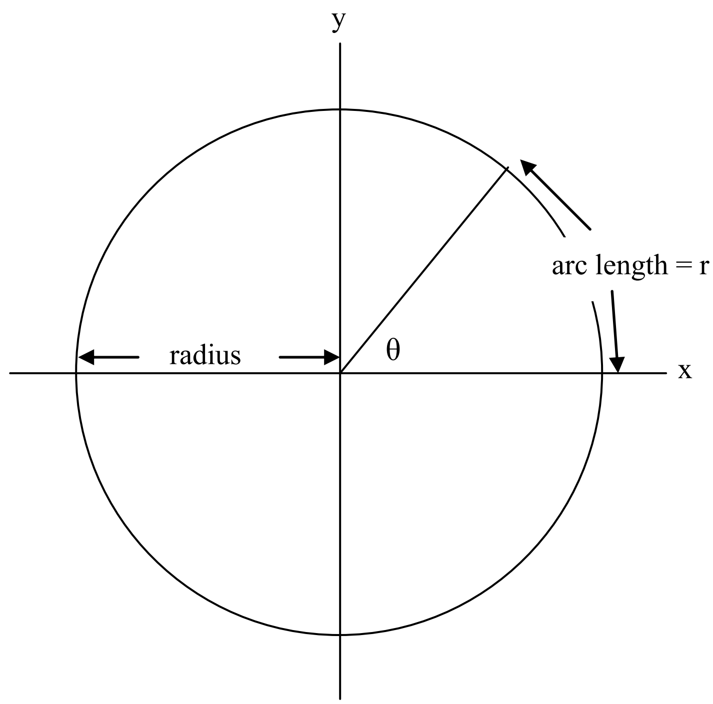
Figure 11-7: Wave with phase shift

Radian Units

It is common in math and science for angles to be expressed in *radians* rather than degrees. What is a radian?

A radian is the angle created by an arc that has the same length as the radius of the circle. See Figure 11-8 for a sketch of this arc and angle.

Figure 11-8: Angle θ = one radian



There are 360º in one full revolution. How many radians are in one full revolution? From geometry, the circumference C (distance around the outside) of a circle is calculated using this equation:

$C = 2\pi r$, where r is the radius of the circle

A radian is created by an arc of radius r. Therefore, $C = 2\pi$ radians

We can set these two expressions for a revolution equal to each other:

$360^\circ = 2\pi$ radians

Solve this expression for a radian:

1 radian = $(360^\circ)/2\pi$

For accuracy, use the value of π that is displayed by your calculator.

1 radian = $(360^\circ)/2\pi$ = approximately 57.3°

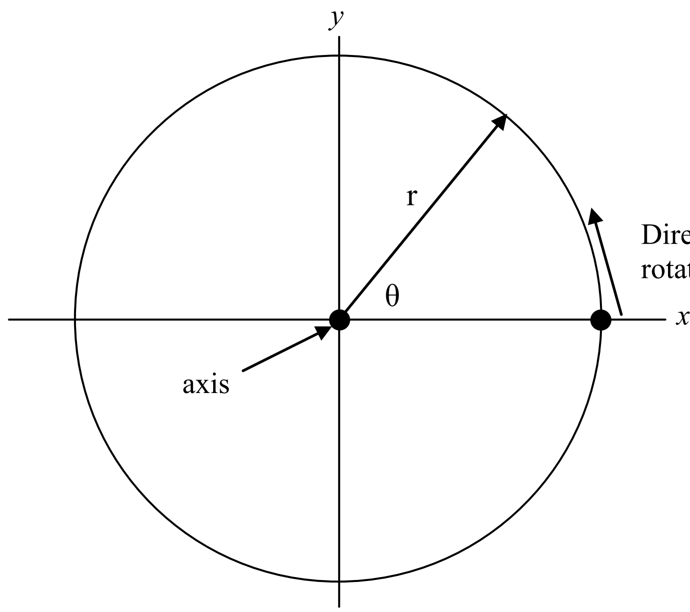
Rotational Motion

Example

Let's use radians to graph an example of *rotational motion*. Rotational motion is circular motion of an object around a fixed axis. The axis is commonly the center of a circle. In contrast, the spring example demonstrates linear, or straight line, motion.

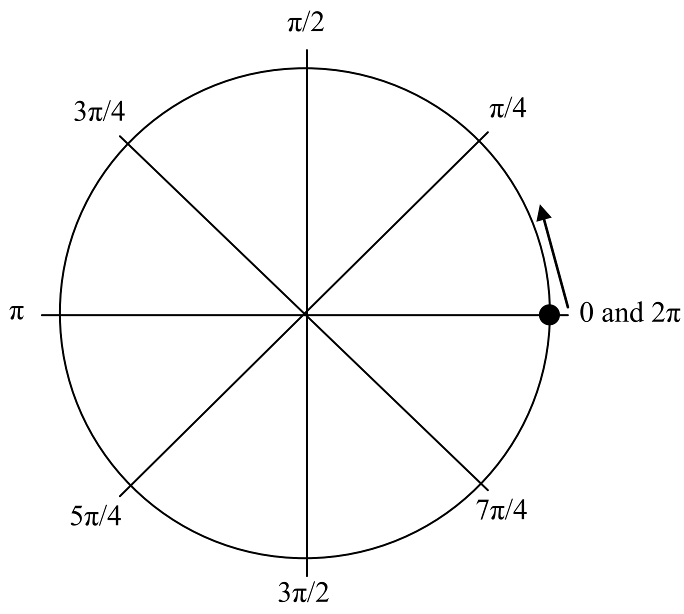
The wheel in Figure 11-9 below has a radius, r of 3.5 cm. All points on the wheel rotate around the axis at the center of the wheel.

Figure 11-9: Rotating wheel



We will graph the *y* value of a point on the outer edge of the wheel. That point is shown in Figure 11-9, along with the direction of rotation. Figure 11-10 below shows the same wheel marked with the angles in radians. In math and science topics, it is common for angles to start from the "easterly" direction and rotate counterclockwise.

Figure 11-10: Radian measures around a circle



Rotational Motion, continued

Example, continued

The time for one rotation of this wheel is 8 seconds. Start with the basic equation, y = sin θ. Modify it to account for the amplitude of 3.5 cm and the wave period T of 8 seconds. We will also substitute 2π radians for a full revolution rather than 360º.

$y = 3.5 \sin(2\pi\,(t/8))$, where the wave period, T = 8 seconds

Before calculating the y values, you must change the angle units in your calculator to radians. We discussed this setting in Chapter 2. Your choices for angle units are degrees, radians, and grads. Make sure that your calculator is set to radians. You will see RAD on the display.

At time t = 1 second the equation becomes $y = 3.5 \sin(2\pi\,(1/8)$. Using your calculator, y = 2.47.

At time t = 2 second the equation becomes $y = 3.5 \sin(2\pi\,(2/8)$. The fraction of 2/8 may be reduced to a fraction of 1/4, as you can see in Table 11-5 below. Using your calculator, y = 3.5.
▲

Exercise 11-5

Complete the rest of the calculations in Table 11-5 below. These are the y values for the point on the outside of the wheel as it rotates.

Table 11-5: Wheel displacement calculations

time, t (seconds)	Calculation	y
0	$y = 3.5 \sin(0)$	0
1	$y = 3.5 \sin(2\pi\,(1/8))$	2.47
2	$y = 3.5 \sin(2\pi\,(1/4))$	3.5
3	$y = 3.5 \sin(2\pi\,(3/8))$	
4	$y = 3.5 \sin(2\pi\,(t/8))$	
5	$y = 3.5 \sin(2\pi\,(t/8))$	
6	$y = 3.5 \sin(2\pi\,(t/8))$	
7	$y = 3.5 \sin(2\pi\,(t/8))$	
8	$y = 3.5 \sin(2\pi\,(t/8))$	

Use Figure 11-11 on the next page to graph the motion of this rotating wheel.

Rotational Motion, continued

Exercise 11-5, continued

Graph the results of the calculations from Table 11-5. This graph shows how a point on the outside of the wheel changes its y position during one rotation.

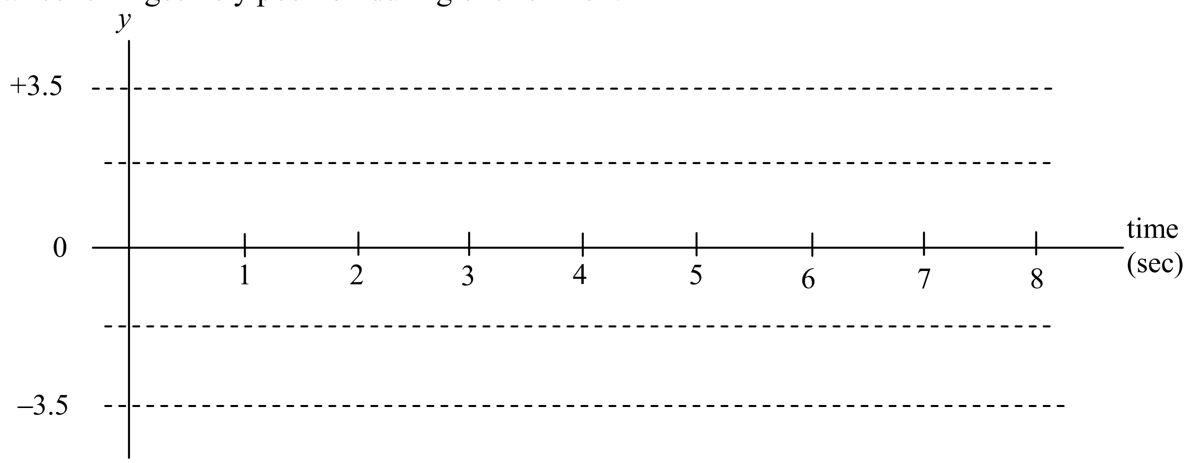


Figure 11-11: Graph of wheel rotation

Exercise 11-6

Here is another example of a rotating wheel, as illustrated in Figure 11-12 below. The following information is given:

- The wheel has a radius of 6.75 inches.
- The direction of rotation is counterclockwise
- The wave period, T = 1.2 seconds. This is the time for one rotation of the wheel.
- A point on the outside of the wheel is selected as the point to graph.
- This point starts at its maximum *y* value of 6.75 inches.
- This results in a phase shift of one-fourth of a rotation.

Create an equation that describes the *y* position of the point on the wheel during a complete revolution of the wheel. Use either radians or degrees.

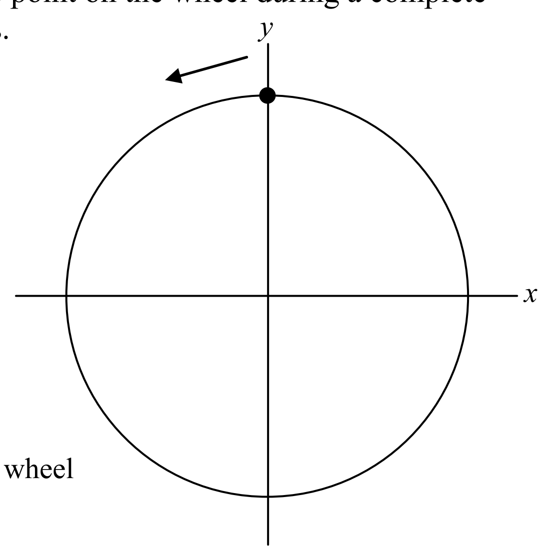


Figure 11-12: Rotating wheel

Sound Waves

Another type of mechanical wave is a sound wave. Sound waves vary in terms of loudness and pitch.

Loudness

Loudness of a sound is caused by a variation in air pressure. The maximum air pressure in the sound is the amplitude of the sound wave. Our hearing senses this air pressure as loudness.

The units of air pressure are pounds per square inch, or psi, in the English system. In the metric system, air pressure is expressed as Newtons per square meter, or Pascals (Pa).

The variation of pressure in a sound wave takes the shape of a sine wave, when graphed.

Pitch

The *pitch* of a sound is caused by the frequency of the wave. *Frequency* measures the number of waves, or cycles, that occur in a period of time. A high frequency sound is sensed as a high pitch.

The units of frequency are cycles per second, often expressed as Hertz. One cycle per second is equal to 1 Hertz (Hz).

Example

Let's create an equation for a sound wave and then graph its variation of pressure over time.

Suppose a sound wave has an amplitude of 3 Pascals (Pa). The frequency of the sound is 100 Hertz, or cycles per second.

The equation for this sound wave is:

$$y = 3 \sin (2\pi \, (t/\mathrm{T}))$$

We do not yet know the wave period, T, the time for one complete wave.

We can calculate T, using the frequency. The wave period is the inverse of the frequency.

$$\text{Wave period, T} = \frac{1}{\text{frequency}} = \frac{1 \text{ second}}{100 \text{ cycles}} = \frac{.01 \text{ second}}{1 \text{ cycle}}$$

The equation for this sound wave then becomes:

$$y = 3 \sin (2\pi \, (t/.01))$$

▲

On the next page, calculate the air pressure at a variety of time intervals. Then, graph the results to see the sound wave.

Sound Waves, continued

Exercise 11-7

Calculate air pressure using the equation: $y = 3 \sin(2\pi(t/.01))$, in Table 11-6 below. We will use time intervals of 0.001 second. These points do not include the maximum positive displacement which occurs at 0.0025 second, and the maximum negative displacement which occurs at 0.0075 second.

Table 11-6: Sound wave pressure

time, t (seconds)	Calculation	y (Pascals)
0	$y = 3 \sin(0)$	
.001	$y = 3 \sin(2\pi(.001/.01))$	
.002		
.003		
.004		
.005		
.006		
.007		
.008		
.009		
.01		

Exercise 11-8

In Figure 11-13, graph the results from Table 11-6 above.

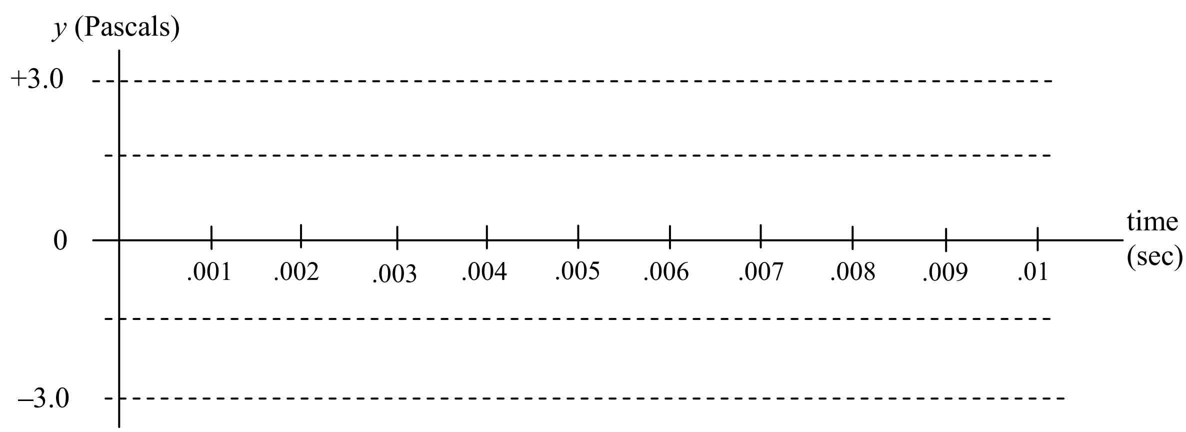


Figure 11-13: Sound wave pressure

Alternating Current

Alternating electrical current also takes the shape of a sine wave.

Voltage

Voltage in alternating electrical current is one way to express the amplitude of the electricity. The voltage varies as it is created by a rotating generator.

The variation of voltage takes the shape of a sine wave, when graphed.

Frequency

The frequency of alternating electricity is an expression of how many complete cycles occur in a period of time. Electrical frequency is expressed in the same way as sound frequency.

The units of frequency are cycles per second, and is often expressed as Hertz. One cycle per second is equal to 1 Hertz (Hz). Typical electrical frequency in the United States is 60 Hertz. Electrical frequency may be different in other countries.

Exercise 11-9

Let's create an equation for an alternating current, and then graph its variation of voltage over time.

An alternating electrical current has a maximum voltage of 170 volts. The frequency of the electricity is 60 Hertz, or 60 cycles per second.

We do not yet know the wave period, T, the time for one complete wave. The wave period is the inverse of the frequency. Calculate the wave period:

$$\text{Wave period, T} = \frac{1}{\text{frequency}} =$$

Using radians as the angular units, create an equation for this alternating current:

In Exercise 11-10, calculate the voltage at a variety of time intervals. Then, in Exercise 11-11, graph the results to see the alternating voltage.

Alternating Current, continued

Exercise 11-10

Calculate voltage in Table 11-7, using your equation from the previous page. Note that the time intervals provided are rounded numbers and may not give you the exact results that you expect.

Table 11-7: Voltage calculations

time, t (seconds)	calculation	*y* (voltage)
0		
.0021		
.0042		
.0063		
.0084		
.0104		
.0125		
.0146		
.0167		

Exercise 11-11

Graph the results from Table 11-7 above.

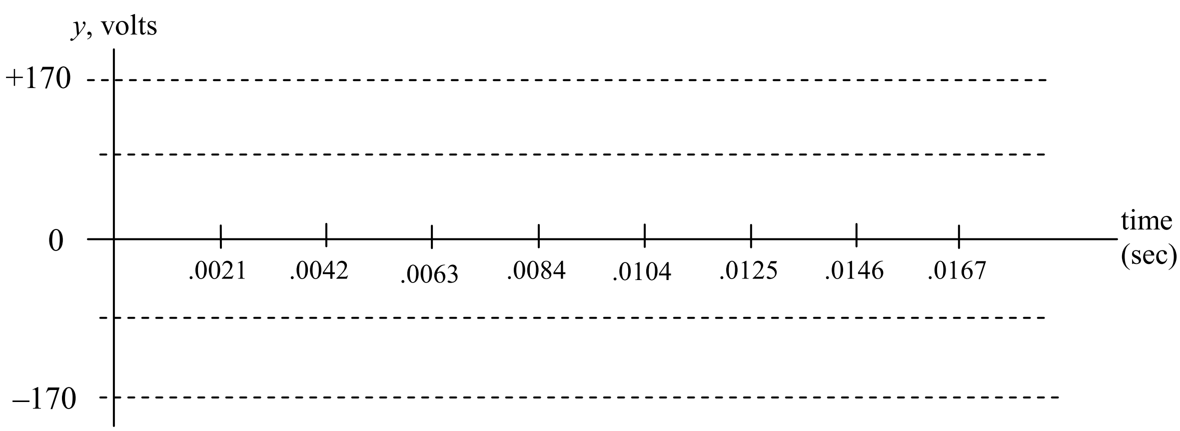


Figure 11-14: Alternating current

Alternating Current, continued

Three-phase current

As you can see from the graph on the previous page, there is a significant variation in voltage during a cycle of alternating current. Some large equipment benefits from using three-phase power. With three phases, each phase reaches its peak voltage at different times, thus reducing the variation in voltage provided to the equipment.

The three phases are shifted in phase by 120º. One phase is the reference phase, and the other two phases are delayed by 120º and 240º.

Exercise 11-12

a. Create an equation for an alternating current that has a peak voltage of 340 volts and has a frequency of 60 Hertz. Use degrees for the angular units. Use the same wave period T as in the previous exercise.

b. Using this first equation, create an equation for the phase which is delayed by 120º from the first phase.

c. Using this first equation, create an equation for the phase which is delayed by 240º from the first phase.

Exercise 11-13

Sketch these three phases in Figure 11-15 below. Try a free-hand sketch, without first calculating individual points. Start with the first phase and then delay the peak of the other two waves by 120º and 240°.

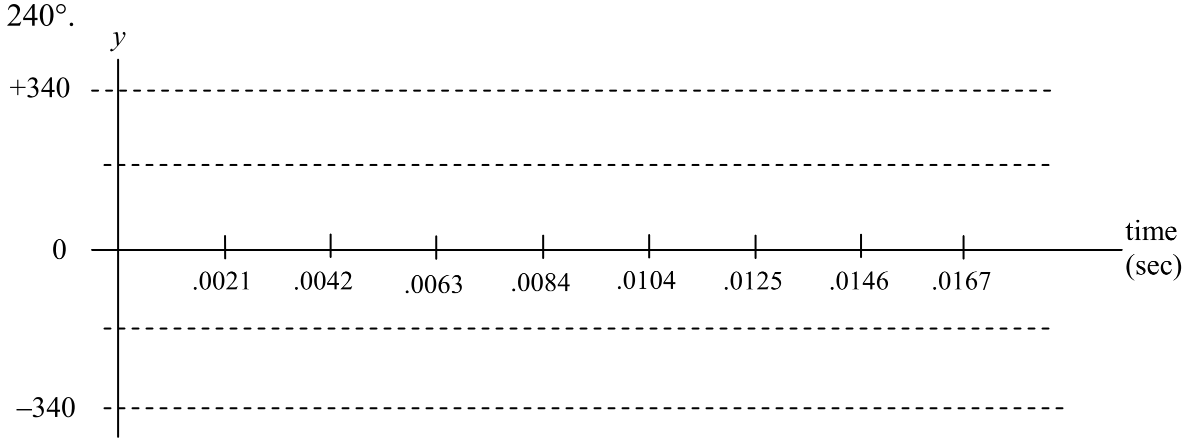


Figure 11-15: Three-phase electricity

Graph of Cosine Function

Let's look at the shape of the cosine function, y = cos θ. We can see how it compares to the shape of the sine function.

Exercise 11-14

We need some points to graph on the Cartesian coordinates. Complete the calculations in Table 11-8.

Table 11-8: Cosine wave points

x-axis (angle θ)	y = cos θ
0°	
45°	
90°	
135°	
180°	
225°	
270°	
315°	
360°	

Exercise 11-15

Now we can plot these points in Figure 11-16.

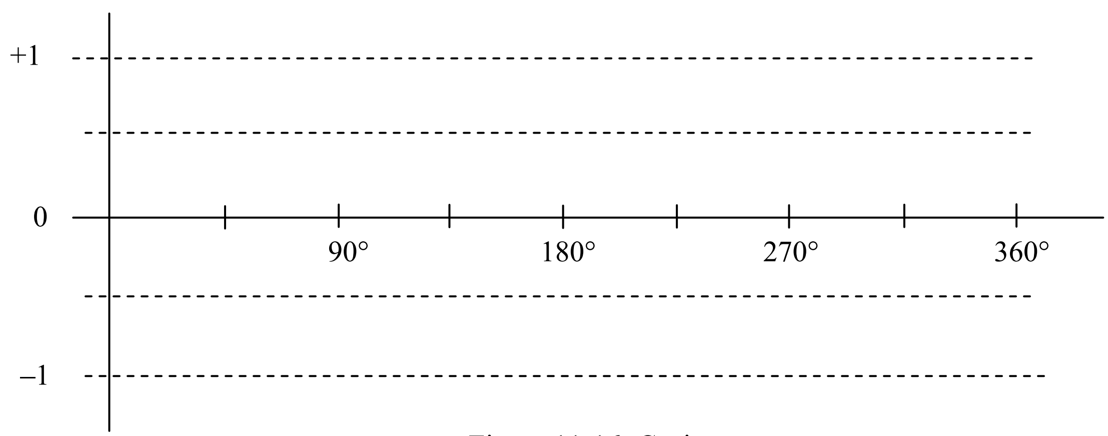


Figure 11-16: Cosine wave

Notice how the shape of the cosine function compares with the sine wave. The cosine wave has the same shape as the sine wave, with a phase shift. Remember that sine and cosine are co-functions and that cos θ = sin (90° – θ).

Therefore, y = cos θ has the same shape as y = sin (θ +90°). The cosine wave is a sine wave that is shifted to the left by 90°.

Graph of Cosine Function, continued

Exercise 11-16

Now we will plot a sine wave and a cosine wave on the same graph, and observe the results.

Here are the equations for these two waves:

- $y = 0.5 \sin(\theta + 45°)$
- $y = 1.25 \cos(\theta - 45°)$

The sine wave has a phase shift of 45° to the left, and the cosine wave has a phase shift of 45° to the right. Calculate the points for each wave in Table 11-9, and then plot the two waves in Figure 11-17 below.

Table 11-9: Cosine wave points

x-axis (angle θ)	$y = 0.5 \sin(\theta + 45°)$	$y = 1.25 \cos(\theta - 45°)$
0°		
45°		
90°		
135°		
180°		
225°		
270°		
315°		
360°		

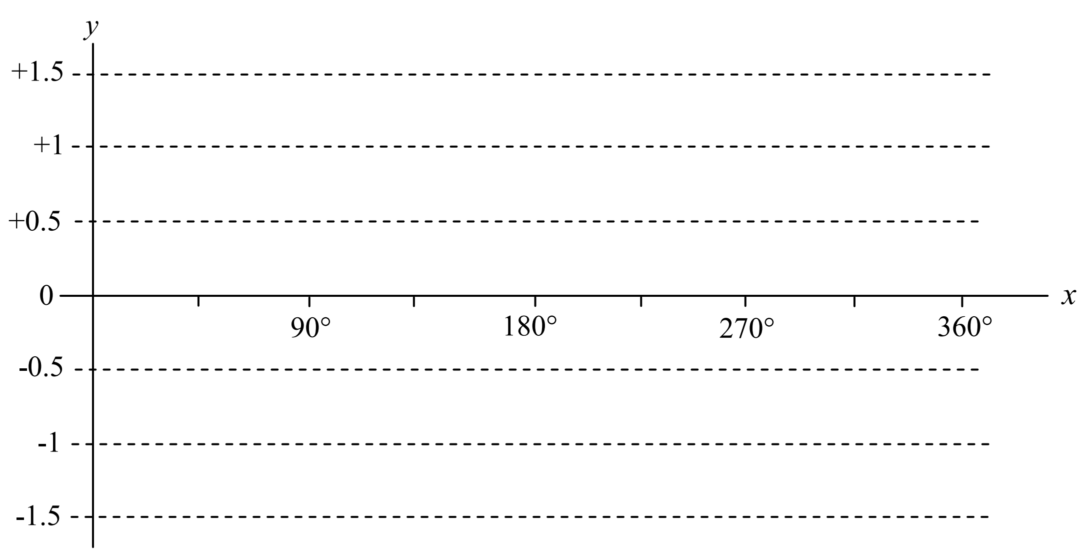
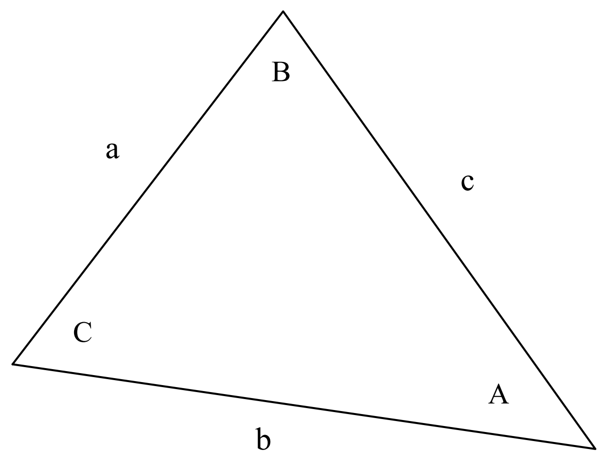


Figure 11-17: Sine wave and cosine wave

Chapter 12: Law of Sines

The Law of Sines is a tool that you can use to help solve for unknowns in an oblique triangle. Remember that an oblique triangle, either obtuse or acute, is a triangle that does not have a right angle.

Figure 12-1: Oblique triangle



Law of Sines Formula

The equation describes the relationship between the sides and angles in an oblique triangle. Refer to Figure 12-1 above for the labeling of the sides and internal angles.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Problem Solving with Law of Sines

To apply the Law of Sines, you must know three dimensions or angles, and at least one of these must be the length of a side.

For example, if you know the value of all three internal angles, you will not be able to solve for the length of the sides unless you know at least one side.

Example

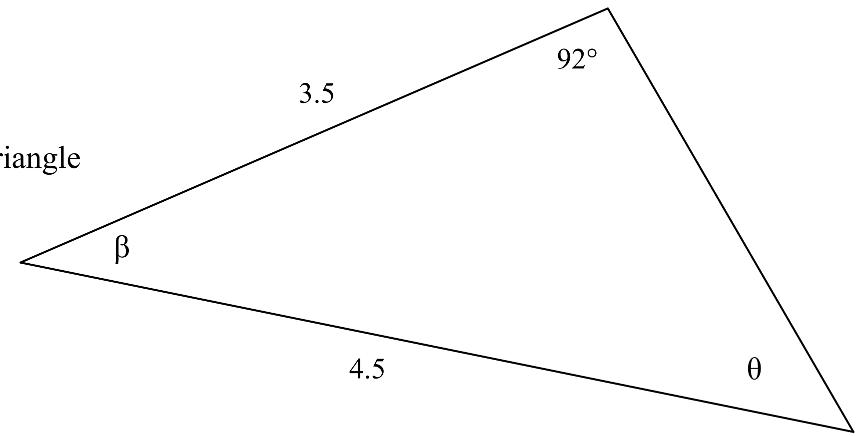


Figure 12-2: Obtuse triangle

Calculate angle θ.

$$\frac{3.5}{\sin \theta} = \frac{4.5}{\sin 92°} \qquad \sin \theta = \frac{(3.5)(\sin 92°)}{4.5} = 0.7773$$

$$\theta = \sin^{-1}(0.7773) = 51°$$

Calculate angle β.

$$B = 180° - 92° - 51° = 37°$$

▲

Example

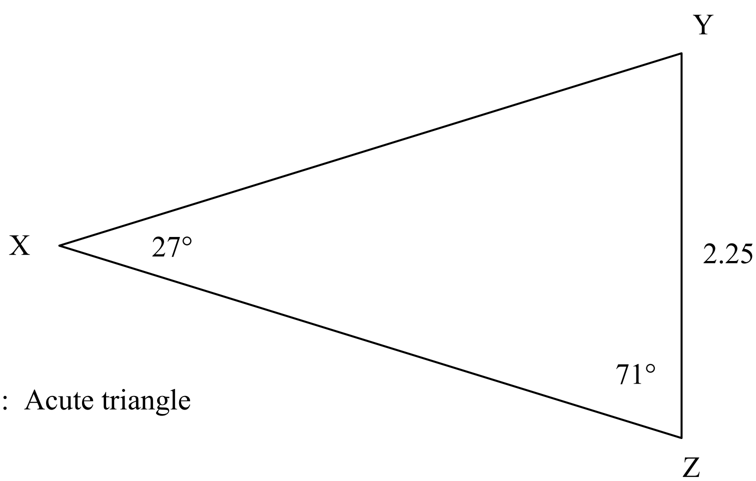


Figure 12-3: Acute triangle

Calculate the length of side XY.

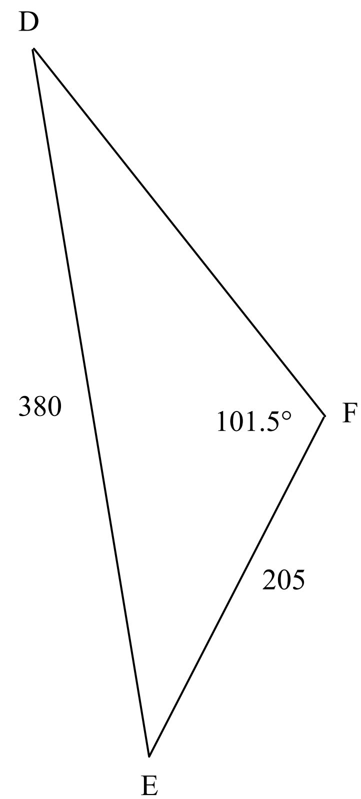
$$\frac{XY}{\sin 71°} = \frac{2.25}{\sin 27°} \qquad XY = \frac{(2.25)(\sin 71°)}{\sin 27°} = 4.69$$

▲

Exercise 12-1

Calculate angle D to the nearest whole degree.

Figure 12-4: Obtuse triangle



Exercise 12-2

Calculate angle F, angle G, and side EF.

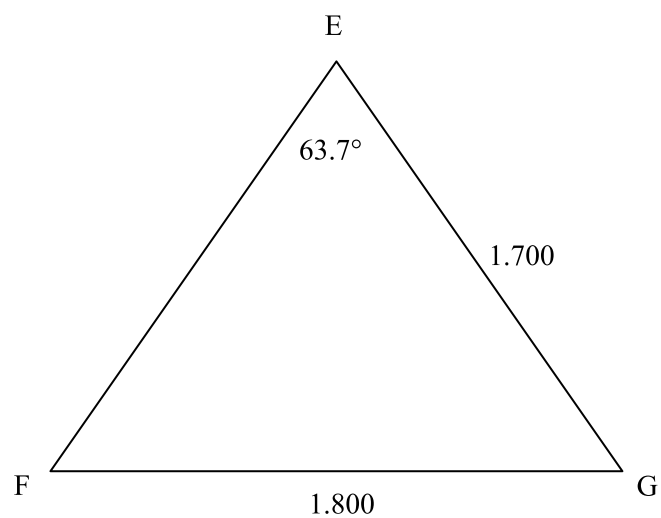


Figure 12-5: Acute triangle

Exercise 12-3

Calculate angle A and side a.

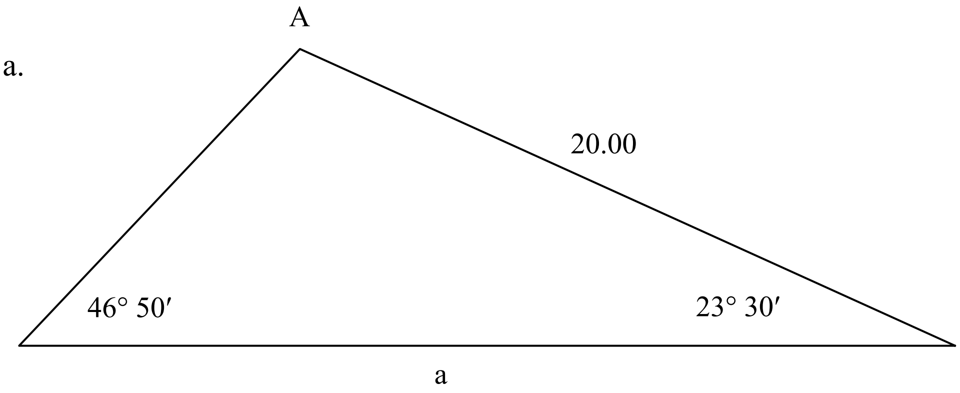


Figure 12-6: Obtuse triangle

Exercise 12-4

Calculate the length of sides PQ and PR

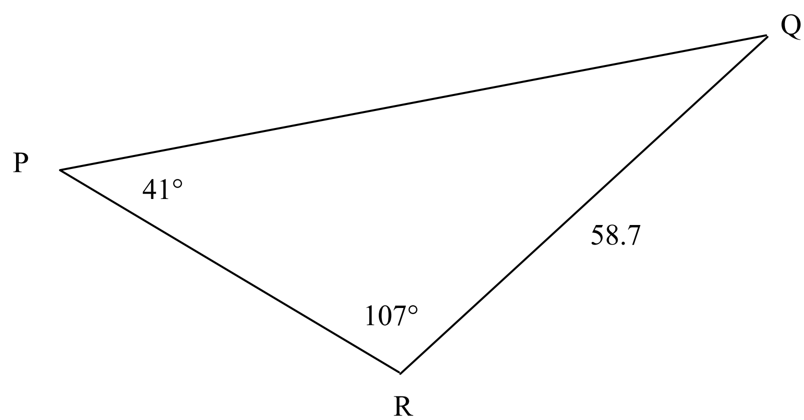


Figure 12-7: Obtuse triangle

Civil Application

Here is a civil example, for a parcel of land, shown in Figure 12-8.

We will calculate the length of side AC, which is identified in Figure 12-8 by the vertices A and C. The lengths of the other two sides are given.

Rather than using internal angles, the line directions are expressed as bearings. We discussed bearings in Chapter 9.

We must first determine the value of the internal angles, using the bearing information.

There are multiple approaches to this problem. We will first convert the bearings to the azimuth format. An azimuth is measured clockwise from North.

We will also find that the *opposite* directions for each azimuth will be helpful. The opposite direction is different from the given direction by 180°.

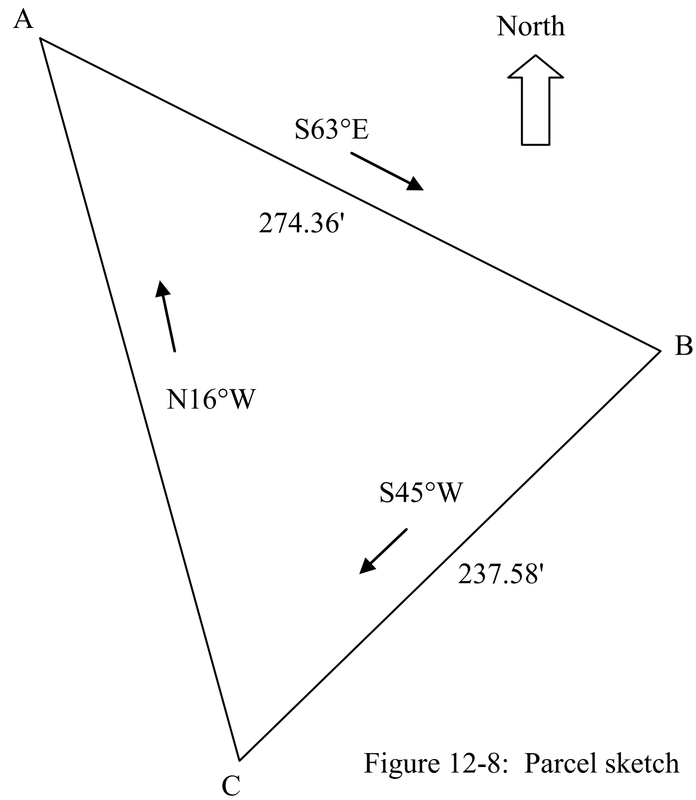


Figure 12-8: Parcel sketch

Convert Bearing to Azimuth

The first step is to convert the bearing information in Figure 12-8 to azimuths. This will make it easier to calculate the internal angles. We will use the method discussed in Chapter 9.

Example

Line AB azimuth
S63°E = 180° – 63° = Azimuth 117°

Line BC azimuth
S45°W = 180° + 45° = Azimuth 225°

Line AC azimuth
N16°W = 360° – 16° = Azimuth 344°

▲

Civil Application, continued

Calculate an Opposite Direction

Next we will calculate the opposite direction for each of these three lines: AB, AC, and BC. The opposite direction must be between 0° and 360°. You may need to add or subtract 180° to achieve an answer in that range.

Example

Line AB
Opposite direction = 117° + 180° = 297°

Line BC
Opposite direction = 225° – 180° = 45°

Line AC
Opposite direction = 344° – 180° = 164°

See these opposite directions in Figure 12-9.

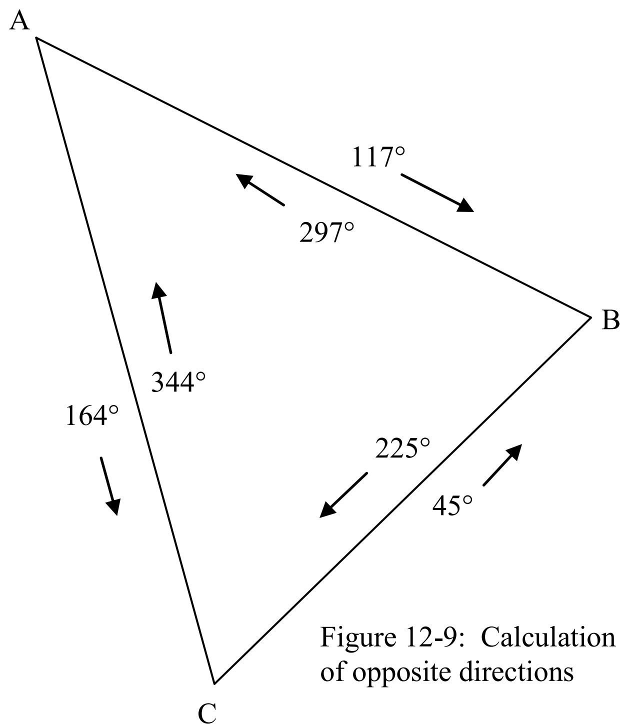


Figure 12-9: Calculation of opposite directions

▲

Calculate Internal Angles

We now have enough information to determine the internal angles in this triangular parcel.

An internal angle represents the *difference in direction* between two lines. The two lines have a common vertex, where the lines meet.

Example

The internal angle A is determined by the difference in direction between lines AB and AC. See Figure 12-10.

Angle A = 164° – 117° = 47°

Note: The internal angle is always a positive number. Ensure a positive result by subtracting the smaller direction from the larger direction.

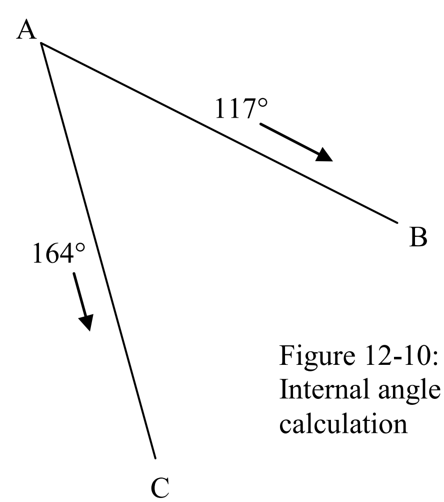


Figure 12-10: Internal angle calculation

▲

Civil Application, continued

Exercise 12-5

Using the method demonstrated on the previous page, calculate the internal angles B and C.

Angle B =

Angle C =

Note: As a check on your calculations, verify that the three internal angles add up to 180°.

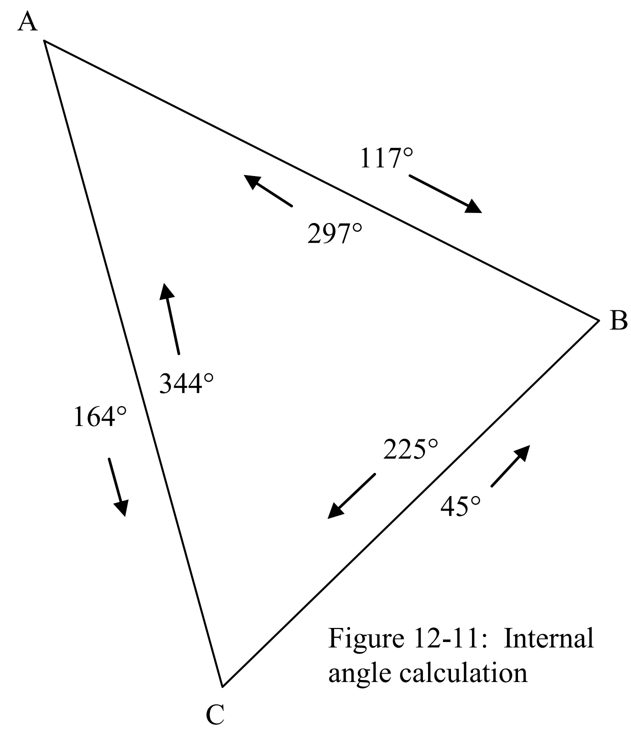


Figure 12-11: Internal angle calculation

Exercise 12-6

Now that we know the values for the internal angles, calculate the length of side AC, using the Law of Sines. Estimate the length of side AC first, so that you will be able to recognize whether your calculated value is reasonable.

Architectural Application

Figure 12-12 shows an example of a roof truss. Recall from Chapter 10 that a roof truss is a *structure* made up of straight boards connected at joints. This dual-pitch truss has a different pitch on each side of the roof. Figure 12-12 will be used for Exercises 12-7 through 12-10.

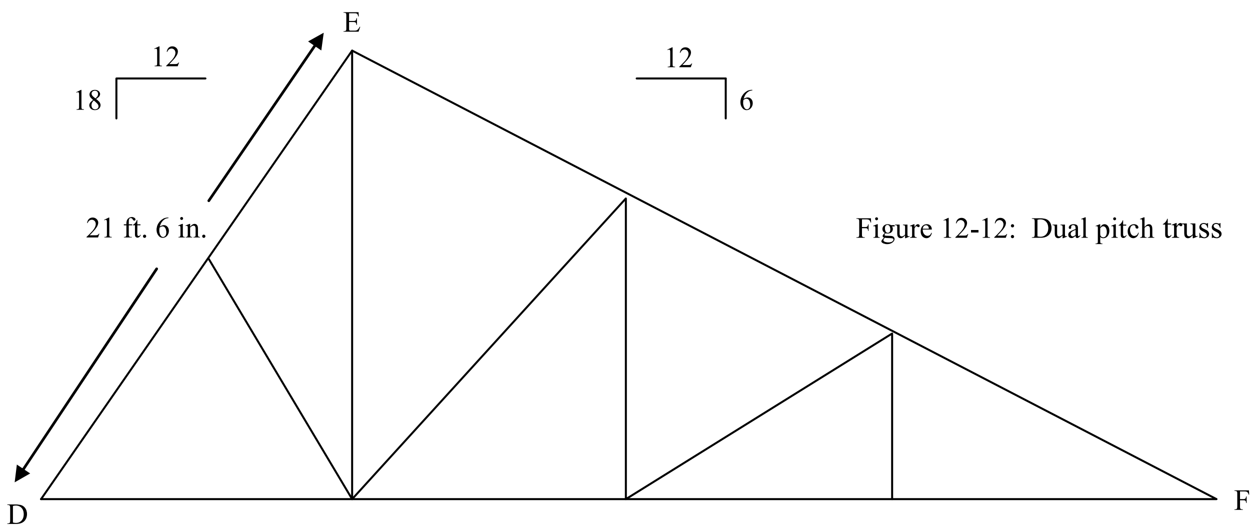


Figure 12-12: Dual pitch truss

Exercise 12-7

Using the given roof pitches, calculate the values for angles D and F. You may express your angles in decimal degrees to two decimal places.

Exercise 12-8

Using the values for angles D and F, calculate the value of angle E.

Architectural Application, continued

Exercise 12-9

Using the Law of Sines, calculate the length of web EF. The length of web DE is 21 ft. 6 in. Express your answer to the nearest 1/4 inch.

Exercise 12-10

Using the Law of Sines, calculate the length of web DF. Express your answer to the nearest 1/4 inch.

Chapter 13: Law of Cosines

The Law of Cosines is another tool that you can use to help solve for unknowns in an oblique triangle.

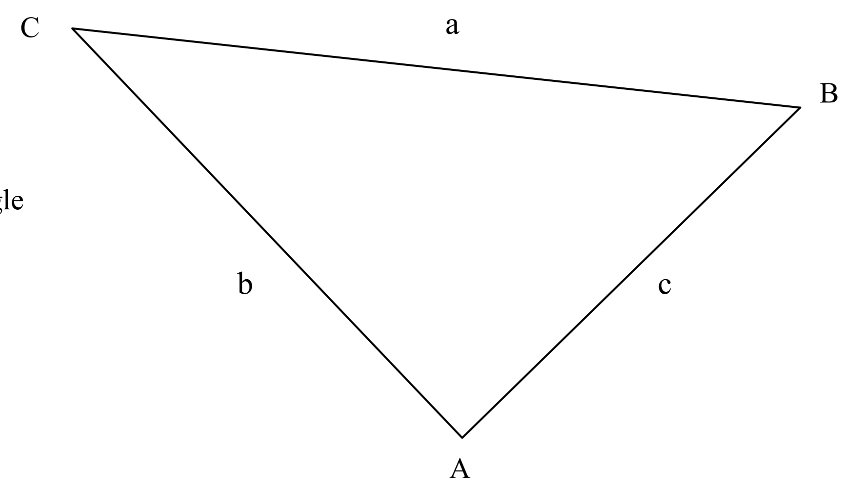


Figure 13-1: Acute triangle

The Law of Cosines is useful if you know the length of two sides and the included angle between those two sides. Refer to Figure 13-1 to understand the formula for this law.

Law of Cosines Formula

$$a^2 = b^2 + c^2 - 2bc\,(\cos A)$$

$$a = \sqrt{b^2 + c^2 - 2bc\,(\cos A)}$$

Law of Cosines Formula, continued

There are two variations of this formula, and they follow the same format:

$$b^2 = a^2 + c^2 - 2ac\ (\cos B)$$

$$b = \sqrt{a^2 + c^2 - 2ac\ (\cos B)}$$

$$c^2 = a^2 + b^2 - 2ab\ (\cos C)$$

$$c = \sqrt{a^2 + b^2 - 2ab\ (\cos C)}$$

Special Case of the Law of Cosines

Let's apply the Law of Cosines to a right triangle to see what happens. Refer to Figure 13-2 below.

We will apply the Law of Cosines to calculate the length of side c.

$$c^2 = a^2 + b^2 - 2ab\ (\cos C)$$

Angle C is a 90° angle. Use your calculator to determine that the value of the cosine of 90° is zero.

This simplifies the equation to the form:

$$c^2 = a^2 + b^2$$

This equation is the Pythagorean Theorem.

Therefore, we can consider the Pythagorean Theorem to be a special case of the Law of Cosines.

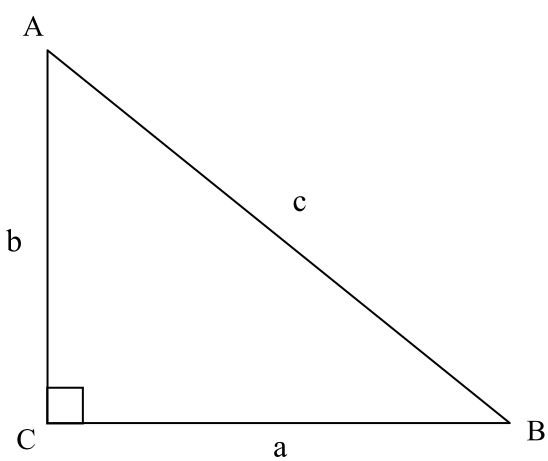


Figure 13-2: Right triangle

Example Calculate the length of side BC in Figure 13-3.

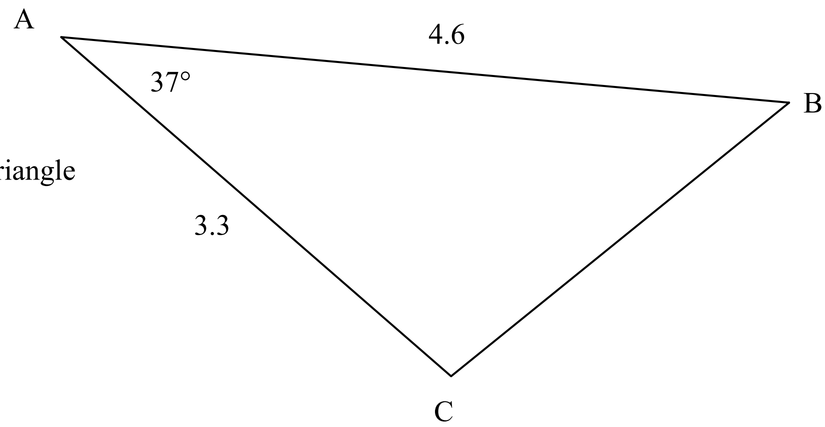


Figure 13-3: Obtuse triangle

$a^2 = b^2 + c^2 - 2bc (\cos A)$

$BC = \sqrt{(3.3)^2 + (4.6)^2 - 2 (3.3) (4.6) (\cos 37°)}$

$BC = 2.8$

▲

Example Calculate the length of side RT in Figure 13-4.

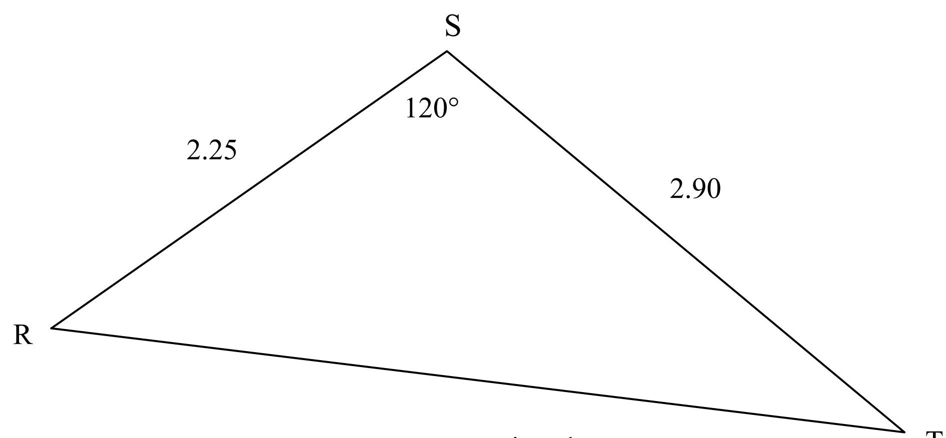


Figure 13-4: Obtuse triangle

$a^2 = b^2 + c^2 - 2bc (\cos A)$

$(RT)^2 = (RS)^2 + (ST)^2 - 2(RS) (ST) (\cos 120°)$

$RT = \sqrt{(2.25)^2 + (2.90)^2 - 2 (2.25) (2.90) (\cos 120°)}$

$RT = 4.47$

▲

Exercise 13-1

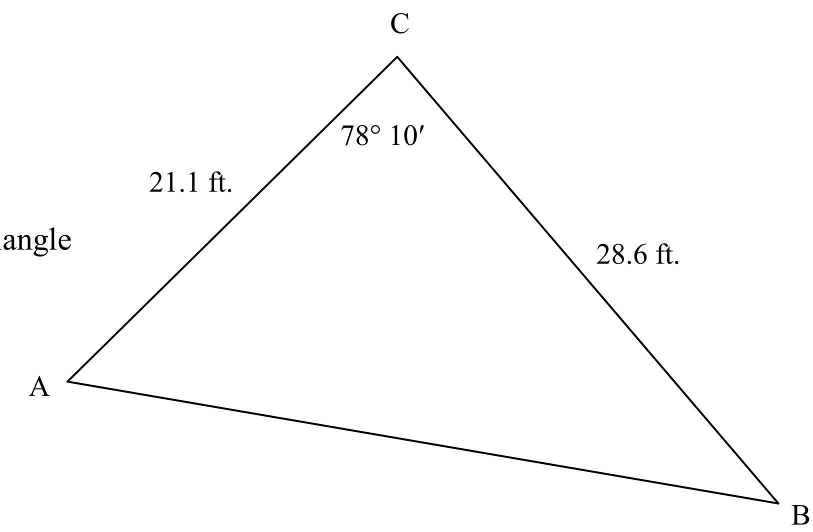


Figure 13-5: Acute triangle

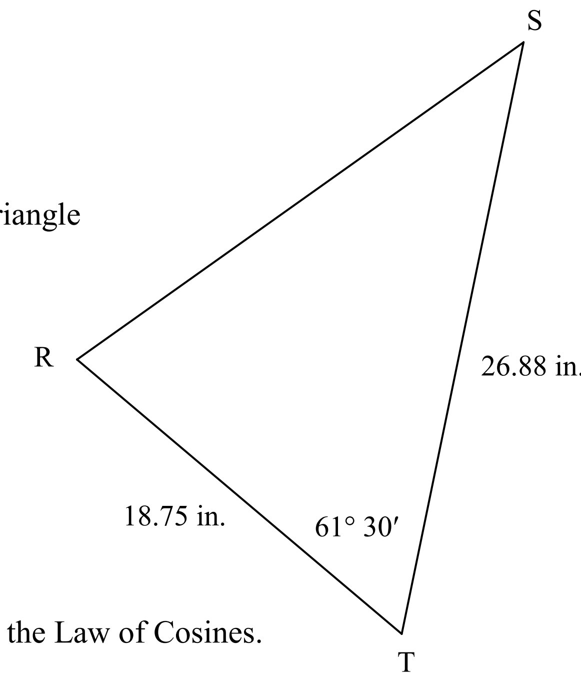
a. Calculate the length of side AB using the Law of Cosines.

b. Calculate the value of angle A using the Law of Sines.

c. Calculate the value of angle B.

Exercise 13-2

Figure 13-6: Acute triangle



a. Calculate the length of side RS using the Law of Cosines.

b. Calculate the value of angle R, using the Law of Sines.

c. Calculate the value of angle S.

Exercise 13-3

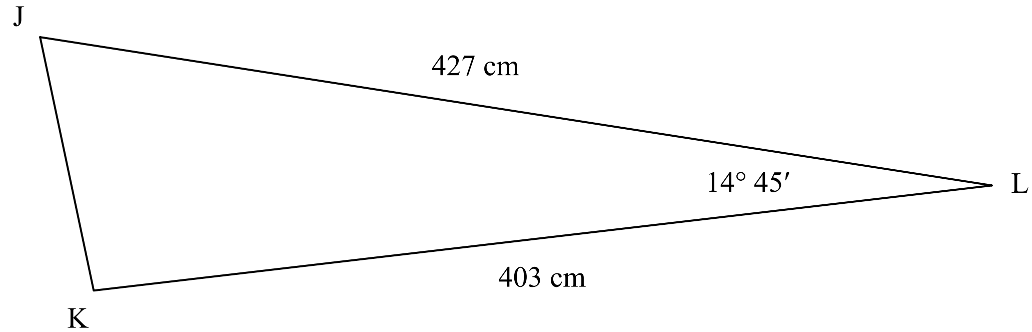


Figure 13-7: Acute triangle

a. Calculate the length of side JK in Figure 13-7 using the Law of Cosines.

b. Calculate the value of angle K, using the Law of Sines.

c. Calculate the value of angle J.

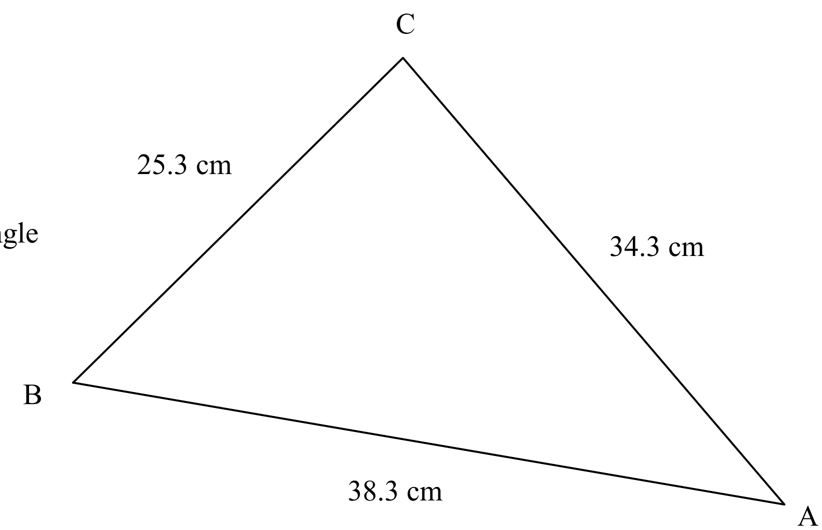
Calculate an Angle with the Law of Cosines

You can also use the Law of Cosines to calculate an angle. You must know the lengths of all three sides.

Example

Calculate angle A in the triangle shown in Figure 13-8.

Figure 13-8: Acute triangle



$a^2 = b^2 + c^2 - 2bc\,(\cos A)$

We will let a = 25.3, b = 34.3, and c = 38.3

First, solve the basic equation for angle A.

Step 1: $-2bc\,(\cos A) = a^2 - b^2 - c^2$

Step 2: $\cos A = \left(\dfrac{a^2 - b^2 - c^2}{-2bc}\right)$

Step 3: $\text{Angle } A = \cos^{-1}\left(\dfrac{a^2 - b^2 - c^2}{-2bc}\right)$

Now substitute the known information and solve for angle A.

$$\text{Angle } A = \cos^{-1}\left(\frac{(25.3)^2 - (34.3)^2 - (38.3)^2}{-2(34.3)(38.3)}\right) = \cos^{-1}\ (0.762466792) = 40.3°$$

$$\text{Angle } A = 40.3°$$

▲

Calculate an Angle with the Law of Cosines, continued

Exercise 13-4

Using the Law of Cosines, calculate angle B in the triangle shown in Figure 13-8.

Practice Worksheet for Chapters 11–13

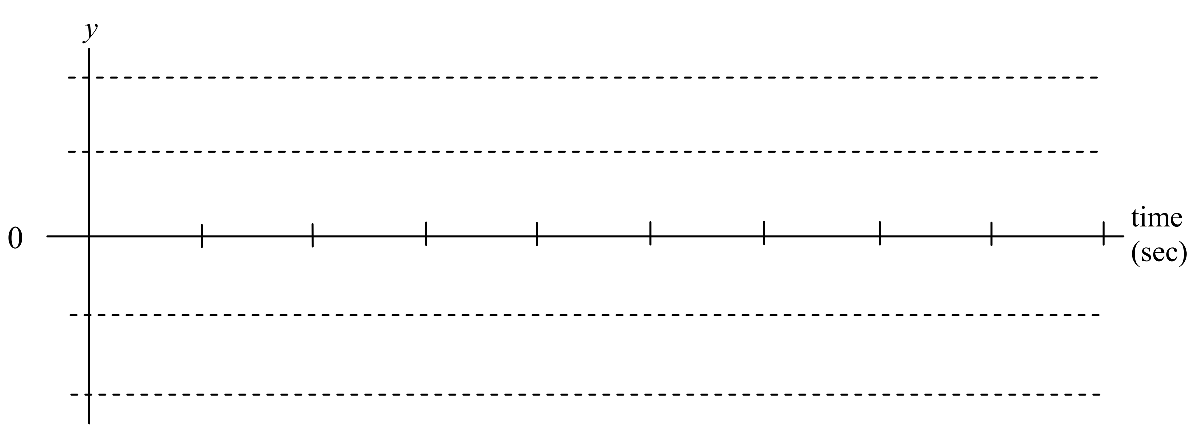
1. Create an equation for a mechanical wave, given the following information:

   - The amplitude is 15 cm.
   - The wave period, T is 12 seconds.
   - The positive peak of the wave is *advanced by one-fourth of a wave* (phase shift).
   - Use degrees for the angular units.

2. Using your equation from problem #1, calculate some *y*-value points for this wave at the following time intervals:

time, t (seconds)	*y*
0	
1.5	
3	
4.5	
6	
7.5	
9	
10.5	
12	

3. Sketch this wave using your *y* values from problem #2. Be sure to label the *y* and *x* axes.



Practice Worksheet for Chapters 11–13, continued

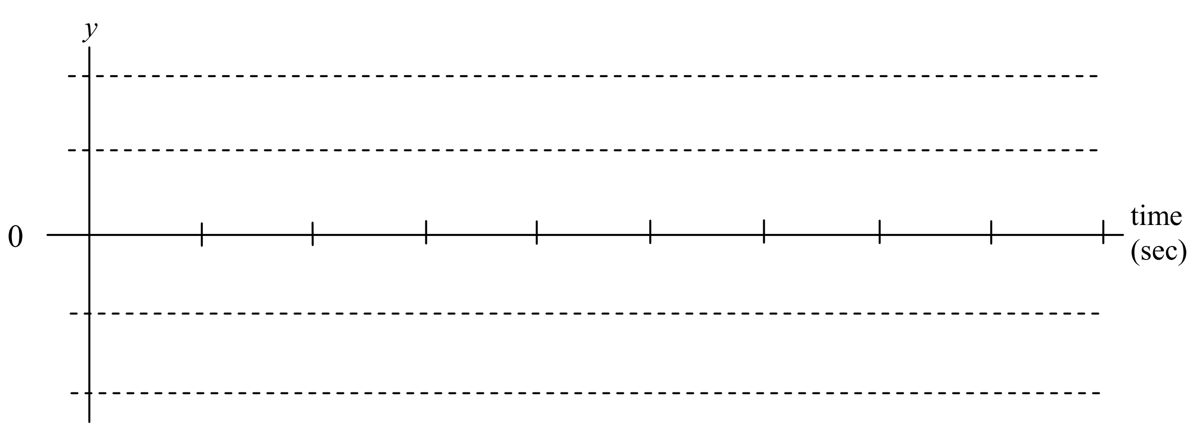
4. Create an equation for a sound wave, given the following information. Before creating the equations, you will need to calculate the wave period T.

   - The amplitude is 1.7 Pascals (Pa).
   - The wave frequency is 500 Hertz.
   - This wave does not have a phase shift.
   - Use radians for the angular units.

5. Using your equation from problem #4, calculate some y-value points for this wave:

time, t (seconds)	Y (Pascals)
0	
.00025	
.0005	
.00075	
.001	
.00125	
.0015	
.00175	
.002	

6. Sketch this wave using your *y* values from problem #5. Be sure to label the *y* and *x* axes.



Practice Worksheet for Chapters 11–13, continued

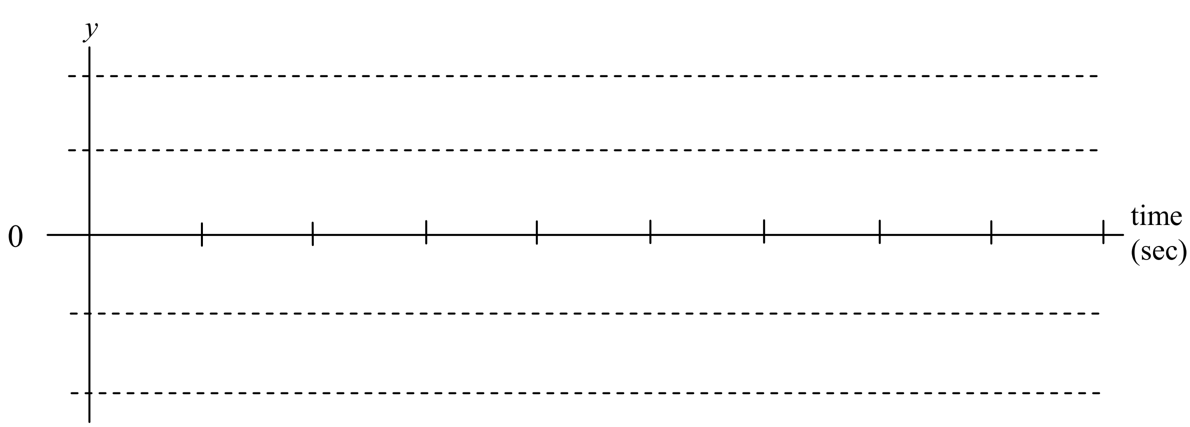
7. Create an equation for alternating electricity, given the following information. Before creating the equation, you will need to calculate the wave period T using the given frequency.

   - The amplitude is 240 volts.
   - The frequency is 50 Hertz.
   - The positive peak of the wave is *delayed by one-eighth of a full wave* (45°).
   - Use degrees for the angular units.

8. Using your equation from problem #7, calculate some *y*-value points for this wave:

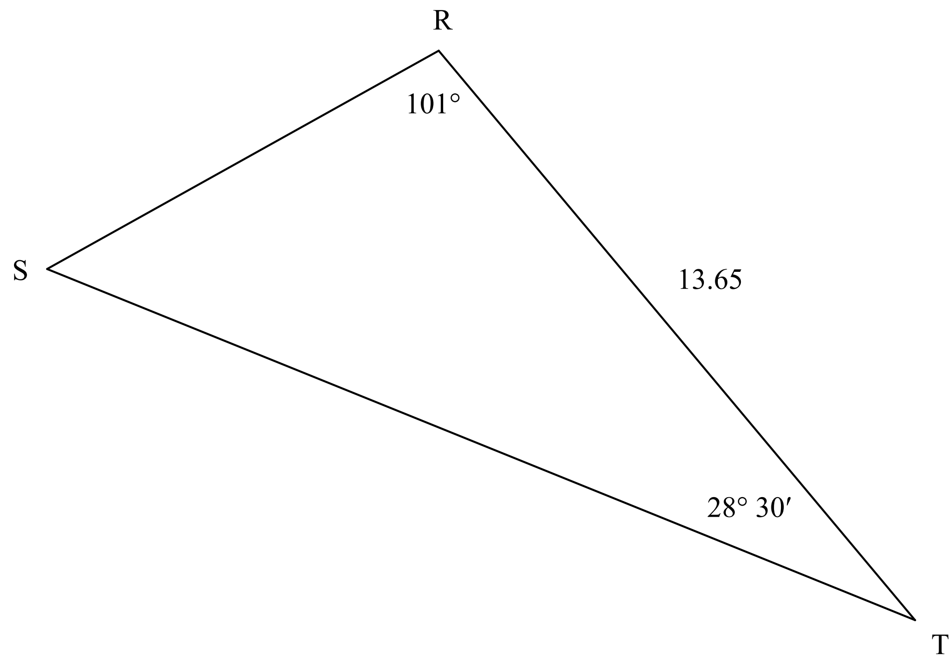
time, t (seconds)	y (volts)

9. Sketch this wave using your *y* values from problem #8. Be sure to label the *y* and *x* axes.

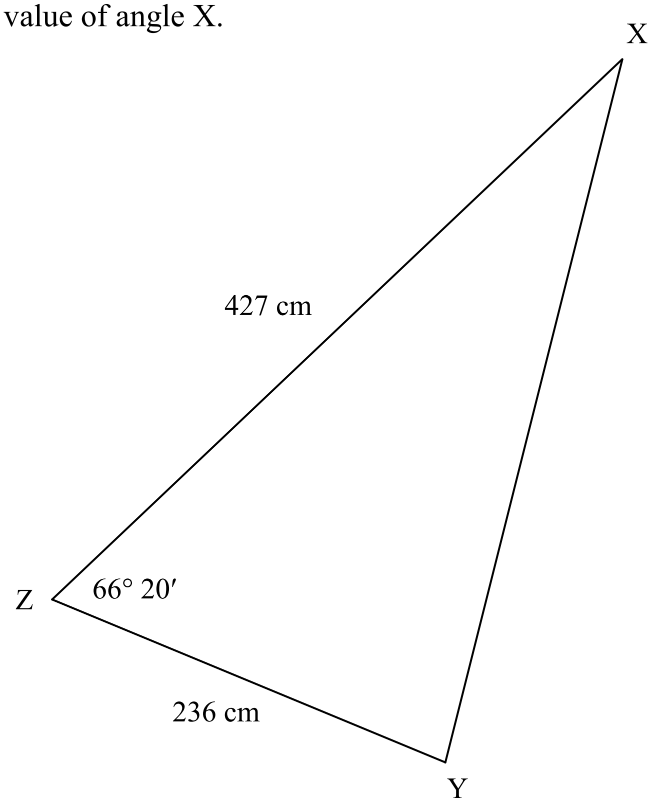


Practice Worksheet for Chapters 11–13, continued

10. Calculate angle S and side ST.



11. Calculate the length of side XY and the value of angle X.



Appendix: Answers to Exercises

Exercise 2-1

1. 240°
2. 150°
3. 105°
4. 120°
5. 90°
6. 45°

Exercise 2-2

1. Vertices are named D, E, and F. The three sides are DE (or ED), DF (or FD), and EF (or FE).
2. $\angle D$, $\angle DEF$, $\angle FED$
3. $\angle F$, $\angle EFD$, $\angle DFE$

Exercise 2-3

1.	$\theta = 118°$, $\alpha = 31°$, $\beta = 31°$
2.	$\theta = 90°$, $\varphi = 64°$, $\beta = 26°$
3.	$\theta = 75°$, $\alpha = 75°$, $\beta = 30°$

Exercise 2-4

1. 41° 34′ 45″
2. 176° 45′ 09″
3. 149° 14′ 42″
4. 75° 03′ 15″
5. 389° 59′ 04″ or simplified to 29° 59′ 04″ by subtracting one full revolution of 360°
6. 1° 0′ 14″
7. 130° 41′ 58″
8. 192° 28′ 03″

Exercise 2-5

1. 93° 24′ 17″
2. 12° 30′ 57″
3. 90° 31′ 17″
4. 23° 50′ 39″
5. 31° 59′ 38″
6. 86° 21′ 47″

Exercise 3-1

1. Find angle θ	$\theta = 180° - 90° - 63° = 27°$
2. Find angle φ	$\varphi = 180° - 39° - 18° = 123°$
3. Find angle β	$\beta = 180° - 76° - 73° = 31°$
4. Find angle θ	All three angles are equal. $\theta = 180° \div 3 = 60°$
5. Find angle φ	$\varphi = 180° - 90° - 21° = 69°$

Exercise 3-2

1. Find angle β	$\beta = 180° - 109° - 49° = 22°$
2. Find angle θ	$\theta = 180° - 82° - 29° = 69°$
3. Find angle φ	$\varphi = (180° - 106°) \div 2 = 37°$ In an isosceles triangle, the two base angles are equal.
4. Find angle β	$\beta = 180° - 24° - 66° = 90°$
5. Find angle θ	$\theta = 180° - 2\,(77°) = 26°$ In an isosceles triangle, the two base angles are equal.

Exercise 3-3

1. Solve for side c	$c = \sqrt{a^2 + b^2}$ $\quad c = \sqrt{(46.5)^2 + (65.0)^2} = 79.9$
2. Solve for side b	$b = \sqrt{c^2 - a^2}$ $\quad b = \sqrt{(8.7)^2 - (7.6)^2} = 4.2$
3. Solve for side a	$a = \sqrt{c^2 - b^2}$ $\quad a = \sqrt{(12.3)^2 - (6.9)^2} = 10.2$
4. Solve for side c	$c = \sqrt{a^2 + b^2}$ $\quad c = \sqrt{(2.12)^2 + (4.56)^2} = 5.03$

continued on next page

Exercise 3-3, continued

5. Solve for side b	$b = \sqrt{c^2 - a^2}$ $b = \sqrt{(10.49)^2 - (7.42)^2} = 7.42$ With a 45° angle, the two legs are equal.
6. Solve for side a	$a = \sqrt{c^2 - b^2}$ $a = \sqrt{(25.00)^2 - (21.65)^2} = 12.50$ The shorter leg is one-half the length of the longest side.

Exercise 3-4

1. Solve for side c	$c = \sqrt{a^2 + b^2}$ $c = \sqrt{(3.5)^2 + (6.5)^2} = 7.4$
2. Solve for side b	$b = \sqrt{c^2 - a^2}$ $b = \sqrt{(0.92)^2 - (0.81)^2} = 0.44$
3. Solve for side a	$a = \sqrt{c^2 - b^2}$ $a = \sqrt{(124.2)^2 - (64.7)^2} = 106.0$
4. Solve for side c	$c = \sqrt{a^2 + b^2}$ $c = \sqrt{(4.472)^2 + (1.744)^2} = 4.800$
5. Solve for side b	$b = \sqrt{c^2 - a^2}$ $b = \sqrt{(1.77)^2 - (1.25)^2} = 1.25$ With a 45° angle, the two legs are equal.
6. Solve for side c	$c = \sqrt{a^2 + b^2}$ $c = \sqrt{(3.50)^2 + (6.06)^2} = 7.00$ The shorter leg is one-half the length of the longest side.

Exercise 4-1

- Side MO corresponds with side PR. Both sides are opposite the 31° angle.
- Side QR corresponds with side MN. Both sides are opposite the 103° angle.

Exercise 4-2

- Side RS corresponds with side XY. Both sides are opposite the 43° angle.
- Side XZ corresponds with side ST. Both sides are opposite the 81° angle.

Exercise 4-3

- The two triangles are similar because they have the same shape. Angle K = 58° and angle T = 32°.
- Side UV corresponds with side JK. Both sides are opposite the 32° angle.
- Side JL corresponds with side TU. Both sides are opposite the 58° angle.

Exercise 4-4

- Yes, the two triangles are similar because they have the same shape. Angle F = 42° and angle H = 77°.
- Side DE corresponds with side GH. Both sides are opposite the 42° angle.
- Side GI corresponds with side DF. Both sides are opposite the 77° angle.

Exercise 4-5

- See Figure 4-7. Are these two triangles similar? Explain your answer.

  No, the two triangles are not similar because they do not have the same shape. Triangle ABC has internal angles of 90°, 63°, and 27°. Triangle MNO has internal angles of 90°, 59°, and 41°.

- Which side in triangle MNO corresponds with side AB in triangle ABC?

  Because the two triangles are not similar, the sides are not identified as corresponding.

Exercise 4-6

The two triangles are similar if the corresponding sides are proportional.

If $\frac{UW}{VW} = \frac{XY}{YZ}$ then the triangles are similar.

$$\frac{UW}{VW} = \frac{0.750}{0.860} = 0.87209$$

$$\frac{XY}{YZ} = \frac{1.125}{1.290} = 0.87209$$

The corresponding sides are proportional and, therefore, the triangles are similar.

Exercise 4-7

If $\dfrac{LM}{MN} = \dfrac{XZ}{YZ}$ then the triangles are similar.

$$\frac{LM}{MN} = \frac{17.5}{12.0} = 1.4583$$

$$\frac{XZ}{YZ} = \frac{21.8}{14.5} = 1.5034$$

The corresponding sides are not proportional and, therefore, the triangles are not similar.

Exercise 4-8

$LN = \sqrt{(17.5)^2 + (12.0)^2} = 21.2$ in. and $XY = \sqrt{(21.8)^2 + (14.5)^2} = 26.2$ in.

If $\dfrac{LN}{MN} = \dfrac{XY}{YZ}$ then the triangles are similar

$$\frac{LN}{MN} = \frac{21.2 \text{ in.}}{12.0 \text{ in.}} = 1.77 \qquad \frac{XY}{YZ} = \frac{26.2}{14.5} = 1.81$$

The corresponding sides are not proportional and, therefore, the triangles are not similar.

Exercise 4-9

$HI = \ (7.500)^2 - (3.750)^2 = 6.495$ cm and $JK = \ (10.500)^2 - (9.093)^2 = 5.250$ cm.

If $\dfrac{GI}{GH} = \dfrac{JL}{JK}$ then the triangles are similar.

$$\frac{GI}{GH} = \frac{7.500}{3.750} = 2.0 \qquad \frac{JL}{JK} = \frac{10.500}{5.250} = 2.0$$

Therefore, the triangles are similar. There are other solutions that would confirm this answer.

Exercise 4-10

In triangle RST,		In triangle XYZ,	
1. The side opposite angle θ is side	RS	5. The side opposite angle θ is side	XZ
2. The side adjacent to angle β is side	RS	6. The side opposite angle β is side	YZ
3. The side adjacent to angle θ is side	ST	7. The side adjacent to angle θ is side	YZ
4. The side opposite angle β is side	ST	8. The side adjacent to angle β is side	XZ

Exercise 4-11

side	angle
1. Side LM is opposite which angle?	β
2. Side LM is adjacent to which angle?	θ
3. Side MO is opposite which angle?	θ
4. Side MO is adjacent to which angle?	β

Chapters 2–4, Practice Worksheet

1. a. $\theta = 31°$ b. $\alpha = 90°$ c. $\beta = 59°$

2. a. $\theta = 31°$ b. $\varphi = 118°$ c. $\beta = 31°$

3. a. $\theta = 26°$ b. $\alpha = 77°$ c. $\beta = 77°$

4. Angle $\varphi = 117°$
5. Angle $\beta = 62°$
6. Angle $\theta = 26°$

7. side x = 8.65

8. side s = 4.3

9. side m = 80.3

10. Is this triangle a right triangle? Justify your answer mathematically.

    If this is a right triangle, the hypotenuse should have a length of 0.821 (using Pythagorean Theorem). Therefore, the triangle is not a right triangle.

11.

If $\dfrac{LN}{MN} = \dfrac{XZ}{YZ}$ then, the triangles are similar.

$$\frac{LN}{MN} = \frac{15.0}{18.0} = 0.833 \qquad \frac{XZ}{YZ} = \frac{20.0}{24.0} = 0.833$$

The corresponding sides are proportional and, therefore, the triangles are similar.

Chapters 2–4, Practice Worksheet, continued

12.

If $\frac{AB}{AC} = \frac{EF}{DE}$ then the triangles are similar.

$\frac{AB}{AC} = \frac{24.6}{14.2} = 1.732 \qquad \frac{EF}{DE} = \frac{17.3}{8.8} = 1.966$

The corresponding sides are not proportional and, therefore, the triangles are not similar.

13.

If $\frac{QS}{RS} = \frac{CD}{DE}$ then the triangles are similar

Using Pythagorean's Theorem, the length of side CD = 57.8 cm.

$\frac{QS}{RS} = \frac{35.9}{15.3} = 2.346 \qquad \frac{CD}{DE} = \frac{57.8}{24.5} = 2.359$

The corresponding sides are not proportional and, therefore, the triangles are not similar.

14.

If $\frac{QP}{QR} = \frac{UT}{ST}$ then, the triangles are similar

$\frac{QP}{QR} = \frac{0.746}{1.600} = 0.466 \qquad \frac{UT}{ST} = \frac{1.493}{3.200} = 0.466$

The corresponding sides are proportional and, therefore, the triangles are similar.

15.

In triangle ABC,		In triangle QRS,	
1. The side adjacent to angle θ is side	AB	5. The side adjacent to angle θ is side	RS
2. The side adjacent to angle β is side	AC	6. The side adjacent to angle β is side	QR
3. The side opposite angle θ is side	AC	7. The side opposite angle θ is side	QR
4. The side opposite angle β is side	AB	8. The side opposite angle β is side	RS

16. 14°55′11″ = 14.91972222°

17. 131.18° = 131°10′48″

Chapters 2–4, Practice Worksheet, continued

18.

$$\begin{array}{rrrr} & 36^\circ & 40' & 37'' \\ + & 115^\circ & 14' & 25'' \\ \hline & 151 & 55' & 02'' \end{array}$$

19.

$$\begin{array}{rrrr} & 107^\circ & 15' & 31'' \\ & 67^\circ & 39' & 07'' \\ \hline & 39^\circ & 36' & 24'' \end{array}$$

Exercise 5-1

	Function			Name of sides		Length		Numerical answer
1.	$\sin\beta$	=	$\dfrac{\text{side opposite angle } \beta}{\text{hypotenuse}}$ =	$\dfrac{BC}{AC}$	=	$\dfrac{4.0\text{ ft.}}{5.0\text{ ft.}}$	=	0.8
2.	$\cos\beta$	=	$\dfrac{\text{side adjacent to angle } \beta}{\text{hypotenuse}}$ =	$\dfrac{AB}{AC}$	=	$\dfrac{3.0\text{ ft.}}{5.0\text{ ft.}}$	=	0.6
3.	$\sin\theta$	=	$\dfrac{\text{side opposite angle } \theta}{\text{hypotenuse}}$ =	$\dfrac{AB}{AC}$	=	$\dfrac{3.0\text{ ft.}}{5.0\text{ ft.}}$	=	0.6
4.	$\cos\theta$	=	$\dfrac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$ =	$\dfrac{BC}{AC}$	=	$\dfrac{4.0\text{ ft.}}{5.0\text{ ft.}}$	=	0.8

Exercise 5-2

$$EF = \sqrt{(24.3)^2 - (9.9)^2} = 22.2 \text{ cm}$$

	Function	Definition		Names of sides		Lengths		Numerical answer
1.	$\cos\beta$	= $\dfrac{\text{side adjacent to angle } \beta}{\text{hypotenuse}}$	=	$\dfrac{EF}{DF}$	=	$\dfrac{22.2\text{ cm}}{24.3\text{ cm}}$	=	0.91
2.	$\sin\theta$	= $\dfrac{\text{side opposite angle } \theta}{\text{hypotenuse}}$	=	$\dfrac{EF}{DF}$	=	$\dfrac{22.2\text{ cm}}{24.3\text{ cm}}$	=	0.91

Exercise 5-3

$$PR = 8.60 \text{ in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} = 21.84 \text{ cm}$$

$$PQ = \sqrt{(15.90)^2 + (21.84)^2} = 27.01 \text{ cm}$$

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	$\sin \beta$	=	$\frac{\text{side opposite angle } \beta}{\text{hypotenuse}}$	=	$\frac{QR}{PQ}$	=	$\frac{15.90 \text{ cm}}{27.01 \text{ cm}}$	=	0.59
2.	$\cos \theta$	=	$\frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$	=	$\frac{QR}{PQ}$	=	$\frac{15.90 \text{ cm}}{27.01 \text{ cm}}$	=	0.59

Exercise 5-4

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	$\tan \theta$	=	$\frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$	=	$\frac{RS}{ST}$	=	$\frac{36 \text{ in.}}{15 \text{ in.}}$	=	2.4
2.	$\tan \beta$	=	$\frac{\text{side opposite angle } \beta}{\text{side adjacent to angle } \beta}$	=	$\frac{ST}{RS}$	=	$\frac{15 \text{ in.}}{36 \text{ in.}}$	=	0.42

Note: What do you observe about these two tangent functions? How are they different? These two tangent functions are the reciprocal of each other.

Exercise 5-5

Determine the value of the tangent functions in Figure 5-9 for both angle θ and angle β.

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	$\tan \theta$	=	$\frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$	=	$\frac{UV}{TU}$	=	$\frac{9.1 \text{ cm}}{12.5 \text{ cm}}$	=	0.73
2.	$\tan \beta$	=	$\frac{\text{side opposite angle } \beta}{\text{side adjacent to angle } \beta}$	=	$\frac{TU}{UV}$	=	$\frac{12.5 \text{ cm}}{9.1 \text{ cm}}$	=	1.37

Exercise 5-6

$$\mathrm{RS} = \sqrt{(25.00)^2 \quad (8.60)^2} = 23.47 \text{ in.}$$

	Function		Definition		Names of sides		Lengths		Numerical answer
1.	$\tan\beta$	=	$\dfrac{\text{side opposite angle } \beta}{\text{side adjacent to angle } \beta}$	=	$\dfrac{\mathrm{RS}}{\mathrm{ST}}$	=	$\dfrac{23.47 \text{ in.}}{8.60 \text{ in.}}$	=	2.73
2.	$\tan\theta$	=	$\dfrac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$	=	$\dfrac{\mathrm{ST}}{\mathrm{RS}}$	=	$\dfrac{8.60 \text{ in.}}{23.47 \text{ in.}}$	=	0.37

Exercise 5-7

$$\mathrm{DB} = \sqrt{(49.37)^2 - (42.32)^2} = 25.42 \text{ ft.}$$

1. $\sin\beta = \dfrac{\mathrm{BC}}{\mathrm{DC}} = \dfrac{42.32 \text{ ft.}}{49.37 \text{ ft.}} = 0.86$

2. $\cos\beta = \dfrac{\mathrm{DB}}{\mathrm{DC}} = \dfrac{25.42 \text{ ft.}}{49.37 \text{ ft.}} = 0.51$

3. $\tan\beta = \dfrac{\mathrm{BC}}{\mathrm{DB}} = \dfrac{42.32 \text{ ft.}}{25.42 \text{ ft.}} = 1.66$

4. $\sin\theta = \dfrac{\mathrm{DB}}{\mathrm{DC}} = \dfrac{25.42 \text{ ft.}}{49.37 \text{ ft.}} = 0.51$

5. $\cos\theta = \dfrac{\mathrm{BC}}{\mathrm{DC}} = \dfrac{42.32 \text{ ft.}}{49.37 \text{ ft.}} = 0.86$

6. $\tan\theta = \dfrac{\mathrm{DB}}{\mathrm{BC}} = \dfrac{25.42 \text{ ft.}}{42.32 \text{ ft.}} = 0.60$

Exercise 5-8

$$\mathrm{XY} = 2.42 \text{ ft.} \times \frac{12 \text{ in.}}{1 \text{ ft.}} = 29.0 \text{ in.}$$

$$\mathrm{XZ} = \sqrt{(87.0)^2 - (29.0)^2} = 82.0 \text{ in.}$$

continued on next page

Exercise 5-8, continued

1. $\tan\theta = \dfrac{XY}{XZ} = \dfrac{29.0\text{ in.}}{82.0\text{ in.}} = 0.35$

2. $\cos\beta = \dfrac{XY}{YZ} = \dfrac{29.0\text{ in.}}{87.0\text{ in.}} = 0.33$

3. $\sin\theta = \dfrac{XY}{YZ} = \dfrac{29.0\text{ in.}}{87.0\text{ in.}} = 0.33$

4. $\tan\beta = \dfrac{XZ}{XY} = \dfrac{82.0\text{ in.}}{29.0\text{ in.}} = 2.83$

5. $\cos\theta = \dfrac{XZ}{YZ} = \dfrac{82.0\text{ in.}}{87.0\text{ in.}} = 0.94$

6. $\sin\beta = \dfrac{XZ}{YZ} = \dfrac{82.0\text{ in.}}{87.0\text{ in.}} = 0.94$

Exercise 6-1

a. $\sin 61° = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{x}{20}$ $\quad x = (20)(\sin 61°)$

b. $\cos 29° = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{a}{12}$ $\quad a = (12)(\cos 29°)$

c. $\tan 29° = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{b}{7}$ $\quad b = (7)(\tan 29°)$

d. $\sin 40° = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{16}{t}$ $\quad t = \dfrac{16}{\sin 40°}$

e. $\tan 50° = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{t}{16}$ $\quad t = (16)(\tan 50°)$

f. $\cos 50° = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{16}{k}$ $\quad k = \dfrac{16}{\cos 50°}$

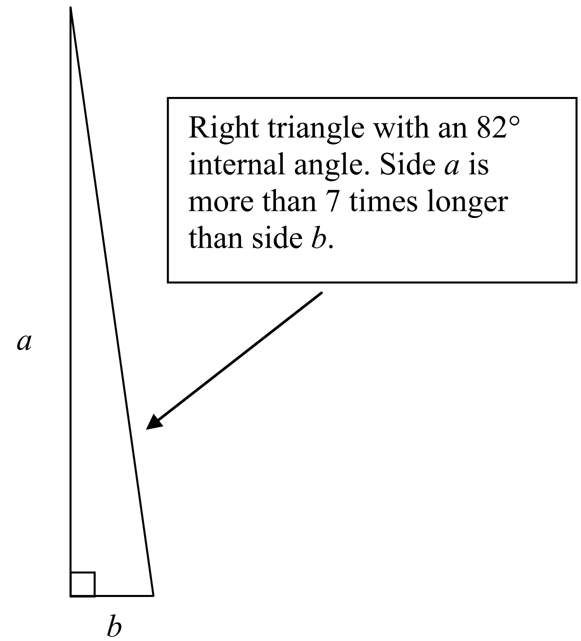
g. $\tan 32° = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{10}{y}$ $\quad y = \dfrac{10}{\tan 32°}$

h. $\cos 32° = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{14}{w}$ $\quad w = \dfrac{14}{\cos 32°}$

i. $\sin 58° = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{17}{x}$ $\quad x = \dfrac{17}{\sin 58°}$

Exercise 6-2

sin 45° =	0.707106781
cos 40° =	0.766044443
tan 53° =	1.327044822
sin 8° =	0.139173101
cos 79° =	0.190808995
tan 82° =	7.115369722
cos 120°	-0.5



Exercise 6-3

Calculate the values of these trigonometry values:

cos 120° =	-0.5
sin 120° =	0.866
tan 120° =	-1.732
cos 235° =	-0.574
sin 235° =	-0.819
tan 235° =	1.428
cos 320° =	0.766
sin 320° =	-0.643
tan 320° =	-0.839

Exercise 6-4

Next, draw these three angles, 120°, 235°, and 320°, on the Cartesian coordinate system. The first angle is done for you. After the line representing the angle is drawn, sketch the right triangle, with the line as the hypotenuse.

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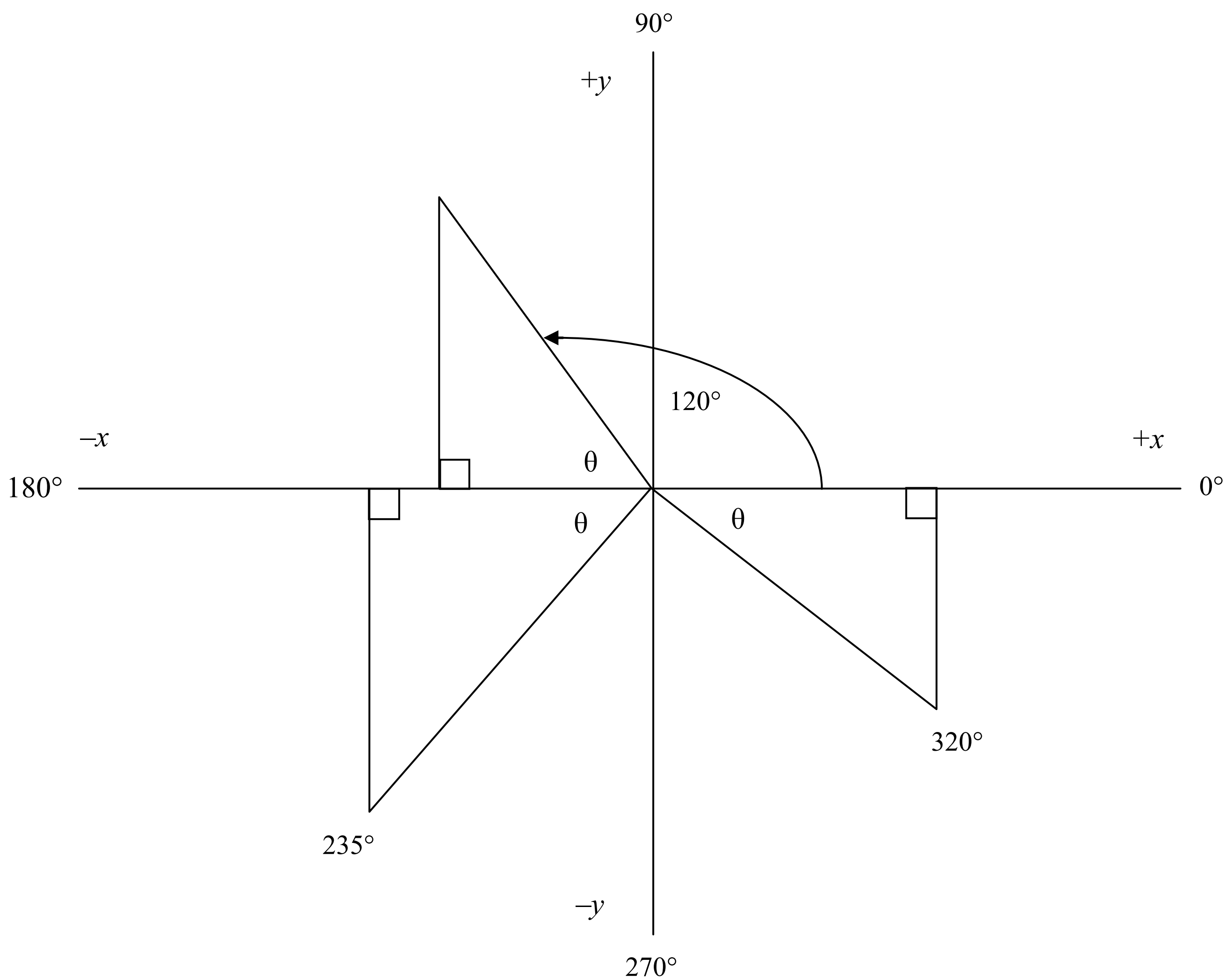
Exercise 6-4, continued



The sine of 235° has a negative value because the opposite side has a negative value (–*y* axis).

The tangent of 320° has a negative value because the opposite side has a negative value (–*y* axis), and the adjacent side has a positive value (+*x* axis).

Exercise 6-5

cos 40°	=	0.766044443	sin 50°	=	0.766044443
sin 31.5°	=	0.522498565	cos 58.5°	=	0.522498565
cos 18°	=	0.951056516	sin 72°	=	0.951056516
sin 8°	=	0.139173101	cos 82°	=	0.139173101
cos 79°	=	0.190808995	sin 11°	=	0.190808995

Exercise 6-6

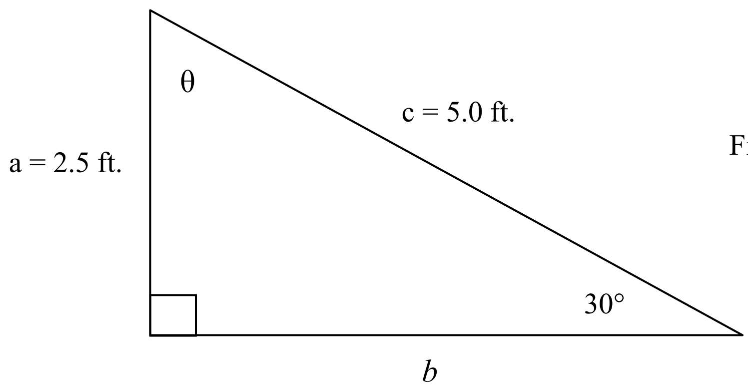


Figure 6-5: Right triangle

- Pythagorean Theorem solution to side *b*.

  $b = \sqrt{c^2 - a^2}$

  $b = \sqrt{(5.0)^2 - (2.5)^2} = 4.3$ ft.

- Using the 30° interior angle and two different trigonometry functions.

  $\cos 30° = \dfrac{b}{5.0} \qquad b = (5.0)(\cos 30°) = 4.3$ ft.

  $\tan 30° = \dfrac{2.5}{b} \qquad b = \dfrac{2.5}{\tan 30°} = 4.3$ ft.

- Angle θ = 180° – 90° – 30° = 60°

- Using angle θ and two different trigonometry functions.

  $\sin 60° = \dfrac{b}{5.0} \qquad b = (5.0)(\sin 60°) = 4.3$ ft.

  $\tan 60° = \dfrac{b}{2.5} \qquad b = (2.5)(\tan 60°) = 4.3$ ft.

Exercise 6-7

Step	Answer
Determine which unknown you need to calculate.	Side *b*
Do you have enough information to solve for this unknown?	Yes, one side and two angles are known, with one of the angles being the right angle.
Next, decide which interior angle you will use as a reference angle.	62° (you could also choose to use the other acute angle, which is 90° – 62° = 28°)
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	Cosine is the right selection for the given angle of 62°. You know the value of the hypotenuse and the unknown is the adjacent side.
Create an equation using this function.	$\cos 62° = \frac{b}{12.2}$
Use algebra to solve for the unknown in the equation.	$b = (12.2)(\cos 62°)$
Use your calculator to solve for the length of side b.	$b = 5.7$ in.

Exercise 6-8

Step	Answer
Determine which unknown you need to calculate.	Side *b*
Do you have enough information to solve for this unknown?	Yes, one side and two angles are known, with one of the angles being the right angle.
Next, decide which interior angle you will use as a reference angle.	20° (you could also choose to use the other acute angle, which is 90° – 20° = 70°)
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	Tangent is the right selection for the given angle of 20°. You know the value of the adjacent side and the unknown is the opposite side.
Create an equation using this function.	$\tan 20° = \frac{b}{7.6}$
Use algebra to solve for the unknown in the equation.	$b = (7.6)(\tan 20°)$
Use your calculator to solve for the length of side b.	$b = 2.8$ cm

Exercise 6-9

Step	Answer
Determine which unknown you need to calculate.	Side *c*
Do you have enough information to solve for this unknown?	Yes, one side and two angles are known, with one of the angles being the right angle.
Next, decide which interior angle you will use as a reference angle.	49° (you could also choose to use the other acute angle, which is 90° – 49° = 41°)
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	Sine is the right selection for the given angle of 49°. You know the value of the opposite side and the unknown is the hypotenuse.
Create an equation using this function.	$\sin 49° = \frac{14.7}{c}$
Use algebra to solve for the unknown in the equation.	$c = \frac{14.7}{\sin 49°}$
Use your calculator to solve for the length of side *c*.	$c = 19.5$ in.

Exercise 6-10

Step	Answer
Determine which unknown you need to calculate.	Side *b*
Do you have enough information to solve for this unknown?	Yes, one side and two angles are known, with one of the angles being the right angle.
Next, decide which interior angle you will use as a reference angle.	67° (you could also choose to use the other acute angle, which is 90° – 67° = 23°)
Decide which trigonometry function to use. Pick the function which relates both the unknown and known sides.	Sine is the right selection for the given angle of 67°. You know the value of the opposite side and the unknown is the hypotenuse.
Create an equation using this function.	$\sin 67° = \frac{15.5}{b}$
Use algebra to solve for the unknown in the equation.	$b = \frac{15.5}{\sin 67°}$
Use your calculator to solve for the length of side *b*.	$b = 16.8$ cm

Exercise 7-1

a. $\cos\theta = \dfrac{16}{19}$ $\qquad \theta = \cos^{-1}\dfrac{16}{19}$

b. $\tan\theta = \dfrac{9}{15}$ $\qquad \theta = \tan^{-1}\dfrac{9}{15}$

c. $\tan\theta = \dfrac{16}{10}$ $\qquad \theta = \tan^{-1}\dfrac{16}{10}$

d. $\tan\theta = \dfrac{4}{9}$ $\qquad \theta = \tan^{-1}\dfrac{4}{9}$

e. $\cos\theta = \dfrac{8}{10}$ $\qquad \theta = \cos^{-1}\dfrac{8}{10}$

f. $\sin\theta = \dfrac{17}{20}$ $\qquad \theta = \sin^{-1}\dfrac{17}{20}$

g. $\sin\theta = \dfrac{7}{8}$ $\qquad \theta = \sin^{-1}\dfrac{7}{8}$

h. $\sin\theta = \dfrac{7}{15}$ $\qquad \theta = \sin^{-1}\dfrac{7}{15}$

i. $\cos\theta = \dfrac{13}{16}$ $\qquad \theta = \cos^{-1}\dfrac{13}{16}$

Exercise 7-2

		Decimal degrees		Degree, minutes, seconds (to the nearest second)
$\sin^{-1} 0.15$	=	8.626926559°	=	8° 37′ 37″
$\cos^{-1} 0.85$	=	31.78833062°	=	31° 47′ 18″
$\tan^{-1} 0.35$	=	19.29004622°	=	19° 17′ 24″
$\sin^{-1} 0.75$	=	48.59037789°	=	48° 35′ 25″
$\cos^{-1} 0.23$	=	76.70292825°	=	76° 42′ 11″
$\tan^{-1} 1.12$	=	48.2397003°	=	48° 14′ 23″
$\sin^{-1} 0.95$	=	71.80512766°	=	71° 48′ 18″
$\cos^{-1} 0.55$	=	56.63298703°	=	56° 37′ 59″
$\tan^{-1} 2.25$	=	66.03751103°	=	66° 02′ 15″

Exercise 7-3

- Solve for side *a* using the Pythagorean Theorem

$a = \sqrt{(2.35)^2 - (1.75)^2} = 1.57$ in.

2. Solve for side *a* using the tangent of angle β (41° 52′ 06″)

$$\tan\beta = \frac{a}{1.75}$$

$$a = (1.75)(\tan 41^\circ\, 52'\, 06'') = 1.57 \text{ in.}$$

3. Solve for angle θ using the inverse sine function.

$$\theta = \sin^{-1}\frac{1.75}{2.35} = 48^\circ\, 07'\, 54''$$

- Solve for angle θ using your knowledge of the sum of interior angles in a triangle.

$\theta = 90^\circ - \beta$

$\theta = 90^\circ - 41^\circ\, 52'\, 06'' = 48^\circ\, 07'\, 54''$

Exercise 7-4

Step	Answer
Determine which unknown you need to calculate.	angle θ
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	yes, the length of two sides are known
Which trigonometry function relates the two known sides (using θ as the reference angle)?	sine
Create an equation for the unknown angle using the inverse function.	$\theta = \sin^{-1}\frac{13.2}{17.5}$
Use your calculator to perform the calculations. Avoid intermediate rounding.	$\theta = 48.96299615^\circ = 48^\circ\, 57'\, 47''$

Exercise 7-5

Step	Answer
Determine which unknown you need to calculate.	angle β
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	yes, the length of two sides are known
Which trigonometry function relates the two known sides (using β as the reference angle)?	tangent
Create an equation for the unknown angle using the inverse function.	$\beta = \tan^{-1} \dfrac{15.5}{7.2}$
Use your calculator to perform the calculations. Avoid intermediate rounding.	$\beta = 65.08437149° = 65° \, 05' \, 04''$

Exercise 7-6

Step	Answer
Determine which unknown you need to calculate.	angle θ
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	yes, the length of two sides are known
Which trigonometry function relates the two known sides (using θ as the reference angle)?	Cosine
Create an equation for the unknown angle using the inverse function.	$\theta = \cos^{-1} \dfrac{5.9}{15.2}$
Use your calculator to perform the calculations. Avoid intermediate rounding.	$\theta = 67.16007331° = 67° \, 09' \, 36''$

Exercise 7-7

Step	Answer
Determine which unknown you need to calculate.	angle β
Do you have enough information to solve for this unknown? You must know two sides that are related to this angle.	yes, the length of two sides are known
Which trigonometry function relates the two known sides (using β as the reference angle)?	Sine
Create an equation for the unknown angle using the inverse function.	$\beta = \sin^{-1} \dfrac{4.495}{7.650}$
Use your calculator to perform the calculations. Avoid intermediate rounding.	$\beta = 35.9855854° = 35° \; 59' \; 08''$

Chapters 6–7 Practice Worksheet

1. Solve for side *c* using two different methods.

   Method 1

   $$\cos 28.5° = \frac{65.2}{c} \qquad c = \frac{65.2}{\cos 28.5°} = 74.2 \text{ in.}$$

   Method 2

   $$\theta = 90° - 28.5° = 61.5°$$

   $$\sin 61.5° = \frac{65.2}{c} \qquad c = \frac{65.2}{\sin 61.5°} = 74.2 \text{ in.}$$

2. Solve for side *b* using two different methods.

   Method 1

   $$\sin 61° = \frac{b}{8.6} \qquad b = (8.6)\,(\sin 61°) = 7.5 \text{ cm}$$

   Method 2

   $$\theta = 90° - 61° = 29°$$

   $$\cos 29° = \frac{b}{8.6} \qquad b = (8.6)\,(\cos 29°) = 7.5 \text{ cm}$$

Chapters 6–7 Practice Worksheet, continued

3. Solve for side *a* using two different methods.

   Method 1

   $$\tan 30.5° = \frac{6.9}{a} \qquad a = \frac{6.9}{\tan 30.5°} = 11.7 \text{ ft.}$$

   Method 2

   $\beta = 90° - 30.5° = 59.5°$

   $$\tan 59.5° = \frac{a}{6.9} \qquad a = (6.9)(\tan 59.5°) = 11.7 \text{ ft.}$$

4. Solve for side *c* using two different methods.

   Method 1

   $$\tan 68° = \frac{c}{2.12} \qquad c = (2.12)(\tan 68°) = 5.25 \text{ in.}$$

   Method 2

   $\varphi = 90° - 68° = 22°$

   $$\tan 22° = \frac{2.12}{c} \qquad c = \frac{2.12}{\tan 22°} = 5.25 \text{ in.}$$

5. Solve for angle θ and side *w*, below.

   Solution for angle θ (DMS format to the nearest second)

   $$\tan\theta = \frac{41.4}{65.0} \qquad \theta = \tan^{-1}\frac{41.4}{65.0} = 32°\,29'\,38''$$

   Solution for side *w* (use two different methods)

   $$w = \sqrt{(41.4)^2 + (65.0)^2} = 77.1 \text{ cm}$$

   or

   $$\cos\theta = \frac{65.0}{w} \qquad w = \frac{65.0}{\cos 32°\,29'\,38''} = 77.1 \text{ cm}$$

Chapters 6–7 Practice Worksheet, continued

6. Solve for angle β and side *d*, below

   Solution for angle β (DMS format to the nearest second)

$$\sin\beta = \frac{6.9}{8.7} \qquad \beta = \sin^{-1}\frac{6.9}{8.7} = 52^\circ\,28'\,35''$$

   Solution for side d (use two different methods)

$$d = (8.7)^2 - (6.9)^2 = 5.3 \text{ in.}$$

   or

$$\tan\beta = \frac{6.9}{d} \qquad d = \frac{6.9}{\tan 52^\circ\,28'\,35''} = 5.3 \text{ in.}$$

7. Solve for angle φ and side *f*, below.

   Solution for angle φ (DMS format to the nearest second)

$$\cos\varphi = \frac{5.1}{12.3} \qquad \varphi = \cos^{-1}\frac{5.1}{12.3} = 65^\circ\,30'\,13''$$

   Solution for side f (use two different methods)

$$f = \sqrt{(12.3)^2 - (5.1)^2} = 11.2 \text{ cm}$$

   or

$$\sin\varphi = \frac{f}{12.3} \qquad f = (12.3)\,(\sin 65^\circ\,30'\,13'') = 11.2 \text{ cm}$$

8. Solve for angle θ and side *x*, below.

   Solution for angle θ (DMS format to the nearest second)

$$\tan\theta = \frac{1.60}{4.65} \qquad \theta = \tan^{-1}\frac{1.60}{4.65} = 18^\circ\,59'\,15''$$

   Solution for side *x* (use two different methods)

$$x = \sqrt{(1.60)^2 + (4.65)^2} = 4.92 \text{ ft.}$$

   or

$$\cos\theta = \frac{4.65}{x} \qquad x = \frac{4.65}{\cos 18^\circ\,59'\,15''} = 4.92 \text{ ft.}$$

Exercise 8-1

Solve for the drill point depth for Hole #2 in Figure 8-1. You can see that the diameter of Hole #2 is .375 and the hole depth is 1.375.

$$\tan 59° = \frac{\text{opposite}}{\text{adjacent}} = \frac{(0.375/2)}{Z}$$

Solve this equation for the unknown side Z. Round your answer to 3 decimal places.

$$Z = \frac{0.1875}{\tan 59°} = 0.113 \text{ in.}$$

Conclusion: In a CNC program, the drill depth will be 1.375 + 0.113 =1.488 in. This depth is necessary to make sure that the full diameter of the drill reaches a depth of 1.375 in., as shown on the drawing.

Exercise 8-2

Calculate the gage block stack necessary to measure the angled surface in the part shown in Fig. 8-7. Express your answer to 4 decimal places.

Assume that the measurement setup will use a 5-inch sine bar (distance between the cylinders).

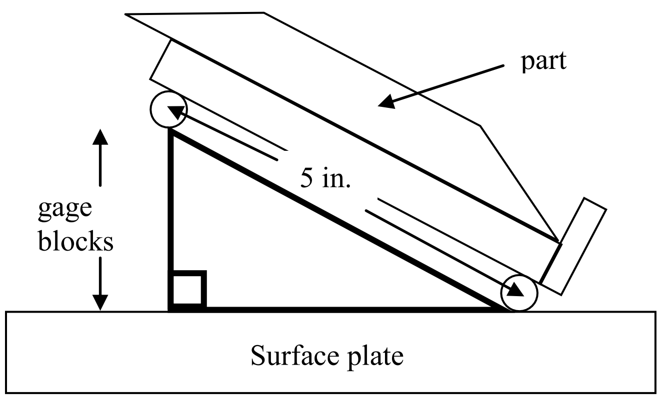


Figure 8-6: Angularity measurement setup

$$\sin 23.5° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{gage blocks}}{5.0 \text{ in.}}$$

Solve this equation for the gage block dimension. Round your answer to 4 decimal places. The gage block set is capable of creating dimensions out to 4 decimal places.

$$\text{gage blocks} = (5.0)(\sin 23.5°) = 1.9937 \text{ in.}$$

Exercise 8-3

Solve for the engraving tool depth, given the following information:

- Tool point angle: 60°
- Tool diameter: 3/16 inch
- Desired text width: 0.125 inch

As part of the solution, sketch and label the tool and the right triangle that you need to solve the problem. Round your answer to 3 decimal places.

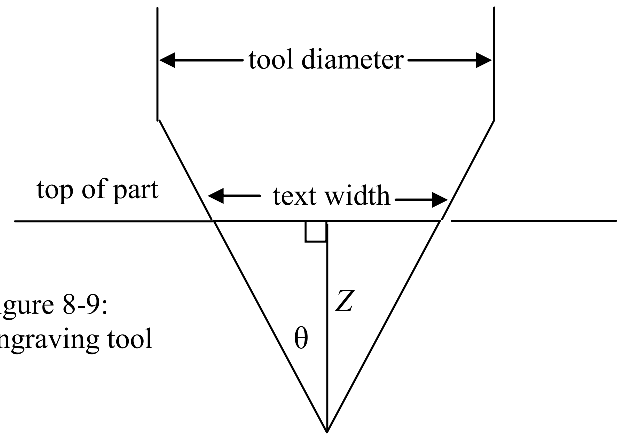


Figure 8-9:
Engraving tool

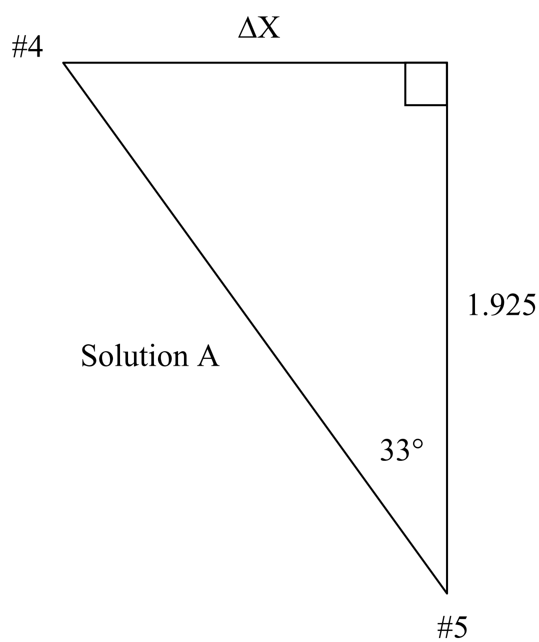
$$\tan (60/2)° = \frac{\text{opposite}}{\text{adjacent}} = \frac{(0.125 /2) \text{ in.}}{Z}$$

$$Z = \frac{(0.125 /2) \text{ in.}}{\tan (60/2)°} = \frac{0.0625 \text{ in.}}{\tan 30°} = 0.108 \text{ in.}$$

This answer means that the tool depth Z is 0.108 in. to make text that is 0.125 in. wide. Notice that the tool diameter was provided, but that information is not required to solve this problem.

Exercise 8-4

The triangle can be sketched in two different ways. Solve for ΔX in Solution A.



$$\tan 33° = \frac{\text{opposite}}{\text{adjacent}} = \frac{\Delta X}{1.925 \text{ in.}}$$

$$\Delta X = (1.925 \text{ in}) (\tan 33°) = 1.250 \text{ in.}$$

From Figure 8-10, you can see that the *X* location of point #5 is 4.500. Therefore, the *X* location of point #4 $= X_4 = X_5 - 1.250 = 4.500 - 1.250 = 3.250$ in.

continued on next page

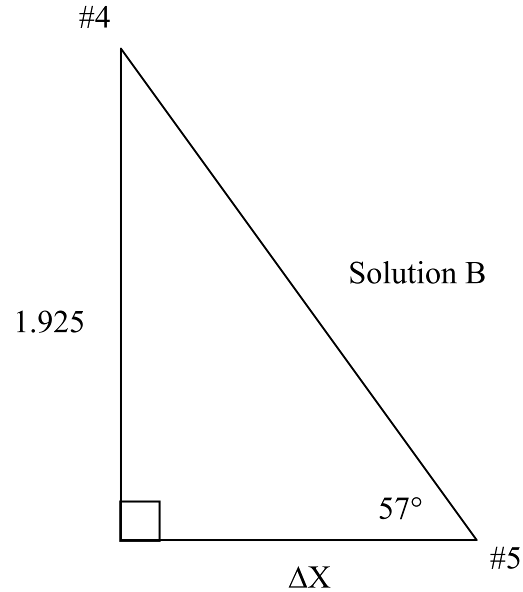
Exercise 8-4, continued

Solve for ΔX in Solution B.

$$\tan 57° = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.925 \text{ in.}}{\Delta X}$$

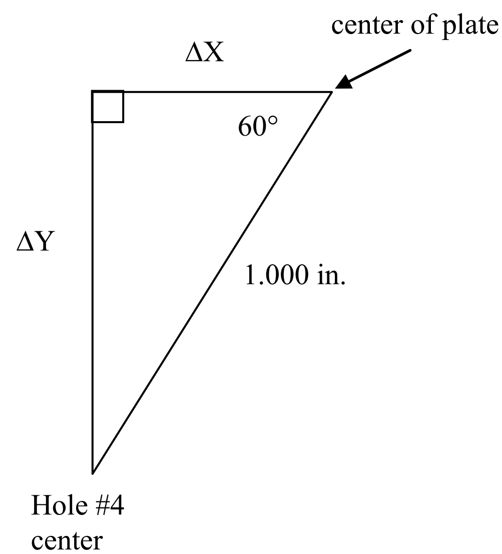
$$\Delta X = \frac{1.925 \text{ in.}}{\tan 57°} = 1.250 \text{ in.}$$

The X location of point 4 = $X_4 = X_5 - 1.250$
= 4.500 – 1.250 = 3.250 in.



Exercise 8-5

Solve for the X location of Hole #4 in Figure 8-13. Include a labeled sketch of the right triangle that you need to help solve the problem.



The hypotenuse of this triangle is the radius of the bolt-hole circle, from the center of the plate to the center of hole #4. The 60° angle represents the angle between the holes, as given in the drawing.

$$\cos 60° = \frac{\text{Adjacent}}{\text{hypotenuse}} = \frac{\Delta X}{1.000}$$

Solve this equation for the unknown side ΔX.

$$\Delta X = (1.000)(\cos 60°) = 0.500 \text{ in.}$$

This calculation locates Hole #4 in the X direction from the center of the circle. Now finish the calculation by determining the X location of Hole #4 from the center of the part which has an X coordinate of 1.500. The center of Hole #4 in the X direction is to the *left* of the center. In other words, it has a less positive, or more negative, value.

X location of hole #4 = X location of the center of the circle – ΔX = 1.500 – 0.500 = 1.000 in.

Exercise 8-6

Solve for the *Y* location of Hole #4 in Figure 8-13. Include a labeled sketch of the right triangle that you need to help solve the problem.

$$\sin 60° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\Delta Y}{1.000}$$

Solve this equation for the unknown side ΔY.

$$\Delta Y = (1.000)(\sin 60°) = 0.866 \text{ in.}$$

This calculation locates Hole #4 in the *Y* direction from the center of the circle. Now finish the calculation by determining the *Y* location of Hole #4 from the center of the part, which has a *Y* coordinate of 1.500. The center of Hole #4 in the *Y* direction is *below* the center. In other words, it has a less positive, or more negative, value.

Y location of hole #4 = *Y* location of the center of the circle – ΔY = 1.500 – 0.866 = 0.634 in.

Exercise 8-7

Complete the solution for the *X* and *Y* locations of all holes in Figure 8-13. Express your answers to three decimals.

Hole	X	Y
1	2.000	2.366
2	2.500	1.500
3	2.000	0.634
4	1.000	0.634
5	0.500	1.500
6	1.000	2.366

Exercise 8-8

Sketch two right triangles on Figure 8-16. These triangles must be helpful in solving for the unknown dimension H. Label the vertices on these triangles so that equations can be created to solve for H.

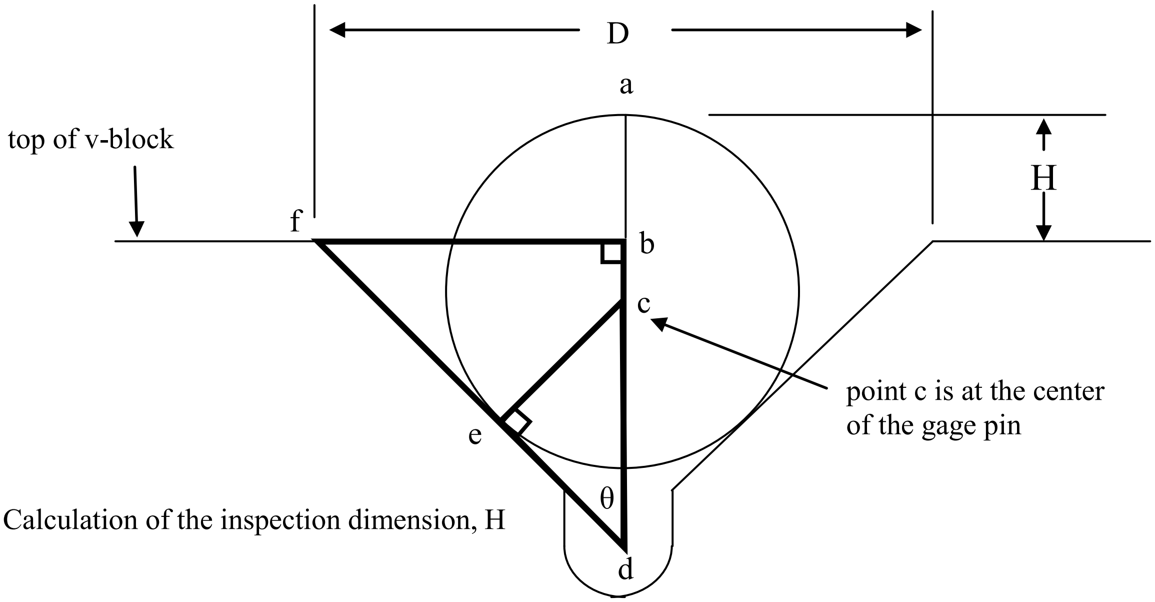


Figure 8-16: Calculation of the inspection dimension, H

Notes:

1. Dimension H represents how far the top of the gage pin extends beyond the top of the v-block.

2. Dimension D is the width of the vee. D is determined from dimensions on the v-block drawing.

3. Line segment ad is vertical, runs through the center of the v-block, and is perpendicular to line fb.

4. Line segment ad bisects the 90° v-block angle into two 45° angles. Therefore, angle $\theta = 45°$.

5. Line segment ad also bisects the dimension D. Therefore, line fb is half of dimension D.

6. Right triangle fbd is an isosceles triangle, since the internal angles are 90°, 45°, and 45°. In an isosceles triangle, the two legs are equal. Therefore, sides fb and bd are equal.

7. In triangle cde, line ce goes from the center of the gage pin, point c, to point e. Point e was selected since it is a "tangent point". Line fd, the side of the vee, is a tangent line that intersects the gage pin at one point, called the tangent point. By definition, a radius of a circle is perpendicular to a tangent line when the radius is drawn to the tangent point. In this problem, point e is a tangent point. Therefore, line ce is perpendicular to line fd.

8. Triangle fbd was created because line bd is an important dimension that we will need.

9. Triangle cde was created because line cd is an another important dimension that we will need.

Exercise 8-9

Create equations that can be used to solve for H.

H = ad – bd

ad = cd + ac = cd + radius of circle

Note: Line ac is also a radius of the circle. This sketch also shows line ce marked with an r for radius. We can determine the radius of the circle (gage pin) from the dimensions on the v-block drawing.

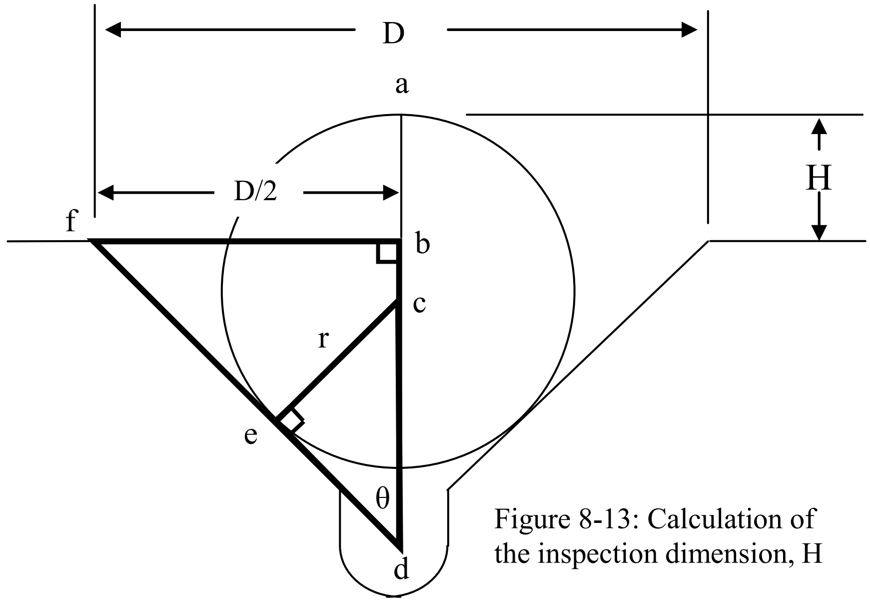


Figure 8-13: Calculation of the inspection dimension, H

Here is an equation we can use to solve for dimension bd:

$$\tan\theta = \frac{D/2}{bd}$$

Here is an equation we can use to solve for dimension cd:

$$\sin\theta = \frac{ce}{cd} = \frac{r}{cd}$$

Exercise 8-10

Calculate dimension H. See Figure 8-15 for the necessary dimensions.

$$\theta = \frac{90°}{2} = 45°$$

On the v-block drawing the gage pin diameter is shown as 0.400 in. The radius of the pin is therefore equal to 0.200 in., which is also the length of lines ce and ac.

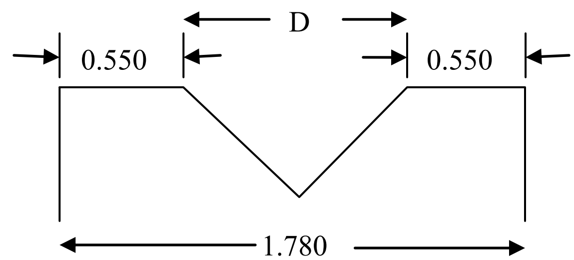
$$\sin 45° = \frac{r}{cd} \qquad cd = \frac{0.200}{\sin 45°} = 0.283$$

ad = cd + ac = cd + radius of circle = 0.283 + 0.200 = 0.483

continued on next page

Exercise 8-10, continued

Dimension D can be determined from the dimensions on the drawing in Figure 8-15. See the sketch on this page. The notation TYP indicates that the 0.550 in. dimension is the same on both sides of the vee. The width of the vee, D, is determined by subtracting the 0.550 in. dimensions from the overall v-block width of 1.780 in.



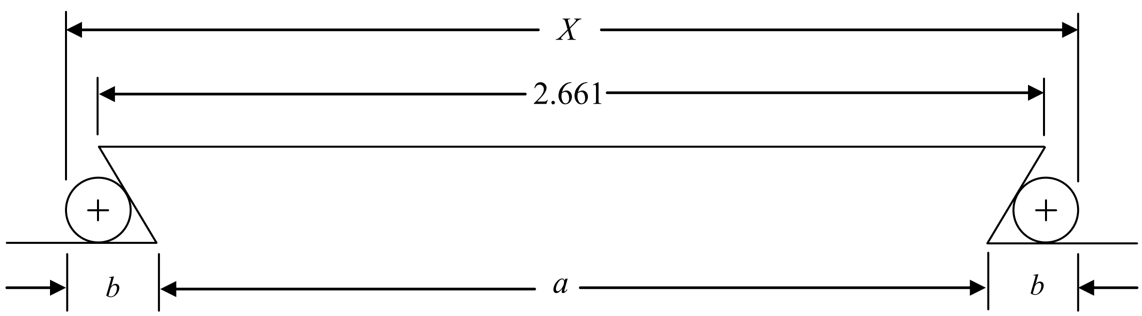
D = 1.780 – 2 (0.550) = 0.680 in.

$$\tan\theta = \frac{D/2}{bd} \qquad bd = \frac{D/2}{\tan\theta} = \frac{(.680/2)}{\tan 45°} = 0.340$$

H = ad – bd = 0.483 – 0.340 = 0.143 in.

Exercise 8-11

Sketch two right triangles on Figure 8-19. These two triangles must be useful in the calculation of the slide's inspection dimension, *X*. Label the vertices of the triangles to help with the solution.



As shown in this sketch, we want to determine the inspection dimension *X*, which is the measurement to the outside of the two gage pins. We will first determine the dimension *a*, and then determine dimension *b*. Dimensions *a* and *b* will be used to calculate dimension *X*.

Hint: As in previous problems, a radius drawn to a tangent line is perpendicular to the tangent line.

continued on next page

Exercise 8-11, continued

Right triangle #1

This triangle is sketched with a vertical line drawn from the top of the dovetail angle to the bottom as shown. This vertical side is marked as dimension *y*. The dovetail angle is 60°, so that angle β in this sketch is 90° – 60° = 30°.

Dimension *x1* in Triangle #1 is used to determine dimension *a* on the sketch above.

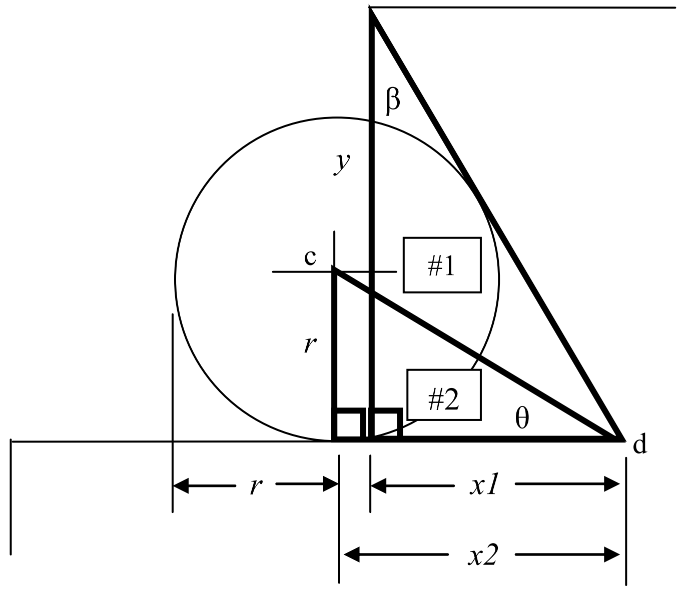


Figure 8-19: Slide detail

Right triangle 2

This triangle is sketched by drawing two lines from the center of the circle, shown as point c. One line is a radius of the circle and is drawn down to the horizontal surface of the slide. This side is marked as *r* for radius. The bottom of this line is at a tangent point and therefore creates a right angle.

The other line is drawn to the inside point of the dovetail angle, marked as point d. Line cd is a *bisector*. That means that this line from the center of the circle will divide the dovetail angle into two equal angles. Therefore, angle θ in triangle #2 is equal to half of 60°, or 30°.

Triangle #2 will be used to determine dimension *x2*. Dimension *x2* and radius of the pin, *r*, will be added to the previously determined dimension *a* to determine the overall inspection dimension, *X*.

Exercise 8-12

Calculate the inspection dimension *X*.

$X = a + 2r + 2(x2)$

$a = 2.661 - 2(x1)$

First, calculate *x1* using Triangle #1. Dimension y = 0.290 in., as shown in Figure 8-18.

$\tan\beta = \dfrac{x1}{y}$ $\qquad x1 = (y)(\tan\beta) = (0.290)(\tan 30°) = 0.1674$ in.

Now calculate dimension *a*.

$a = 2.661 - 2(0.1674) = 2.326$ in.

continued on next page

Exercise 8-12, continued

Next, calculate $x2$ using Triangle #2. The gage pin diameter is 0.220 in. as shown in Figure 8-18.

$$\tan\theta = \tan 30° = \frac{r}{x2} = \frac{0.110}{x2} \qquad x2 = \frac{0.110}{\tan 30°} = 0.1905 \text{ in.}$$

Finally, determine the value of X.

$$X = a + 2r + 2(x2) = 2.326 + 2(0.110) + 2(0.1905) = 2.927 \text{ in.}$$

Exercise 8-13

On Figure 8-23, sketch two right triangles that will help to calculate the included angle of the taper, α.

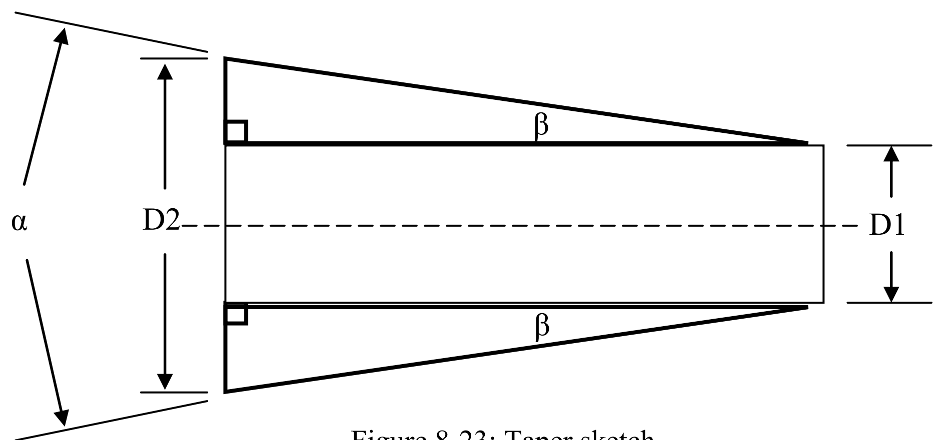
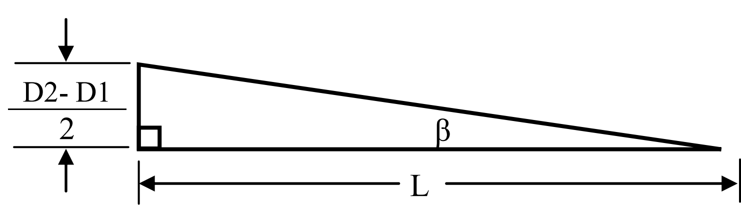


Figure 8-23: Taper sketch

Exercise 8-14

Calculate the included angle, α, for the taper shown in Figure 8-22. Express your answer in DMS format to the nearest second.



D2 = largest taper diameter = 1.0000 in.

D1 = smallest taper diameter = 0.7938 in.

L = 4.1250 in. from the taper drawing in Figure 8-22

continued on next page

Exercise 8-14, continued

The side opposite angle β is calculated using the largest and smallest diameters of the taper. The difference between D2 and D1 is equally distributed between these two sketched triangles, because these are equal triangles. The opposite side is equal to (D2 – D1)/2.

We can use the tangent function to calculate angle β since we know the opposite and adjacent sides.

$$\beta = \tan^{-1}\frac{\left(\dfrac{D2 - D1}{2}\right)}{L} = \tan^{-1}\frac{\dfrac{1.0000 - 0.7938}{2}}{4.1250} = 1°\ 25'\ 54.5''$$

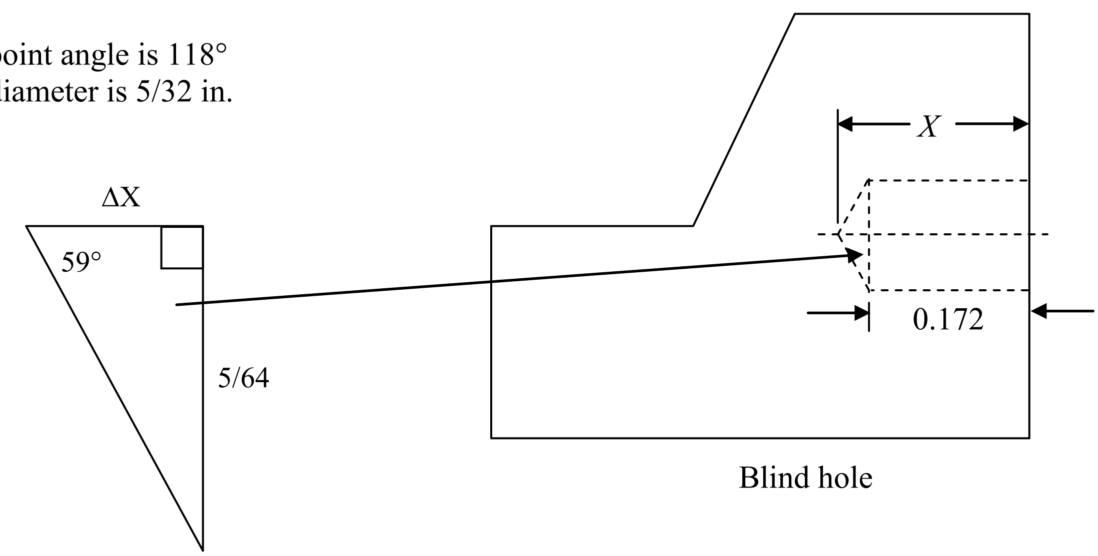
Finally, the included angle, α = 2β, since this angle is equally distributed over the two sketched triangles.

$$\alpha = 2\beta = 2\,(1°\ 25'\ 54.5'') = 2°\ 51'\ 49''$$

Chapter 8 Practice Worksheet

1. Solve for the drilling depth X. for the blind hole in this part. First, solve for the height of the drill point, ΔX, and then add that value to the depth dimension, shown as 0.172 in. on the drawing.

5 The drill point angle is 118°
6 The drill diameter is 5/32 in.



Blind hole

Right triangle sketch

First, calculate the drill point height, ΔX

$$\tan 59° = \frac{5/64}{\Delta X} \qquad \Delta X = \frac{5/64}{\tan 59°} = 0.047 \text{ in.}$$

The actual drilling depth to achieve the depth dimension, 0.172 in., is calculated as follows:

X = depth dimension + drill point height = 0.172 + 0.047 = 0.219 in.

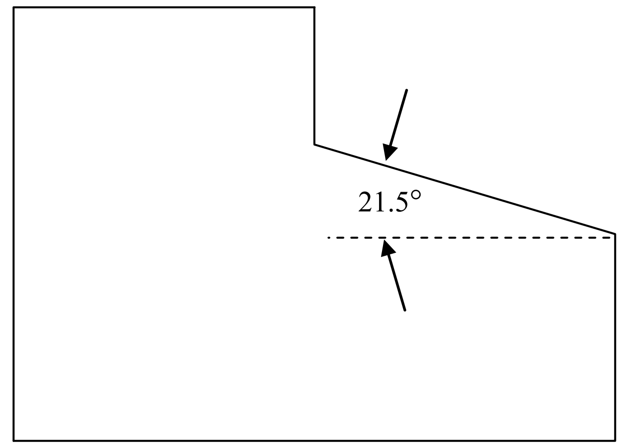
Chapter 8 Practice Worksheet, continued

2. Calculate the size of the gage block stack necessary to measure the angled surface in the part shown below. Express your answer to 4 decimal places.

1. Assume that the measurement setup will use a 5-inch sine bar (distance between the cylinders).

1. The stack of gage blocks will elevate the sine bar at the same angle (21.5°) as the angled surface on the part.

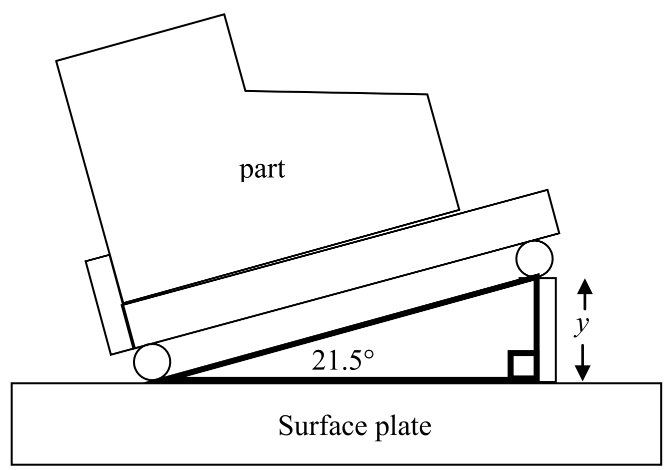
2. Inspection of the angled surface may proceed when the part is then placed on the elevated sine bar.



Part with an angled surface

1. The *y* dimension in this setup represents the height of the gage block stack.

2. The hypotenuse of this sketched triangle has a length of 5 inches. This length is the size of the sine bar, which is supporting the part.



Angularity measurement setup

$$\sin 21.5° = \frac{y}{5}$$

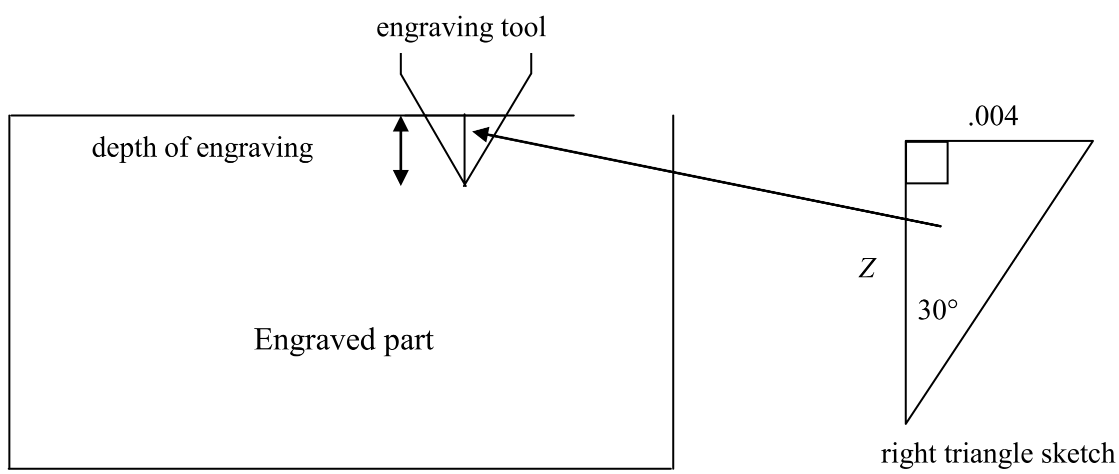
$$y = 5\,(\sin 21.5°) = 1.8325 \text{ in.}$$

The size of the gage block stack is 1.8325 in.

Chapter 8 Practice Worksheet, continued

3. Solve for the engraving tool depth, given the following information:

- Tool point angle: 60°
- Tool diameter: 3/16 inch
- Desired text width: 0.008 inch

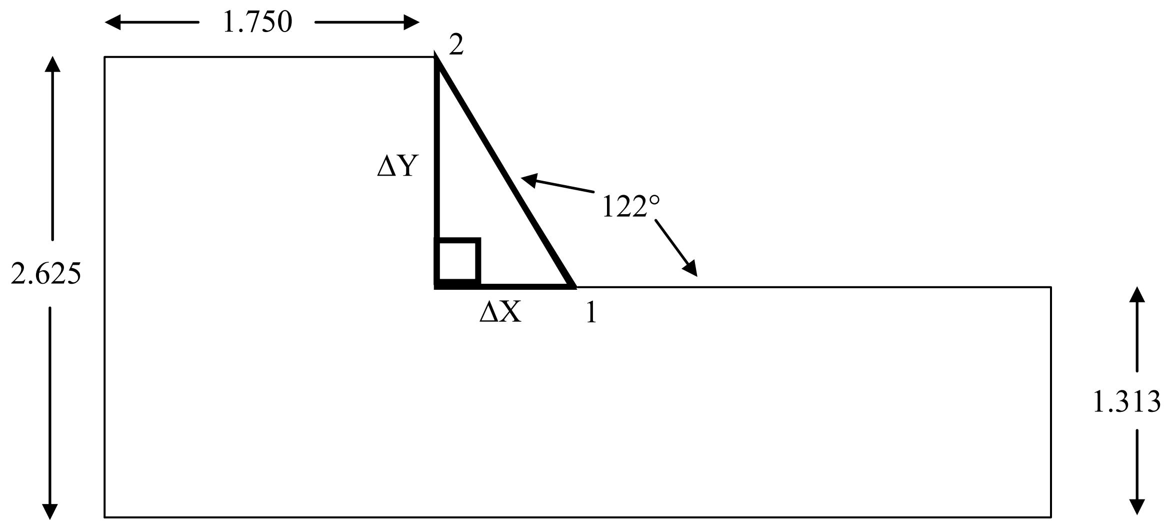


$$\tan 30° = \frac{0.004}{Z} \qquad Z = \frac{0.004}{\tan 30°} = 0.007 \text{ in.}$$

This answer means that the tool depth Z is 0.007 in. to make text that is 0.008 in. wide. Notice that the tool diameter was provided, but that information is not required to solve this problem.

Chapter 8 Practice Worksheet, continued

4. Determine the *X* location of point #1 on this part. As part of your solution, sketch and label the right triangle that you need to solve the problem. Answer to 3 decimals.



The *X* value at point 1 is equal to the *X* value at point 2, plus the ΔX dimension on the sketched right triangle.

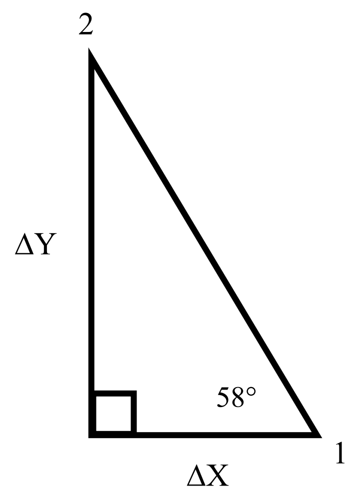
We can solve this triangle for ΔX if we know the value of an angle and a side. The internal angle at point 1 is equal to 58°. This angle and the given 122° angle add up to 180°.

Angle 1 = 180° – 122° = 58°

We can determine the ΔY dimension from the given dimensions on the part drawing.

ΔY = *Y* value at point 2 – *Y* value at point 1

ΔY = 2.625 – 1.313 = 1.312



sketched right triangle

Now solve the sketched triangle for ΔX.

$$\tan 58^\circ = \frac{1.312}{\Delta X} \qquad \Delta X = \frac{1.312}{\tan 58^\circ} = 0.820 \text{ in.}$$

Now solve for the *X* location of point 1

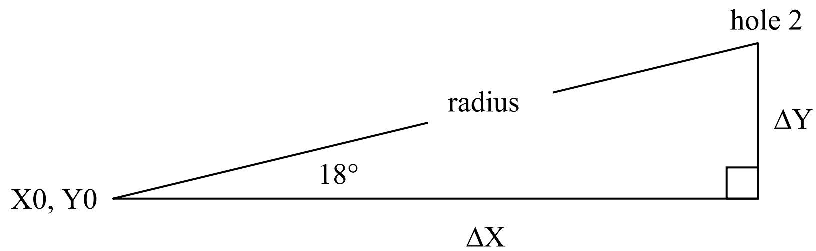
$X = 1.750 + \Delta X = 1.750 + 0.820 = 2.570$ in.

The *X* location of point #1 is 2.570 in.

Chapter 8 Practice Worksheet, continued

5. Solve for the *X* and *Y* locations of holes #2 and #4 in this part.

1. The center of the part has *X* and *Y* coordinates of X0, Y0 (part origin).

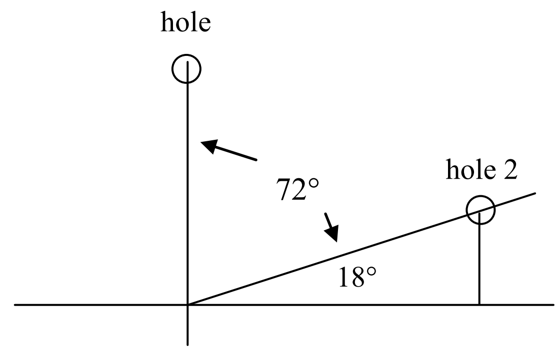


Sketched triangle to solve for hole #2

Hole #2

Radius = diameter/2 = 2.750/2 = 1.375 in.

The internal angle of 18° is determined as shown in this sketch. The angle between the "North" and "East" directions is 90°. The internal angle = 90° – 72° = 18°.



Determining internal angle in sketch

$$\sin 18° = \frac{\text{OPP}}{\text{HYP}} = \frac{\Delta Y}{1.375}$$

$\Delta Y = (1.375)\ (\sin 18°) = 0.425$ in.

$$\cos 18° = \frac{\text{ADJ}}{\text{HYP}} = \frac{\Delta X}{1.375}$$

$\Delta X = (1.375)\ (\cos 18°) = 1.308$ in.

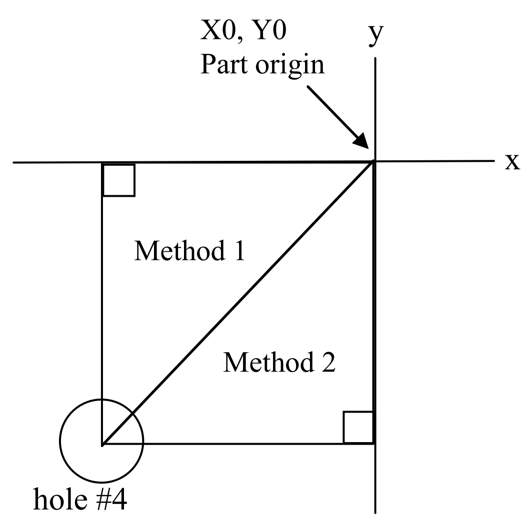
Therefore, X2 = 1.308 in. and Y2 = 0.425 in. In this problem, the center of the part has coordinates of X0 and Y0. The ΔX and ΔY answers represent the actual hole location.

Hole #4

The sketched triangles may be created in two different ways.

In Method 1, the triangle is sketched so that the ΔX dimension is aligned with the x-axis of the Cartesian coordinates. The ΔY dimension is parallel to the y-axis but is not aligned with it.

In Method 2, the triangle is sketched so that the ΔY dimension is aligned with the y-axis of the Cartesian coordinates. The ΔX dimension is parallel to the x-axis but is not aligned with it.



Sketching methods

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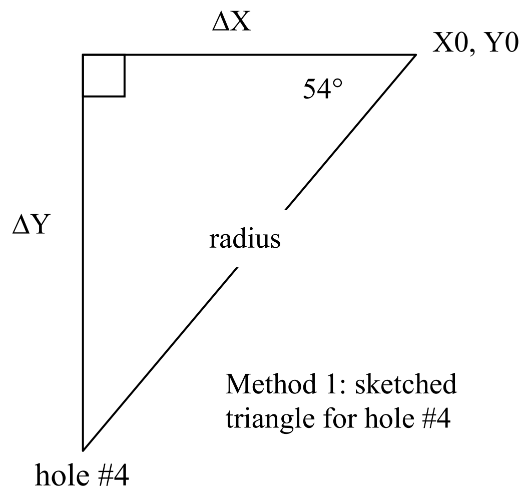
Chapter 8 Practice Worksheet, continued

Problem 5, continued

This solution will illustrate the steps for only the Method 1 triangle. The solution for the Method 2 triangle is similar.

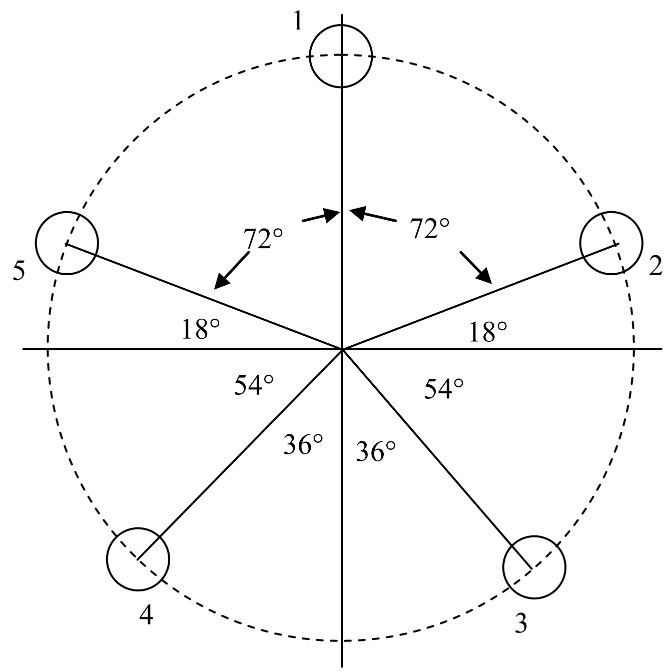


Method 1: sketched triangle for hole #4

The second sketch below illustrates how to determine the internal angles in your sketched triangles. The part drawing indicated a 72° angle between each hole. The TYP notation means that all of these angles between holes are equal.

Notice how the y axis divides the 72° angle between holes 3 and 4 into two equal angles.

For each of the sketched triangles, the hypotenuse is equal to the radius of the bolt-hole circle which is 1.375 in. in this problem.



Determining the value of the internal angles

$$\sin 54° = \frac{\text{OPP}}{\text{HYP}}$$

$$\sin 54° = \frac{\Delta Y}{1.375}$$

$$\Delta Y = 1.375 (\sin 54°) = 1.112 \text{ in.}$$

$$\cos 54° = \frac{\text{ADJ}}{\text{HYP}}$$

$$\cos 54° = \frac{\Delta X}{1.375}$$

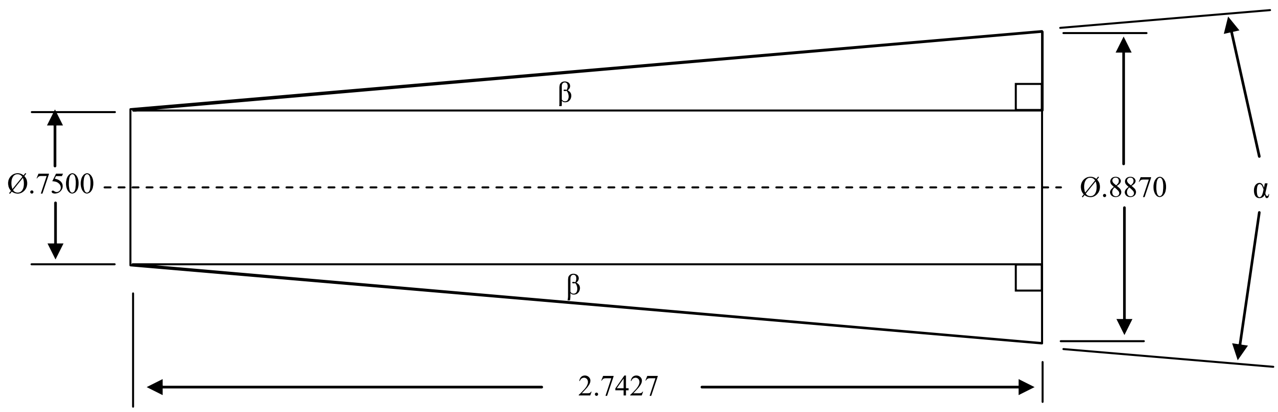
$$\Delta X = 1.375 (\cos 54°) = 0.808 \text{ in.}$$

Notice that hole #4 is in the third quadrant of the Cartesian coordinate system, where the *X* and *Y* values are both negative.

Therefore, $X = -0.808$ in. and $Y = -1.112$ in. for hole #4.

Chapter 8 Practice Worksheet, continued

6. Calculate the included angle, α, in DMS format to the nearest second. All dimensions in inches.

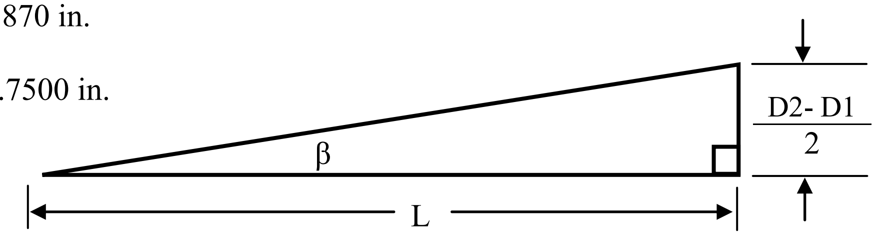


Right triangles sketched on taper

D2 = largest taper diameter = 0.8870 in.

D1 = smallest taper diameter = 0.7500 in.

L = 2.7427 in.



The side opposite angle β is calculated using the largest and smallest diameters of the taper. The difference between D2 and D1 is equally distributed between these two sketched triangles because these are equal triangles. The opposite side is equal to (D2 – D1)/2.

We can use the tangent function to calculate angle β because we know the opposite and adjacent sides.

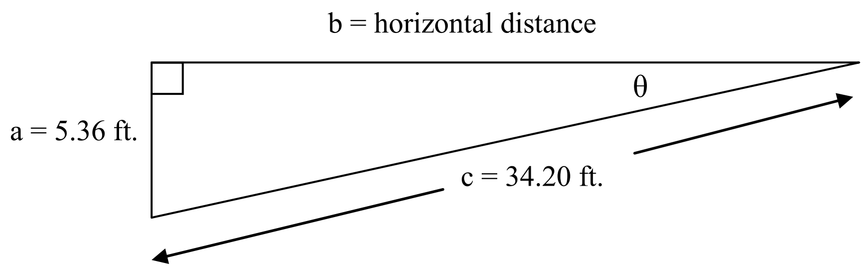
$$\beta = \tan^{-1} \frac{\left(\frac{D2 - D1}{2}\right)}{L} = \tan^{-1} \frac{\frac{0.8870 - 0.7500}{2}}{2.7427} = 1.43068697^\circ$$

$$\alpha = 2\beta = 2\,(1.43068697^\circ) = 2.86137394^\circ = 2^\circ\ 51'\ 41''$$

Exercise 9-1

Referring to Figure 9-2, use the Pythagorean Theorem to calculate the horizontal distance. Compare your result to our calculation on the previous page.

First, draw a sketch of the right triangle that will help you to solve the problem.



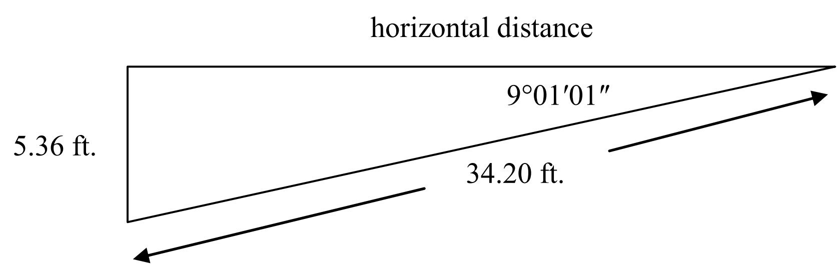
$c^2 = a^2 + b^2$, then using algebra, $b = \sqrt{c^2 - a^2}$

$c = \sqrt{(34.20)^2 - (5.36)^2} = 33.78$ ft.

This is the same result as the calculation that used the tangent function.

Exercise 9-2

In the example on the previous page, we used the tangent function to calculate horizontal distance. Try using the cosine function to calculate the horizontal distance. Compare your result to the previous calculations.



$$\cos 9°01'01'' = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{horizontal distance}}{34.20}$$

Solve this equation for the horizontal distance. Round your answer to 2 decimal places.

$$\text{horizontal distance} = (34.20)(\cos 9°01'01'') = 33.78 \text{ ft.}$$

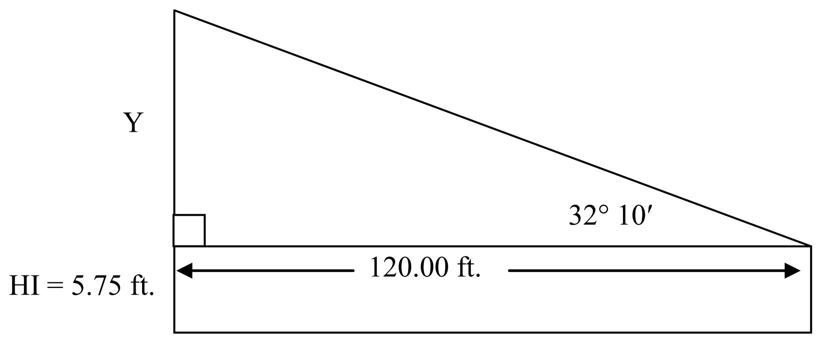
This is the same result as the previous calculations.

Exercise 9-3

Repeat this problem for a different building, using the following measurements:

- The horizontal distance from the building to the transit = 120.00 ft.
- The HI = 5.75 ft.
- The vertical angle is measured as 32° 10′.

Include a labeled sketch of the right triangle that you need to help solve the problem.



First, we will solve for the side Y of the triangle. Then to find the overall height of the building, we will add the HI, height of instrument, to Y.

$$\tan 32° 10' = \frac{\text{opposite}}{\text{adjacent}} = \frac{Y}{\text{horizontal distance}} = \frac{Y}{120.00 \text{ ft.}}$$

$$Y = (120.00 \text{ ft.})(\tan 32° 10') = 75.47 \text{ ft.}$$

Building height = Y + HI = 75.47 ft. + 5.75 ft. = 81.22 ft.

Exercise 9-4

Refer to the sketch in Figure 9-7. Calculate the distance between points *A* and *B*. Use the following additional information:

- A third point *C* is located at a distance of 90.00 feet from point *A*.
- Point *C* is located in a way that creates a right triangle with points *A*, *B*, and *C*.
- The angle measured by the surveyor from point *C* is 67° 37′ 14″ (sight on point *A* and turn towards point *B*).

As part of your solution, sketch the new point C and the resulting right triangle on Figure 9-7.

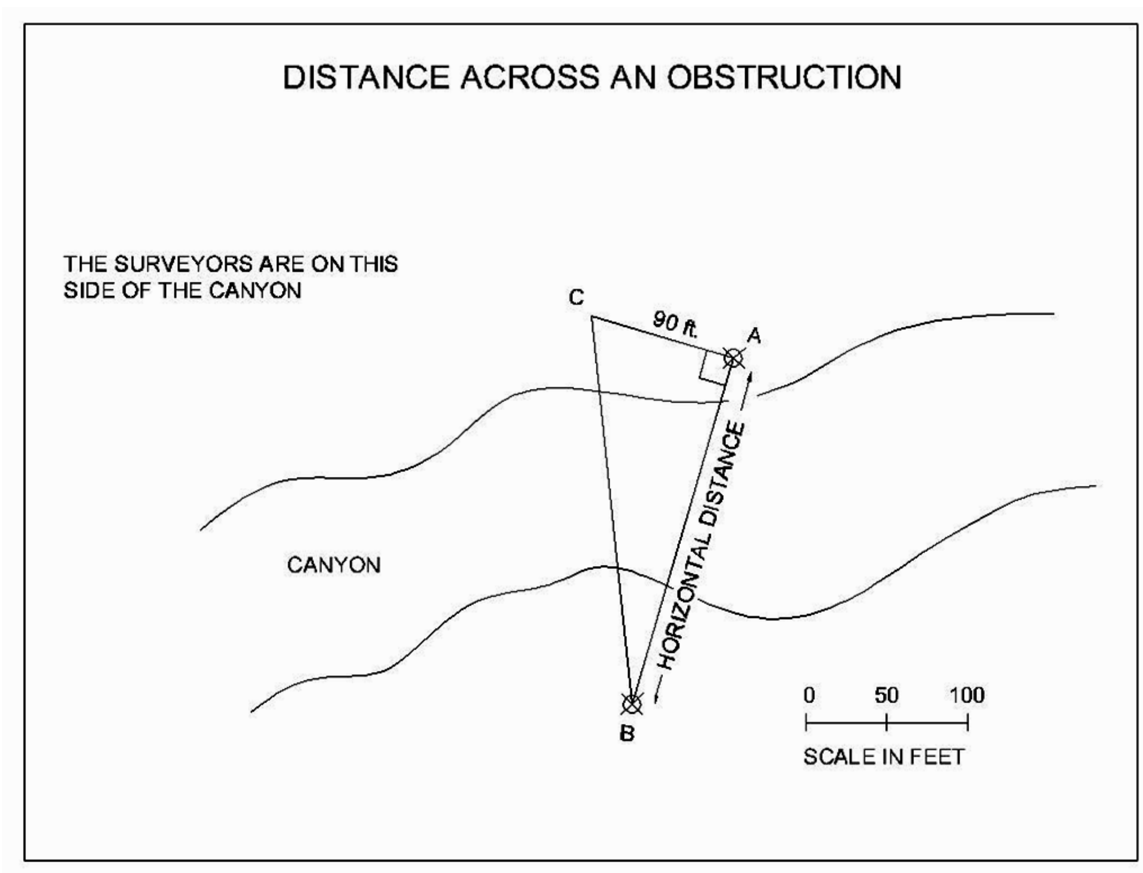


Figure 9-7: Distance across a canyon

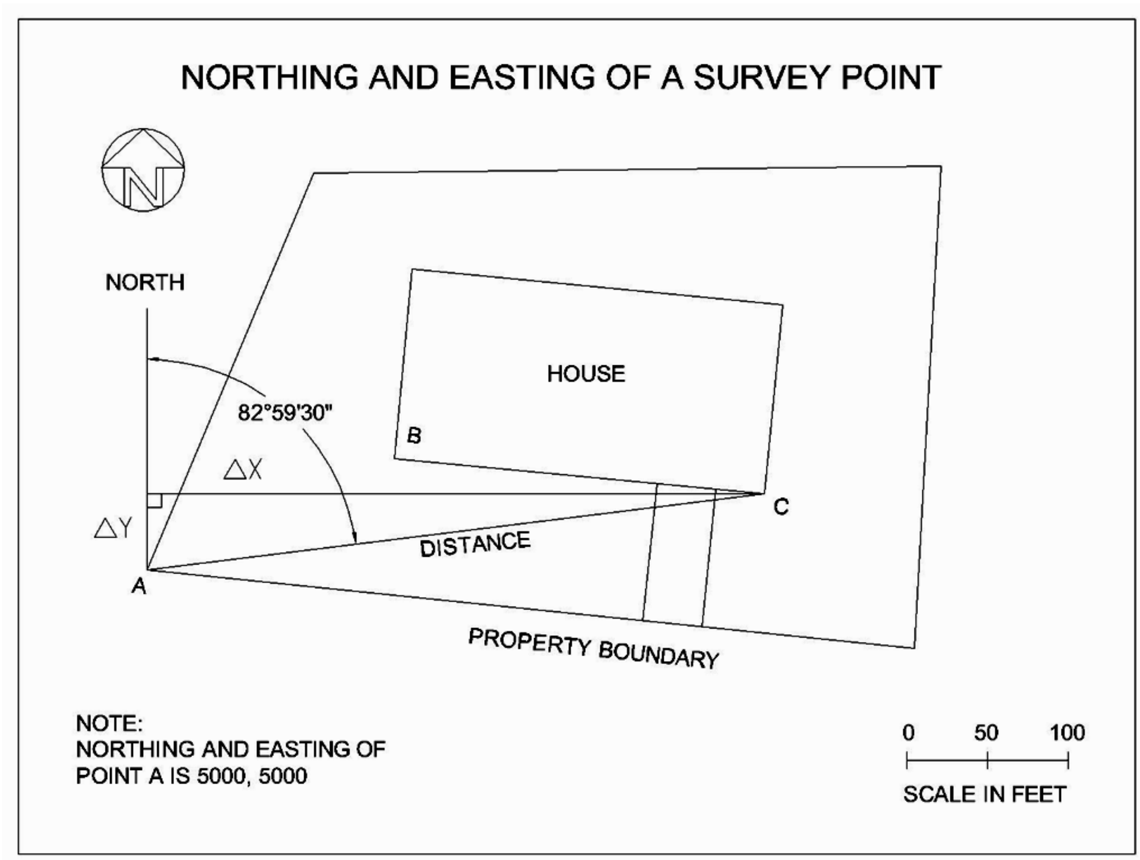
$$\tan 67° \, 37' \, 14'' = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{horizontal distance}}{90.00 \text{ ft.}}$$

$$\text{horizontal distance} = (90.00 \text{ ft.})(\tan 67° \, 37' \, 14'') = 218.58 \text{ ft.}$$

Exercise 9-5

Use Figure 9-11 below to calculate the Northing and Easting coordinates of point *C*, which is the southeastern corner of the house. Use the following information in your calculations:

- Point *A* has a Northing and Easing of 5000, 5000
- The distance from Point *A* to Point *C* = 384.29 feet
- The azimuth from Point *A* to Point *C* = angle θ = 82° 59′ 30″



As part of your solution, sketch the necessary right triangle in Figure 9-11.

Solve for Northing

$$\cos 82^\circ\, 59'\, 30'' = \frac{\Delta Y}{\text{horizontal distance}} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\Delta Y}{384.29\text{ ft.}}$$

$$\Delta Y = (384.29\text{ ft.})\,(\cos 82^\circ\, 59'\, 30'') = 46.89\text{ ft.}$$

Northing of point B = Northing of point A + ΔY = 5000 + 46.89 = 5046.89 ft.

Exercise 9-5, continued

Solve for Easting

$$\sin 82° \, 59' \, 30'' = \frac{\Delta X}{\text{horizontal distance}} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\Delta X}{384.29 \text{ ft.}}$$

$$\Delta X = (384.29 \text{ ft.})(\sin 82° \, 59' \, 30'') = 381.42 \text{ ft.}$$

Easting of point B = Easting of point A + ΔX = 5000 + 381.42 = 5381.42 ft.

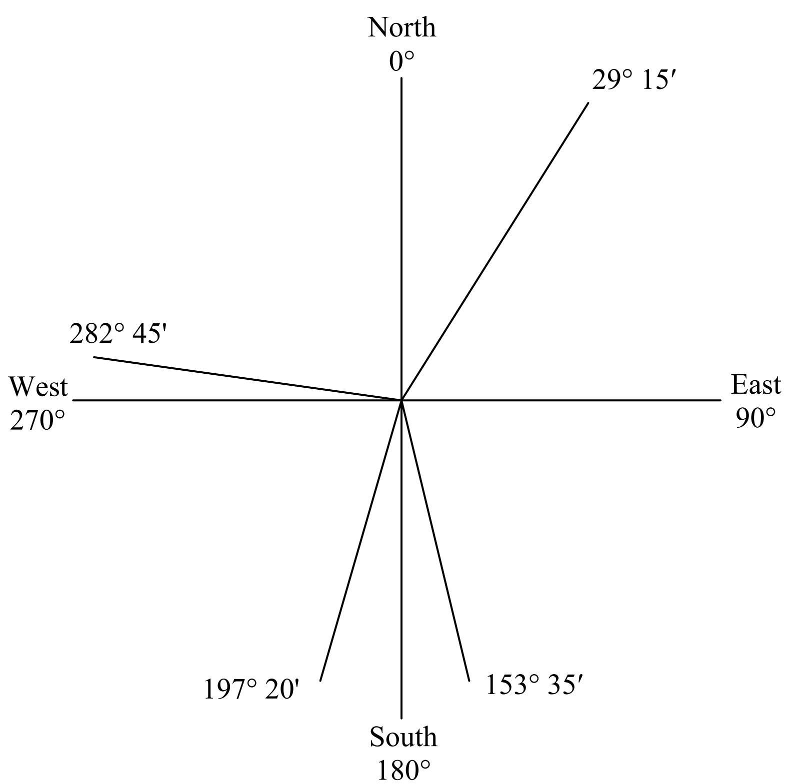
Exercise 9-6

Express the following azimuths as bearings, and the following bearings as azimuths.

Azimuth	Bearing
282° 45'	N77° 15′W
197° 20'	S17° 20′W
153° 35′	S26° 25′E
29° 15′	N29° 15′E

Exercise 9-7

Sketch the four bearings from the table above.



Exercise 9-8

Now calculate the *grade* between points *A* and *B* in Figure 9-13. Express your answer in percent to one decimal.

$$\text{grade} = \frac{25 \text{ ft.}}{263 \text{ ft.}} \times 100\% = 9.5\% \text{ grade}$$

Exercise 9-9

Calculate the slope angle between points *C* and *D* in Figure 9-13. Express your answer in DMS format to the nearest second.

Elevation difference between points *C* and *D* = 130 – 100 = 30 ft.

$$\text{horizontal distance} = \text{measured distance} \times \text{scale} = 1\ 5/8 \text{ in.} \times \frac{100 \text{ ft.}}{1 \text{ inch}} = 163 \text{ ft.}$$

$$\text{slope angle} = \theta = \tan^{-1} \frac{30 \text{ ft.}}{163 \text{ ft.}} = 10.42852848° = 10° 25' 43''$$

Exercise 9-10

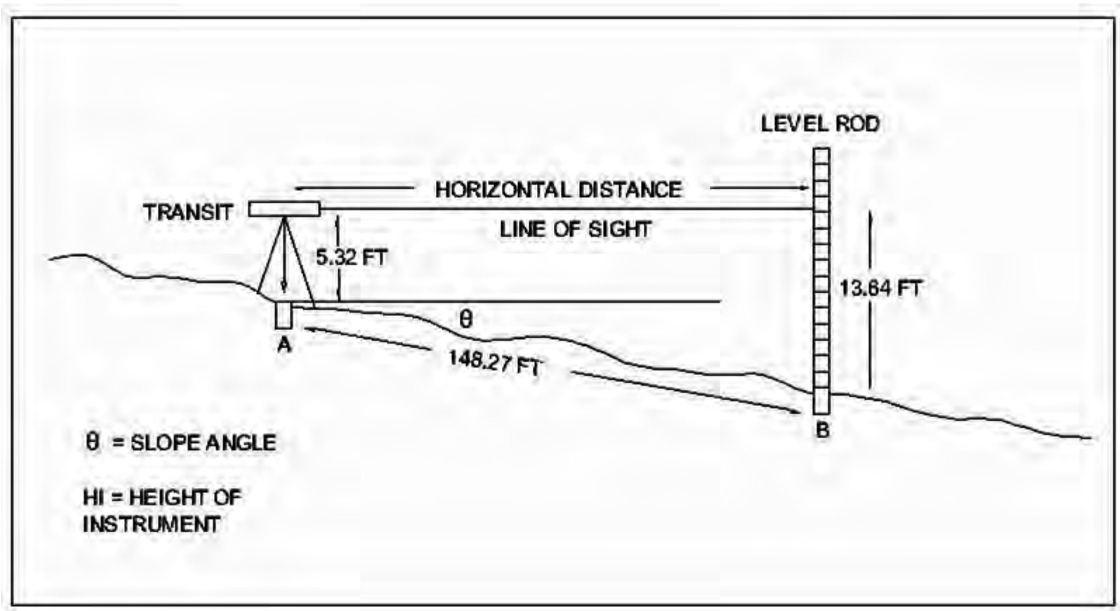
Calculate the grade between points *C* and *D* in Figure 9-13. Express your answer in percent to one decimal

$$\text{grade} = \frac{30 \text{ ft.}}{163 \text{ ft.}} \times 100\% = 18.4\% \text{ grade}$$

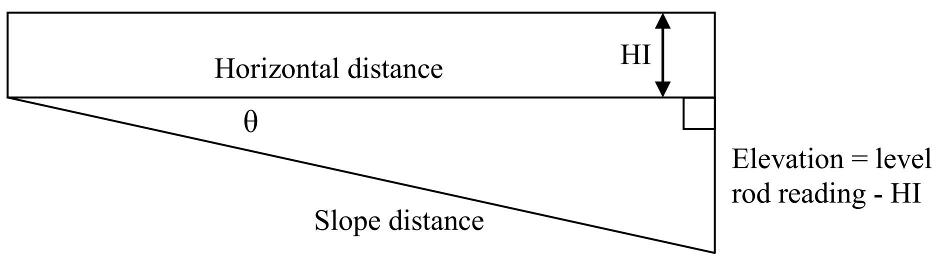
Chapter 9 Practice Worksheet

1. Calculate the horizontal distance and slope angle.

First, draw a sketch of the right triangle that will help you to solve the problem.



Horizontal distance



There are several solutions for horizontal distance.
This solution uses Pythagorean's Theorem.

Horizontal distance $= \sqrt{(148.27)^2 - (8.32)^2} = 148.04$ ft.

Slope angle

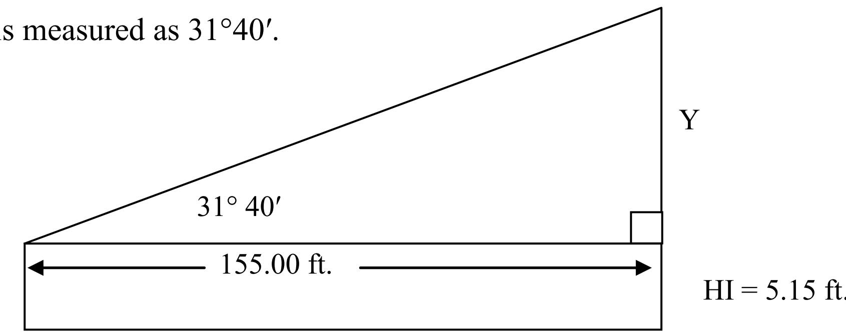
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{elevation}}{\text{slope distance}} = \frac{(13.64 - 5.32)}{148.27} = \frac{8.32}{148.27}$$

$$\theta = \sin^{-1}\frac{8.32}{148.27} = 3.216776222° = 3°\,13'\,00''$$

Chapter 9 Practice Worksheet, continued

2. Calculate the height of this building, using the following measurements:

- The vertical angle θ is measured as 31°40′.



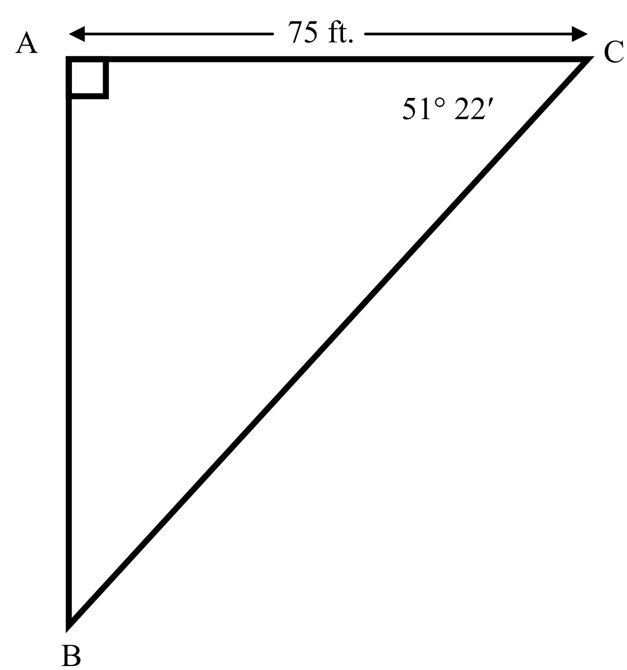
First, we will solve for the side Y of the triangle. Then to find the overall height of the building, we will add HI, height of instrument, to Y.

$$\tan 31°\,40' = \frac{\text{opposite}}{\text{adjacent}} = \frac{Y}{\text{horizontal distance}} = \frac{Y}{155.00\text{ ft.}}$$

$$Y = (155.00\text{ ft.})(\tan 31°\,40') = 95.61\text{ ft.}$$

Building height = Y + HI = 95.61 ft. + 5.15 ft. = 100.76 ft.

3. Calculate the distance between points *A* and *B*.

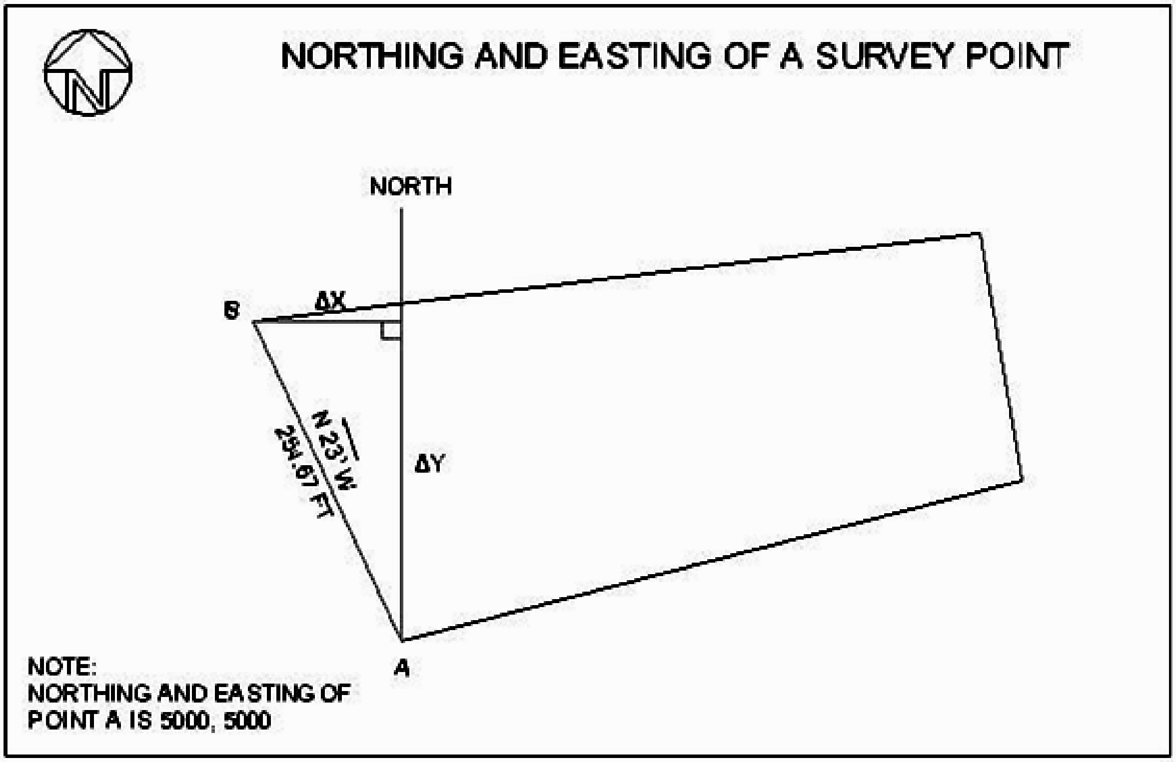


$$\tan 51°\,22' = \frac{AB}{75.00}$$

$$AB = (75.00\text{ ft.})(\tan 51°\,22') = 93.84\text{ ft.}$$

Chapter 9 Practice Worksheet, continued

4. Calculate the Northing and Easting coordinates for point *B* in the following sketch.



Solve for Northing

Azimuth = θ = 360° – 23° = 337°

$$\cos 337^\circ = \frac{\Delta Y}{AB} = \frac{\Delta Y}{254.67 \text{ ft.}}$$

$$\Delta Y = (254.67 \text{ ft.})(\cos 337^\circ) = 234.42 \text{ ft.}$$

Northing of point *B* = Northing of point *A* + ΔY = 5000 + 234.42 = 5234.42 ft.

Solve for Easting

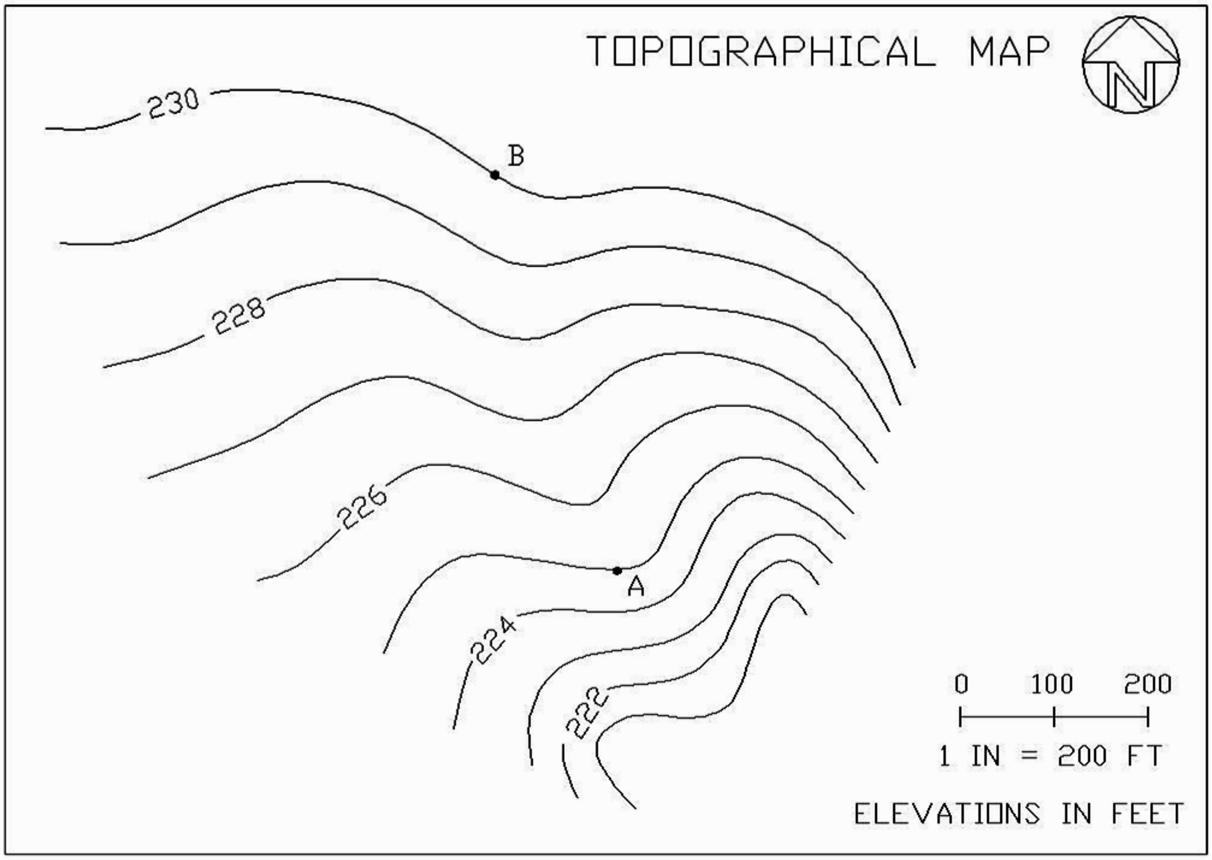
$$\sin 337^\circ = \frac{\Delta X}{AB} = \frac{\Delta X}{254.67 \text{ ft.}}$$

$$\Delta X = (254.67 \text{ ft.})(\sin 337^\circ) = -99.51 \text{ ft.}$$

Easting of point *B* = Easting of point *A* + ΔX = 5000 – 99.51 = 4900.49 ft.

Chapter 9 Practice Worksheet, continued

5. Calculate the slope angle and grade between points *A* and *B* on this topographical map.



The elevation rise between points A and B is read from the contour lines. The rise between *A* and *B* is 230 feet – 225 feet = 5 feet.

$$\text{horizontal distance} = \text{measured distance} \times \text{scale} = 2\ 1/8 \text{ in.} \times \frac{200 \text{ ft.}}{1 \text{ inch}} = 425 \text{ ft.}$$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{elevation difference between A and B}}{\text{horizontal distance between A and B}} = \frac{5 \text{ ft.}}{425 \text{ ft.}}$$

$$\text{slope angle} = \theta = \tan^{-1} \frac{5 \text{ ft.}}{425 \text{ ft.}} = 0° \, 40' \, 27''$$

$$\text{grade} = \frac{5 \text{ ft.}}{425 \text{ ft.}} \times 100\% = 1.18\% \text{ grade}$$

Exercise 10-1

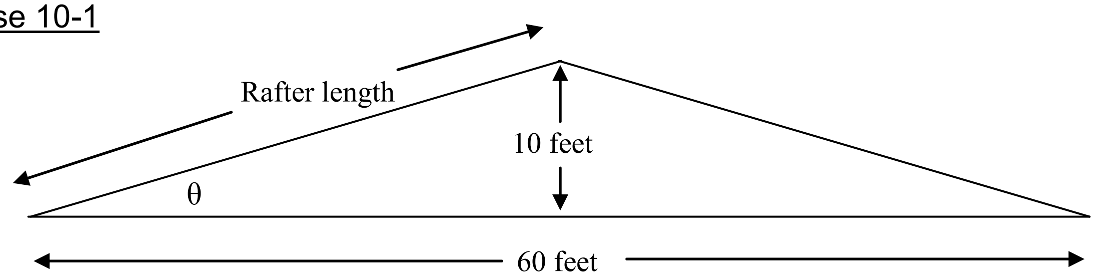


Figure 10-2: Roof pitch calculation

The overall truss width is given as 60 feet. The right triangle that we will use to solve the problem represents only one-half of this truss. We will therefore use 30 feet as the length of the side that is adjacent to angle θ.

Roof pitch = rise/run = 10/30

Roof pitch is typically expressed as a fraction or ratio with 12 as the denominator. We will use a proportion to convert roof pitch to this format.

$$\frac{10}{30} = \frac{X}{12} \qquad X = \frac{(10)\,(12)}{30} = 4$$

Therefore, the roof pitch = 4/12 or 4:12

Roof Angle

$$\text{Roof angle} = \theta = \tan^{-1}\frac{4}{12} = 18.43°$$

Exercise 10-2

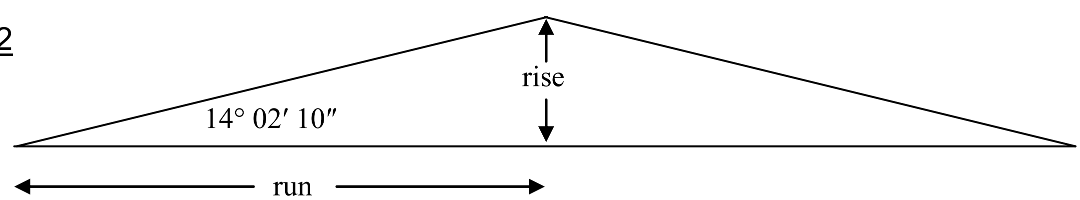


Figure 10-3: Roof pitch calculation

$$\tan 14°\, 02'\, 10'' = 0.25 = \frac{\text{rise}}{\text{run}} = \frac{1}{4}$$

Roof pitch is typically expressed as a fraction or ratio with 12 as the denominator. We will use a proportion to convert roof pitch to this format.

$$\frac{1}{4} = \frac{X}{12} \qquad X = \frac{(1)\,(12)}{4} = 3$$

Therefore, the roof pitch = 3/12 or 3:12

Exercise 10-3

Calculate the length of web DE. Use the value of the roof angle that we calculated and the triangle which is sketched in Figure 10-5.

$$\tan 22.62° = \frac{DE}{15 \text{ ft.}}$$

$$DE = (15.0 \text{ ft.})(\tan 22.62°) = 6.25 \text{ ft.} = 6 \text{ ft. } 3 \text{ in.}$$

Alternate solution:

$$\tan 41.76° = \frac{DE}{CE} = \frac{DE}{7.0 \text{ ft.}}$$

$$DE = (7.0 \text{ ft.})(\tan 41.76°) = 6.25 \text{ ft.} = 6 \text{ ft. } 3 \text{ in.}$$

Exercise 10-4

Calculate the length of rafter ABD. Use the value of the roof angle that we calculated and the triangle that is sketched in Figure 10-5. Remember to add the 1-foot overhang shown in Figure 10-4.

$$\cos 22.62° = \frac{ACE}{ABD} = \frac{15 \text{ ft.}}{ABD} \qquad ABD = \frac{15 \text{ ft.}}{\cos 22.62°} = 16.25 \text{ ft.} = 16 \text{ ft. } 3 \text{ in.}$$

Total rafter length = ABD + 1-foot overhang = 17 ft. 3 in.

Exercise 10-5

Calculate angle BAC from the roof pitch, which is 4:12. You will need this angle to help calculate the unknown lengths in the truss. Express your answer in decimal degree to two decimals.

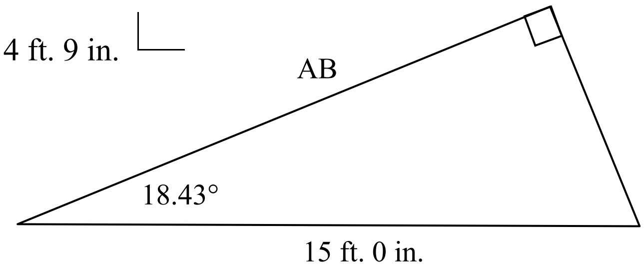
$$\text{Angle } BAC = \tan^{-1} \frac{4}{12} = 18.43°$$

Exercise 10-6

Calculate the length of web BC and also the length of AB. As part of your solution, sketch the right triangle that you need to solve for the unknown length.

$$\sin 18.43° = \frac{BC}{15}$$

$$BC = 15(\sin 18.43°) = 4.743 \text{ ft.} = 4 \text{ ft. } 9 \text{ in.}$$



$$\cos 18.43° = \frac{AB}{15}$$

$$AB = 15(\cos 18.43°) = 14.230 \text{ ft.} = 14 \text{ ft. } 3 \text{ in.}$$

Exercise 10-7

Calculate the length of web CD. As part of your solution, sketch the right triangle that you need to solve for the unknown length.

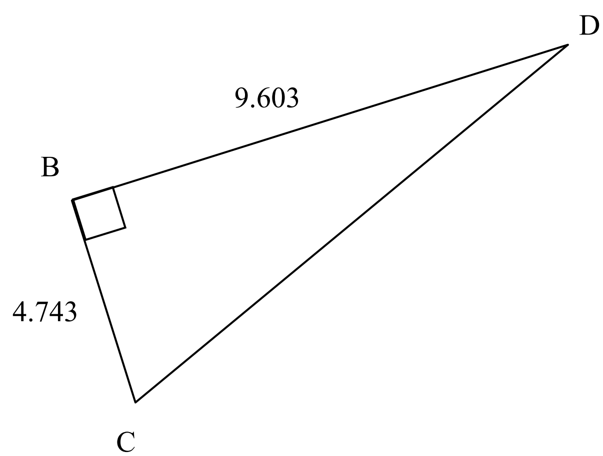
Use the Pythagorean Theorem

BD = 23′ 10″ – AB = 23.833 – 14.230 = 9.603 ft.

Because we know that side BC = 4.743, we can use the Pythagorean Theorem to solve for side CD.

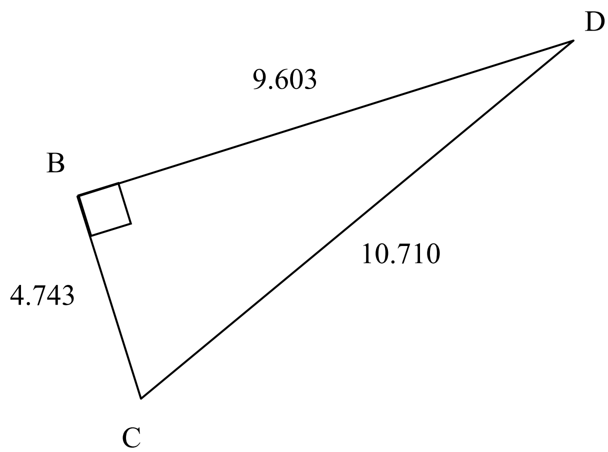
$$CD = \sqrt{(BC)^2 + (BD)^2}$$

$$CD = \sqrt{(4.743)^2 + (9.603)^2} = 10.710 \text{ ft.} = 10 \text{ ft. } 9 \text{ in.}$$



Exercise 10-8

Calculate the value of angles BCD and BDC. Express your answer in DMS format to the nearest minute. As part of your solution, sketch the right triangle that you need to solve for the unknown angles.



Note: There are a number of other possible solutions, other than what is shown below.

$$\text{Angle BCD} = \sin^{-1}\frac{9.603}{10.710} = 63.7197° = 63°\,43'\,11''$$

$$\text{Angle BDC} = \cos^{-1}\frac{9.603}{10.710} = 26.2803° = 26°\,16'\,49''$$

Exercise 10-9

Using the Pythagorean Theorem, confirm that a triangle with leg lengths of 3 and 4 has a hypotenuse with a length of 5.

$$\text{Hypotenuse} = \sqrt{(3)^2 + (4)^2} = 5$$

Exercise 10-10

Using the Pythagorean Theorem, confirm that a triangle with leg lengths of 5 and 12 has a hypotenuse with a length of 13.

$$\text{Hypotenuse} = \sqrt{(5)^2 + (12)^2} = 13$$

Exercise 10-11

In Figure 10-9, label the 3-4-5 triangles with multiples of this ratio.

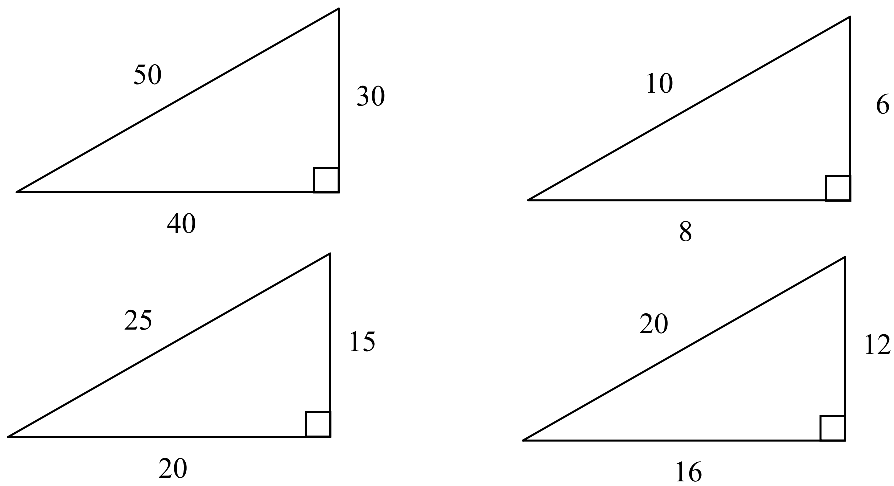


Figure 10-9: Variations of the 3-4-5 triangle

Exercise 10-12

Using the angle θ and the riser height given in Figure 10-11, calculate the tread depth (horizontal dimension). Express your answer to the nearest ¼ inch.

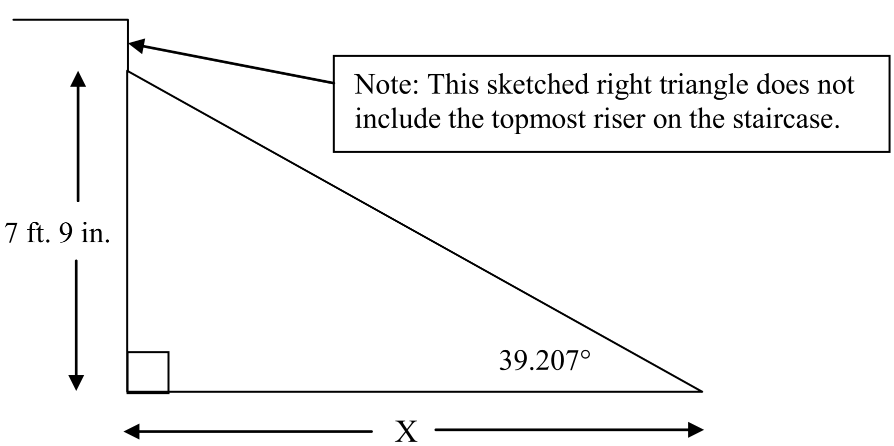
$$\tan 39.207° = \frac{\text{riser}}{\text{tread}} = \frac{7.75\text{ in.}}{\text{tread}} \qquad \text{tread} = \frac{7.75\text{ in.}}{\tan 39.207°} = 9.5\text{ in.} = 9\ 1/2\text{ in.}$$

Exercise 10-13

Vertical side of the right triangle = 8 ft. 4 3/4 in. – the height of one riser (see note below)

Vertical side of the right triangle = 8 ft. 4 3/4 in. – 7 3/4 in. = 7 ft. 9 in.

$$\tan 39.207° = \frac{7\text{ ft. }9\text{ in.}}{X} = \frac{7.75\text{ ft.}}{X} \qquad X = \frac{7.75\text{ ft.}}{\tan 39.207°} = 9.5\text{ ft.} = 9\text{ ft. }6\text{ in.}$$



Exercise 10-14

Using the riser and tread dimensions, calculate the stair angle θ.

$$\theta \quad = \quad \tan^{-1} \frac{8}{9} \quad = \quad 41.6335° \quad = \quad 41° \, 38' \, 01''$$

Exercise 10-15

Calculate dimension Y, the vertical distance from the first floor to the second floor. Express your answer in feet and inches to the nearest inch. Use the stair angle θ to perform the calculation. Check your answer using the riser dimension.

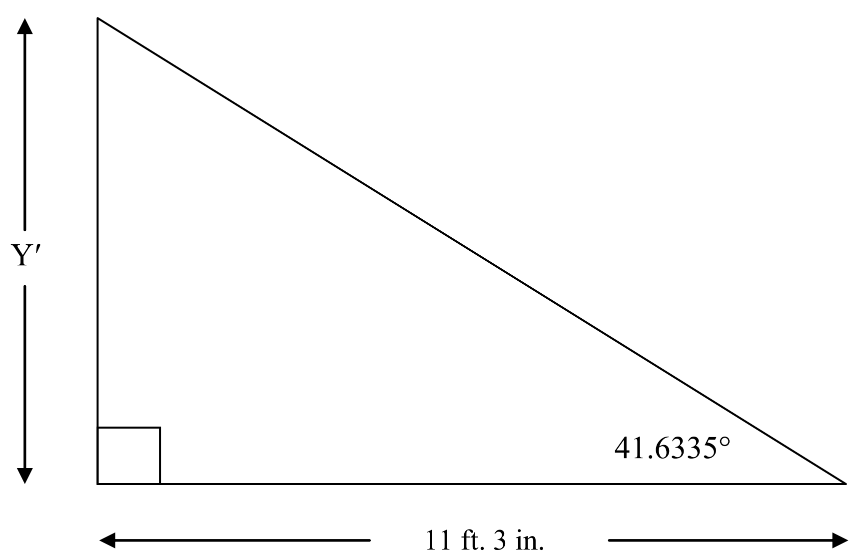
$$\tan 41° \, 38' \, 01'' \quad = \frac{Y'}{11.25 \text{ ft.}}$$

$$Y' \quad = \quad (11.25 \text{ ft.}) \, (\tan 41° \, 38' \, 01'') \quad = \quad 10.00 \text{ ft.} \quad = \quad 10 \text{ ft. } 0 \text{ in.}$$

Y = Y′ + one riser = 10 ft. 0 in. + 8 in. = 10 ft. 8 in.

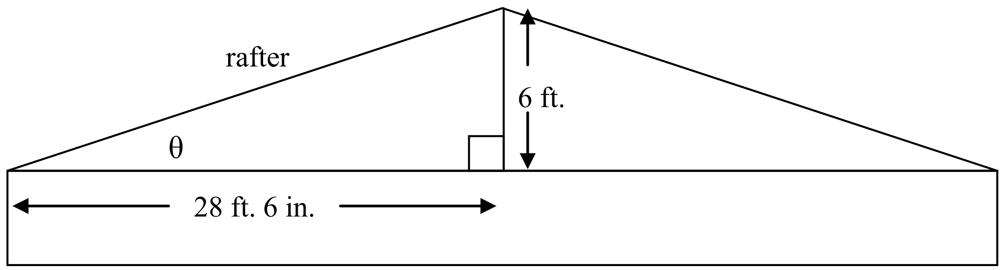
Check:

Y = (16 risers) (8 in. per riser) = 128 in. = 10 ft. 8 in.



Chapter 10 Practice Worksheet

1. Calculate the roof angle, θ, and also the rafter length. Express the angle in DMS format to the nearest second. Express the rafter length in feet and inches to the nearest 1/4 inch.



Roof angle

$$\tan\theta = \frac{6\text{ ft.}}{28.5\text{ ft.}}$$

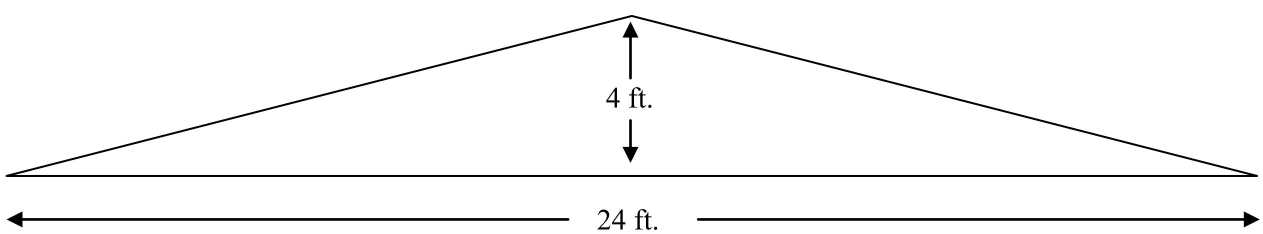
$$\theta = \tan^{-1}\frac{6\text{ ft.}}{28.5\text{ ft.}} = 11.88865804° = 11° 53' 19''$$

Rafter length

$$\cos\theta = \frac{28.5\text{ ft.}}{\text{rafter}}$$

$$\text{rafter} = \frac{28.5}{\cos\theta} = \frac{28.5}{\cos(11.88865804°)} = 29.12473176\text{ ft.} = 29\text{ ft. } 1\tfrac{1}{2}\text{ in.}$$

2. Calculate the roof pitch for the roof shown below. Assume that the roof is symmetrical. Also, calculate the roof angle, θ, in decimal degrees, to two decimals.



Roof pitch

$$\text{Roof pitch} = \frac{\text{rise}}{\text{run}} = \frac{4\text{ feet}}{12\text{ feet}} = \frac{4}{12} = 4/12 = 4{:}12$$

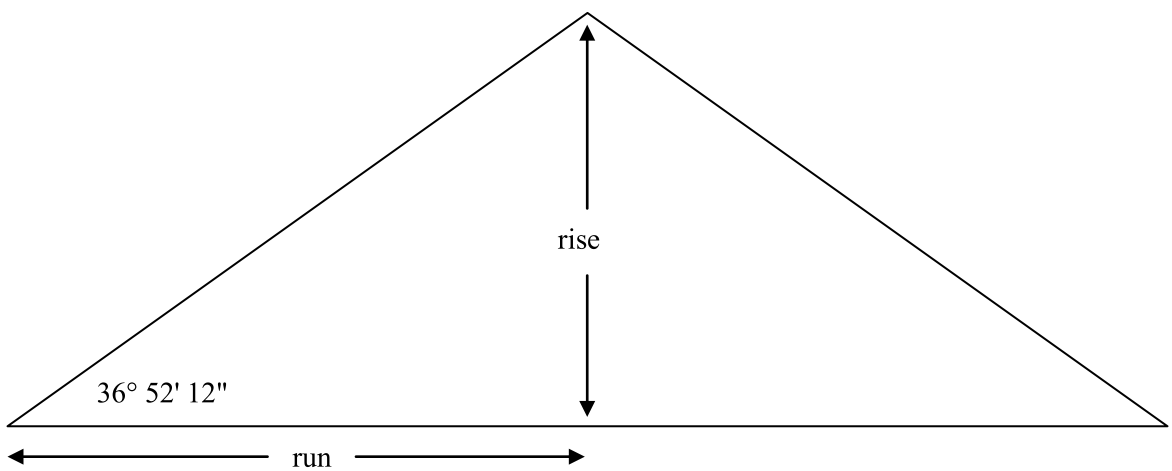
Chapter 10 Practice Worksheet, continued

Roof angle

$$\tan \theta = \frac{4 \text{ ft.}}{12 \text{ ft.}}$$

$$\theta = \tan^{-1} \frac{4}{12} = 18.43°$$

3. In the figure below, the roof angle is given but the roof pitch is not known. Using the given angle, calculate the roof pitch. Express the roof pitch as a ratio with a denominator of 12 (x:12).



$$\tan \theta = \frac{\text{rise}}{\text{run}}$$

$$\tan 36° 52' 12'' = 0.75$$

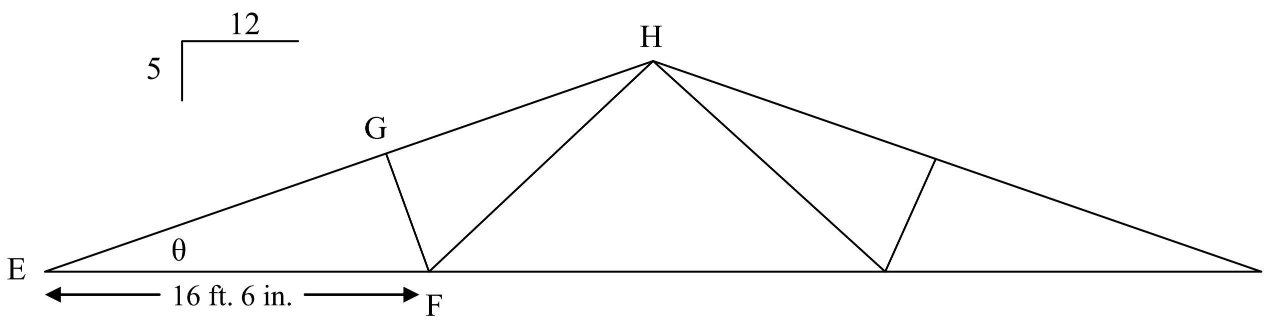
$$\frac{75}{100} = \frac{X}{12}$$

$$X = \frac{(12)(75)}{100} = 9$$

$$\text{pitch} = \frac{9}{12} = 9{:}12$$

Chapter 10 Practice Worksheet, continued

4. Calculate the length of web FG. Express the web length in feet and inches to the nearest 1/2 inch.
   Note: Triangles EFG and FGH are right triangles.



First, calculate the roof angle, θ.

$$\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{5}{12}$$

$$\theta = \tan^{-1}\frac{5}{12} = 22.62°$$

Then, calculate the length of web FG.

First, convert 16 ft. 6 in. to a decimal so that it is more easily used in an equation. The 6 inches is converted to feet as follows:

$$6 \text{ in.} \times \frac{1 \text{ ft.}}{12 \text{ in.}} = 0.5 \text{ ft.}$$ Then, add the 0.5 ft. to the 16 ft. to get 16.5 ft.

$$\sin 22.62° = \frac{FG}{16.5 \text{ ft.}}$$

FG = (16.5 ft.) (sin 22.62°) = 6.346 ft.

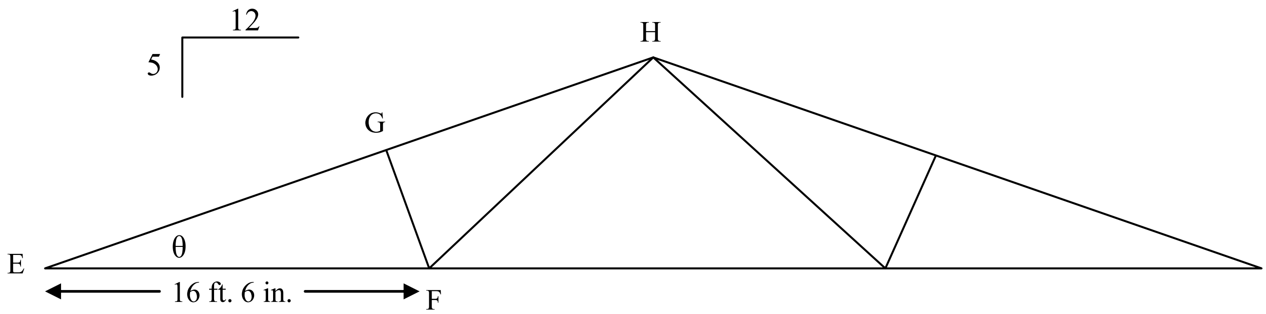
Convert the decimal feet to feet and inches as follows:

$$0.346 \text{ ft.} \times \frac{12 \text{ in.}}{1 \text{ ft.}} = 4.152 \text{ in.}$$ This answer is 4 in., expressed to the nearest 1/2 inch.

Therefore, FG = 6.346 ft. = 6 ft. 4 in.

Chapter 10 Practice Worksheet, continued

5. Calculate the length of rafter EGH as shown on the previous problem. Express the rafter length in feet and inches to the nearest ½ inch.



Given: ∠GHF = 24.5°

EGH = EG + GH

Recall from the previous problem that ∠GEF = 22.62° and FG = 6.346 ft.

First, calculate the length of web EG.

$$\cos 22.62° = \frac{EG}{16.5 \text{ ft.}}$$

EG = (16.5) (cos 22.62°) = 15.23 ft.

Then, calculate the length of web GH.

$$\tan 24.5° = \frac{6.346 \text{ ft.}}{GH}$$

$$GH = \frac{6.346 \text{ ft.}}{\tan 24.5°} = 13.925 \text{ ft.}$$

EGH = EG + GH = 15.23 + 13.925 = 29.155 ft. = 29 ft. 2 in.

Chapter 10 Practice Worksheet, continued

6. Use the staircase sketch shown below to solve the following exercises.

   a. Calculate the tread depth (horizontal dimension), to the nearest 1/8 inch.

$$\tan 38.752^\circ = \frac{\text{riser}}{\text{tread}} = \frac{7.625\text{ in.}}{\text{tread}} \qquad \text{tread} = \frac{7.625\text{ in.}}{\tan 38.752^\circ} = 9.5\text{ in.} = 9\ 1/2\text{ in.}$$

   b. How many risers are required to reach the second floor (nearest whole number)?

   First we can convert the overall staircase height of 8 ft. 10 ¾ in. to 106.75 in.

$$\#\text{ risers} = \frac{\text{overall staircase height}}{\text{riser height}} = \frac{106.75\text{ in.}}{7.625\text{ in. per riser}} = 14\text{ risers}$$

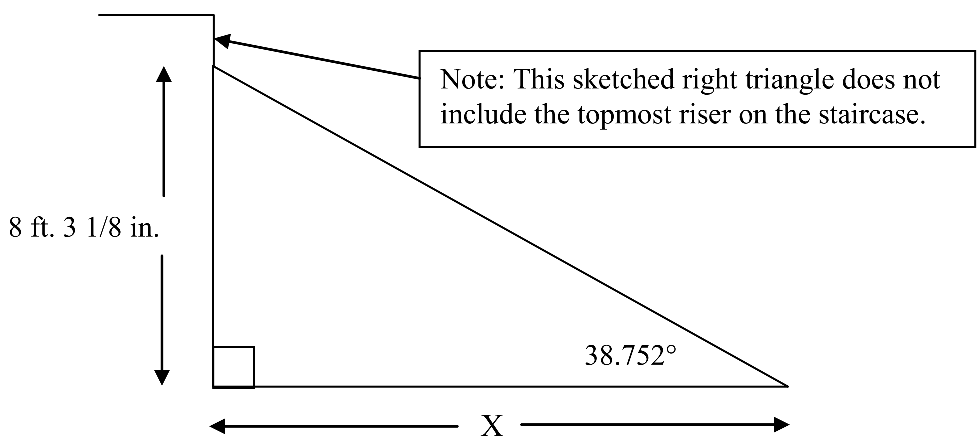
   c. Calculate the dimension X, the dimension that the stairs take up on the floor plan.

   Vertical side of the right triangle = 8 ft. 10 3/4 in. – the height of one riser

   Vertical side of the right triangle = 8 ft. 10 3/4 in. – 7 5/8 in. = 8 ft. 3 1/8 in.

$$\tan 38.752^\circ = \frac{8\text{ ft. } 3\ 1/8\text{ in.}}{X} = \frac{8.2604\text{ ft.}}{X} \qquad X = \frac{8.2604\text{ ft.}}{\tan 38.752^\circ} = 10.29\text{ ft.}$$

   X = 10.29 ft. = 10 ft. 3 1/2 in.



   d. Calculate how many treads are required in this staircase (nearest whole number).

   First, we can convert the overall horizontal dimension of 10 ft. 3 1/2 in. to 123 1/2 in.

$$\#\text{ treads} = \frac{\text{overall horizontal dimension}}{\text{tread depth}} = \frac{123.5\text{ in.}}{9.5\text{ in.}} = 13\text{ treads}$$

Exercise 11-1

Using the basic equation of y = A sin θ, let's graph a wave with the equation y = 2 sin θ. The first step is to calculate the points that you will plot on the graph. Complete the following Table 11-2.

Table 11-2: Sine wave points

x-axis (angle θ)	y = 2 sin θ
0°	0
45°	1.414
90°	2
135°	1.414
180°	0
225°	-1.414
270°	-2
315°	-1.414
360°	0

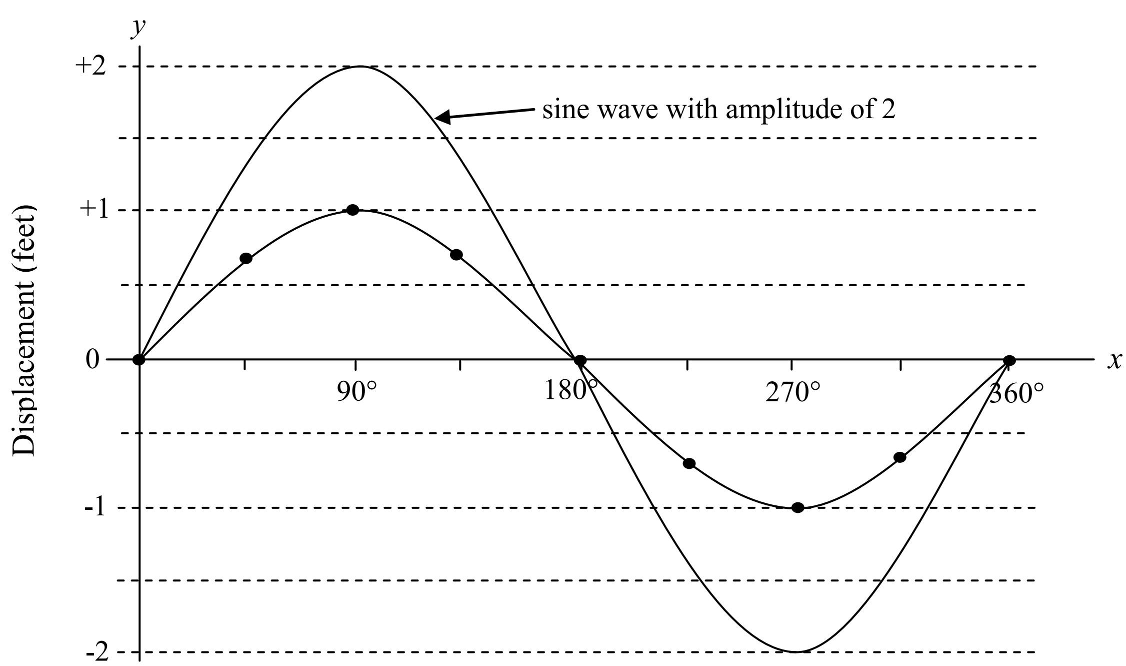


Figure 11-2: Sine wave

Exercise 11-2

Here is one more wave to plot on the Figure 11-2 graph. Use the equation y = 0.5 sin θ. Calculate the points for this graph in Table 11-3 below.

Once you have the points calculated, plot the points on the graph above, and observe how the amplitude of this wave compares with the other two waves.

Table 11-3: Sine wave points

x-axis (angle θ)	y = 0.5 sin θ
0°	0
45°	0.354
90°	0.5
135°	0.354
180°	0
225°	-0.354
270°	-0.5
315°	-0.354
360°	0

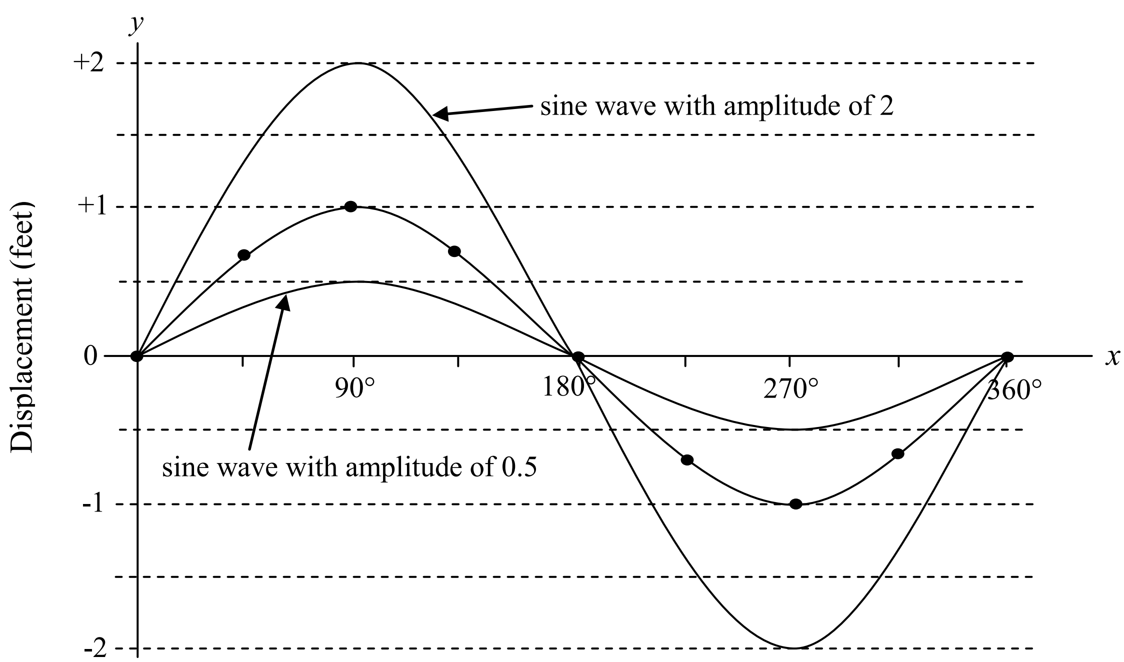


Figure 11-2: Sine wave

Exercise 11-3

Create an equation for a sine wave, similar to the previous example. Use the following information about the wave:

- Amplitude of the wave = 2.5 centimeters
- The wave period, T = 1.5 seconds
- The wave has a phase shift of 45°, shifted to the left. In other words, the peak of the wave occurs 45° earlier than a wave that does not have a phase shift.

$$y = 2.5 \sin\left(360^\circ\,(t/1.5) - 90^\circ\right)$$

phase shift

wave period

amplitude

Exercise 11-4

Create an equation for the wave shown in the Figure 11-7 below. Use the figure to determine the amplitude and phase shift. Let wave period, T = 3.2 seconds.

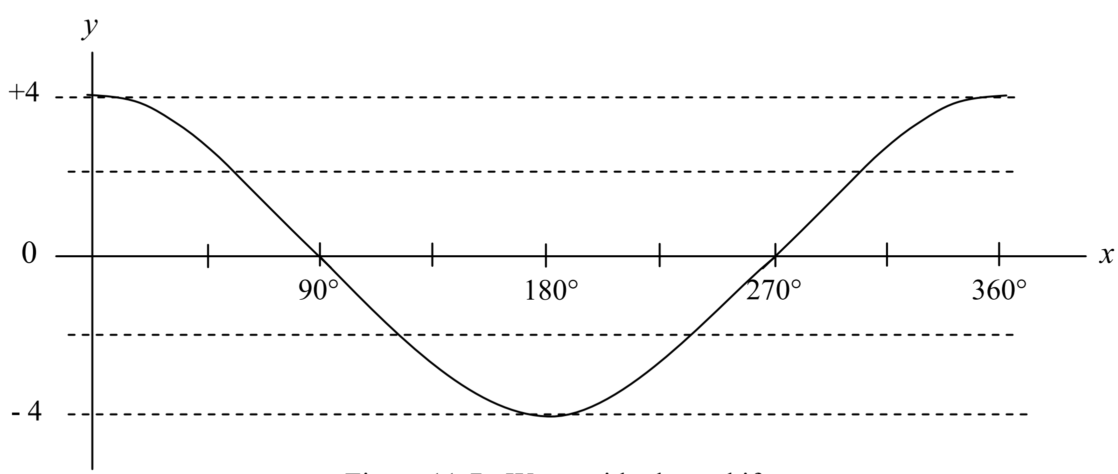


Figure 11-7: Wave with phase shift

From the graph:

- Amplitude = 4
- Phase shift = + 90° since the peak amplitude of 4 occurs 90° earlier than a sine wave without a phase shift

$$y = 4 \sin\left(360^\circ\,(t/3.2) + 90^\circ\right)$$

phase shift

wave period

amplitude

Exercise 11-5

Table 11-5: Wheel displacement calculations

time, t (seconds)	calculation	y
0	$y = 3.5 \sin(0)$	0
1	$y = 3.5 \sin(2\pi (1/8)$	2.47
2	$y = 3.5 \sin(2\pi (1/4)$	3.5
3	$y = 3.5 \sin(2\pi (3/8)$	2.47
4	$y = 3.5 \sin(2\pi (4/8)$	0
5	$y = 3.5 \sin(2\pi (5/8)$	-2.47
6	$y = 3.5 \sin(2\pi (6/8)$	-3.5
7	$y = 3.5 \sin(2\pi (7/8)$	-2.47
8	$y = 3.5 \sin(2\pi (8/8)$	0

Graph the results of the calculations in Table 11-5. This graph shows how a point on the outside of the wheel changes its y position during one rotation.

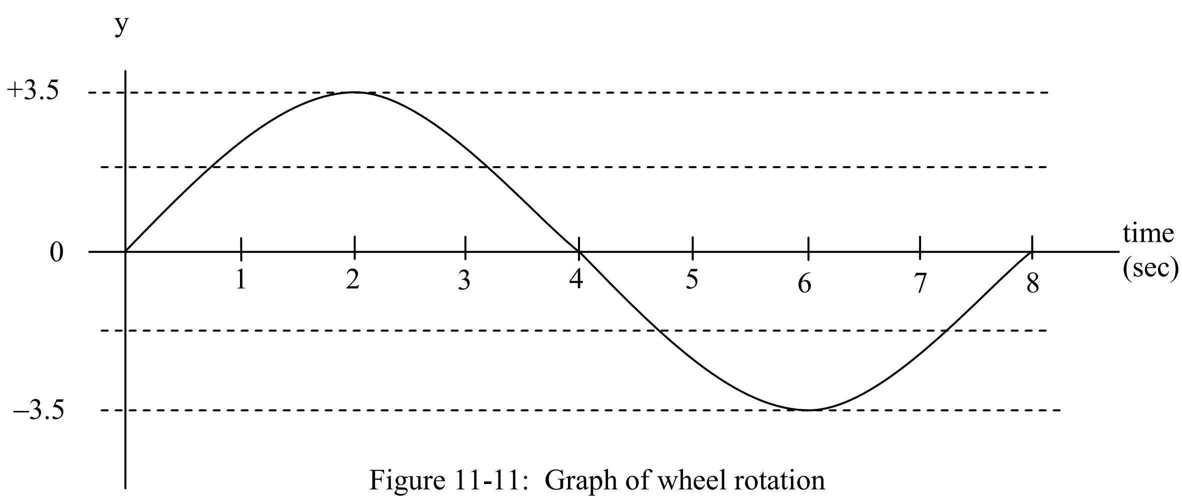


Figure 11-11: Graph of wheel rotation

Exercise 11-6

Here is another example of a rotating wheel. The following information is given:

- The wheel has a radius of 6.75 inches.
- The direction of rotation is counterclockwise.
- The wave period, T = 1.2 seconds. This is the time for one rotation of the wheel.
- A point on the outside of the wheel is selected as the point to graph.
- This point starts at its maximum y value of 6.75 inches.
- This results in a phase shift of one-fourth of a rotation.

Create an equation that describes the y position of the point on the wheel during a complete revolution of the wheel. Use either radians or degrees.

$y = 6.75 \sin (360^\circ (t/1.2) + 90^\circ)$

or

$y = 6.75 \sin (2\pi (t/1.2) + \pi/2)$

Exercise 11-7

Table 11-6: Sound wave pressure

time, t (seconds)	calculation	Y (Pascals)
0	$y = 3 \sin (0)$	0
.001	$y = 3 \sin (2\pi (.001/.01))$	1.76
.002	$y = 3 \sin (2\pi (.002/.01))$	2.85
.003	$y = 3 \sin (2\pi (.003/.01))$	2.85
.004	$y = 3 \sin (2\pi (.004/.01))$	1.76
.005	$y = 3 \sin (2\pi (.005/.01))$	0
.006	$y = 3 \sin (2\pi (.006/.01))$	-1.76
.007	$y = 3 \sin (2\pi (.007/.01))$	-2.85
.008	$y = 3 \sin (2\pi (.008/.01))$	-2.85
.009	$y = 3 \sin (2\pi (.009/.01))$	-1.76
.01	$y = 3 \sin (2\pi (.010/.01))$	0

Exercise 11-8

Graph the results from Table 11-6 above.

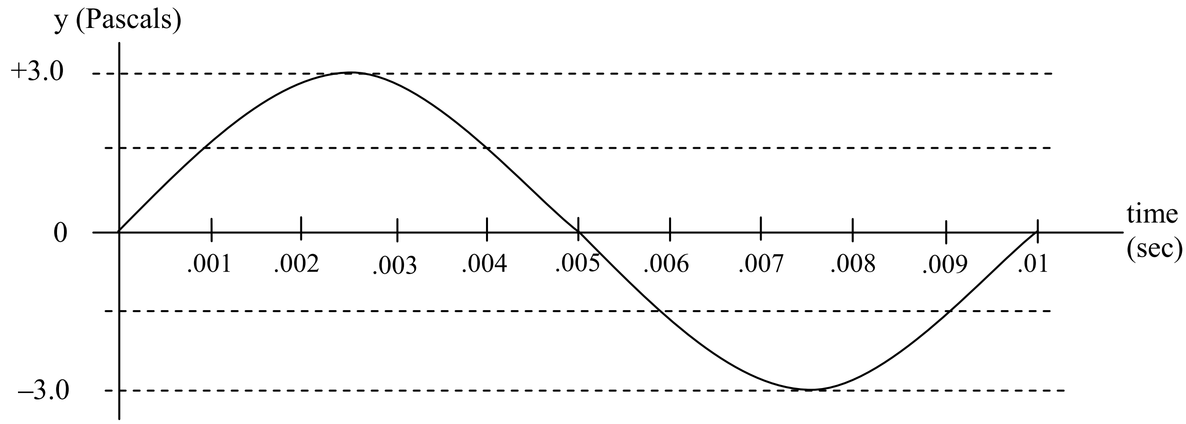


Figure 11-13: Sound wave pressure

Exercise 11-9

An alternating electrical current has a maximum voltage of 170 volts. The frequency of the electricity is 60 Hertz, or 60 cycles.

We do not yet know the wave period, T, the time for one complete wave. The wave period is the inverse of the frequency. Calculate the wave period:

$$\text{Wave period, T} = \frac{1}{\text{frequency}} = \frac{1\text{ second}}{60\text{ cycles}} = \frac{.0167\text{ second}}{1\text{ cycle}}$$

Using radians as the angular units, create an equation for this alternating current:

$y = 170 \sin(2\pi\,(t/.0167))$

Exercise 11-10

Calculate voltage using your equation from the previous page. Note that the time intervals provided are rounded numbers and may not give you the exact results that you expect.

Table 11-7: Voltage calculations

time, t (seconds)	calculation	y (voltage)
0	$y = 170 \sin(2\pi\,(t/.0167))$	0
.0021	$y = 170 \sin(2\pi\,(t/.0167))$	120.8
.0042	$y = 170 \sin(2\pi\,(t/.0167))$	170.0
.0063	$y = 170 \sin(2\pi\,(t/.0167))$	118.5
.0084	$y = 170 \sin(2\pi\,(t/.0167))$	0
.0104	$y = 170 \sin(2\pi\,(t/.0167))$	-118.5
.0125	$y = 170 \sin(2\pi\,(t/.0167))$	-170.0
.0146	$y = 170 \sin(2\pi\,(t/.0167))$	-120.8
.0167	$y = 170 \sin(2\pi\,(t/.0167))$	0

Exercise 11-11

Graph the results from Table 11-7.

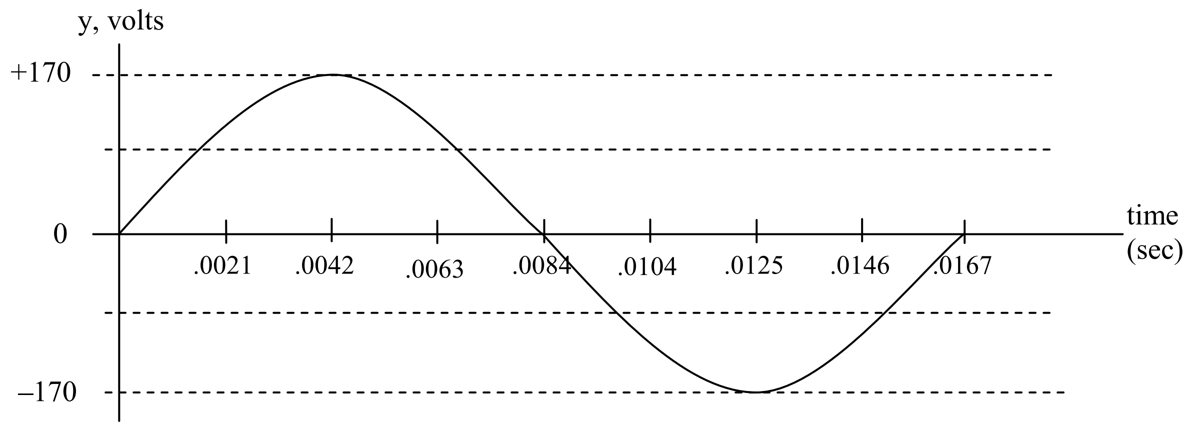


Figure 11-14: Alternating current

Exercise 11-12

a. Create an equation for an alternating current that has a peak voltage of 340 volts and has a frequency of 60 Hertz. Use degrees for the angular units. Use the same wave period, T as in the previous exercise.

$V = 340 \sin((360°)(t/.0167))$

b. Using this first equation, create an equation for the phase which is delayed by 120º from the first phase.

$V = 340 \sin((360°)(t/.0167) - 120°)$

c. Using this first equation, create an equation for the phase which is delayed by 240º from the first phase.

$V = 340 \sin((360°)(t/.0167) - 240°)$

Exercise 11-13

Sketch these three phases on the axes below. Try a free-hand sketch, without first calculating individual points. Start with the first phase and then delay the peak of the other two waves by 120º and 240°.

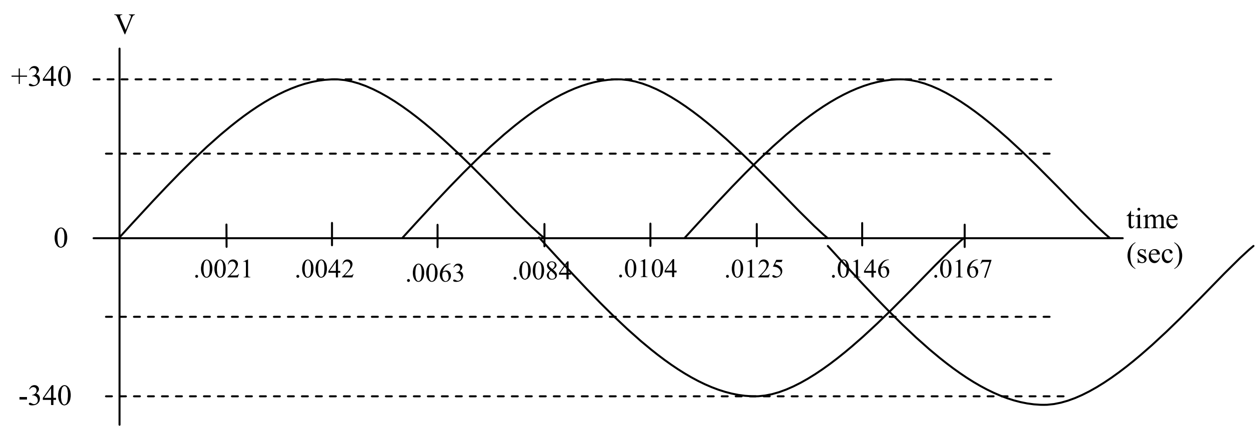


Figure 11-15: Three phase electricity

Exercise 11-14

Substituting different values in our equation y = cos θ, complete the calculations in Table 11-8.

Table 11-8: Cosine wave points

x-axis (angle θ)	y = cos θ
0°	1
45°	.707
90°	0
135°	-.707
180°	-1
225°	-.707
270°	0
315°	.707
360°	1

Exercise 11-15

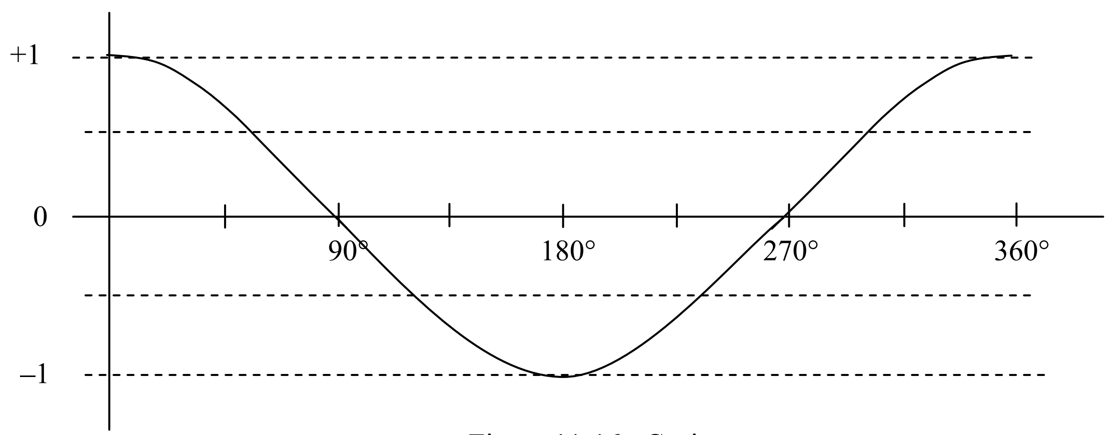


Figure 11-16: Cosine wave

Exercise 11-16

Now we will plot a sine wave and a cosine wave on the same graph, and observe the results.

Here are the equations for these two waves:

- $y = 0.5 \sin(\theta + 45°)$
- $y = 1.25 \cos(\theta - 45°)$

The sine wave has a phase shift of 45° to the left, and the cosine wave has a phase shift of 45° to the right. Calculate the points for each wave in Table 11-9, and then plot the two waves in Figure 11-17 below.

Table 11-9: Sine and cosine wave points

x-axis (angle θ)	$y = 0.5 \sin(\theta + 45°)$	$y = 1.25 \cos(\theta - 45°)$
0°	0.35	0.88
45°	0.5	1.25
90°	0.35	0.88
135°	0	0
180°	-0.35	-0.88
225°	-0.5	-1.25
270°	-0.35	-0.88
315°	0	0
360°	0.35	0.88

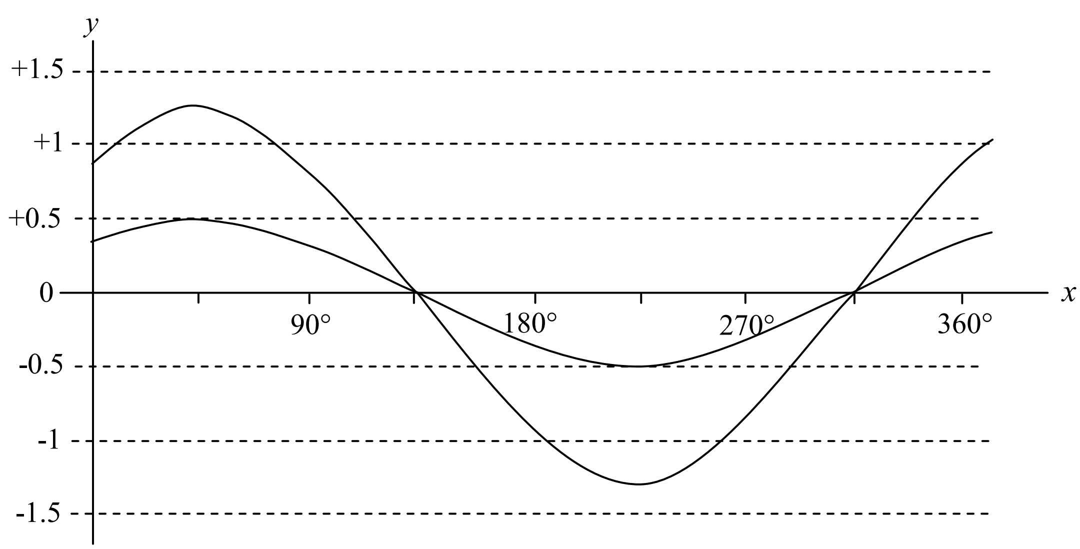


Figure 11-17: Sine wave and cosine wave

Exercise 12-1: Calculate angle D to the nearest whole degree.

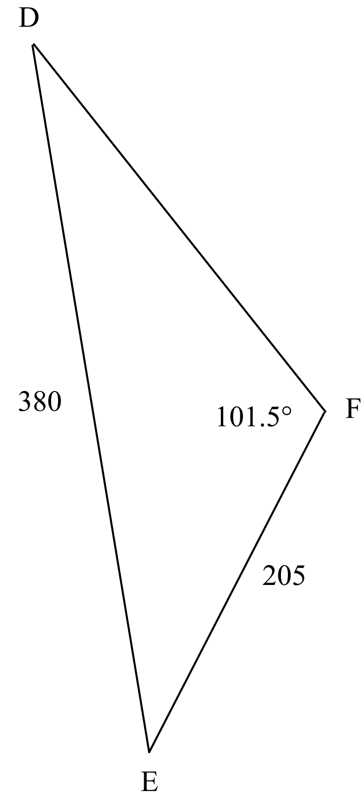


Figure 12-4: Obtuse triangle

$$\frac{205}{\sin D} = \frac{380}{\sin 101.5°}$$

$$\sin D = \frac{(205)(\sin 101.5°)}{380} = 0.5286$$

angle D = $\sin^{-1}(0.5053) = 31.9° \approx 32°$

Exercise 12-2: Calculate angle F, angle G, and side X.

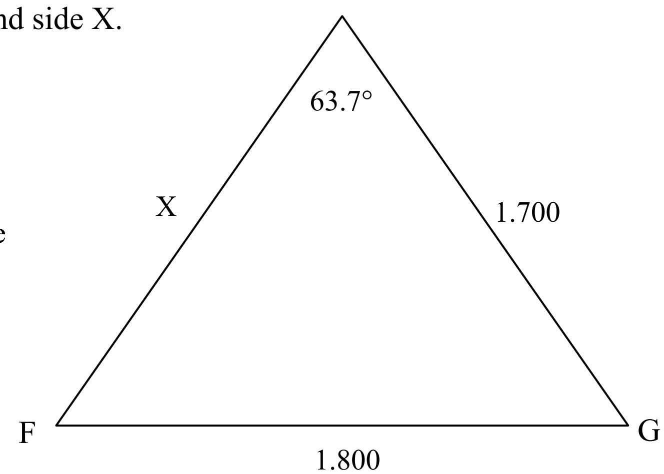


Figure 12-5: Acute triangle

$$\frac{1.700}{\sin F} = \frac{1.800}{\sin 63.7°}$$

$$\sin F = \frac{(1.700)(\sin 63.7°)}{1.800} = 0.84668$$

angle F = $\sin^{-1}(0.84668) = 57.9°$

angle G = 180° – 63.7° – 57.9° = 58.4°

$$\frac{\sin 58.4°}{X} = \frac{\sin 63.7°}{1.800} \qquad X = \frac{(1.800)(\sin 58.4°)}{\sin 63.7°} = 1.710$$

Exercise 12-3: Calculate angle A and side a.

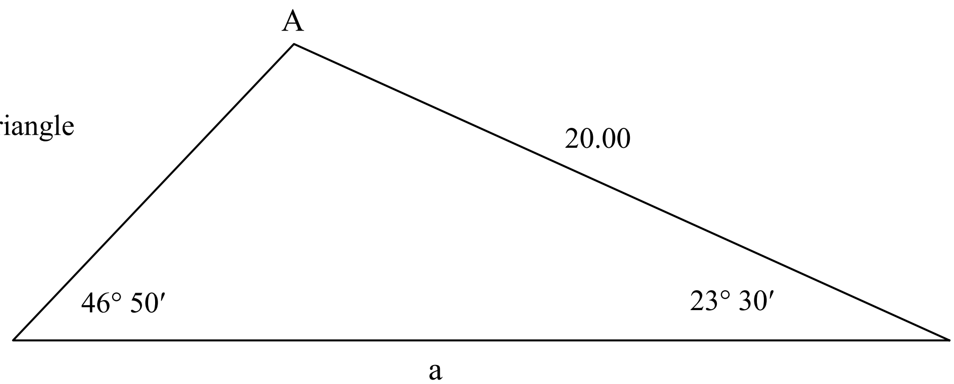


Figure 12-6: Obtuse triangle

angle A = 180° – 46° 50′ – 23° 30′ = 109° 40′

$$\frac{a}{\sin 109^\circ 40'} = \frac{20.00}{\sin 46^\circ 50'} \qquad a = \frac{(20.00)(\sin 109^\circ 40')}{\sin 46^\circ 50'} = 25.82$$

Exercise 12-4

Calculate the length of sides PQ and PR.

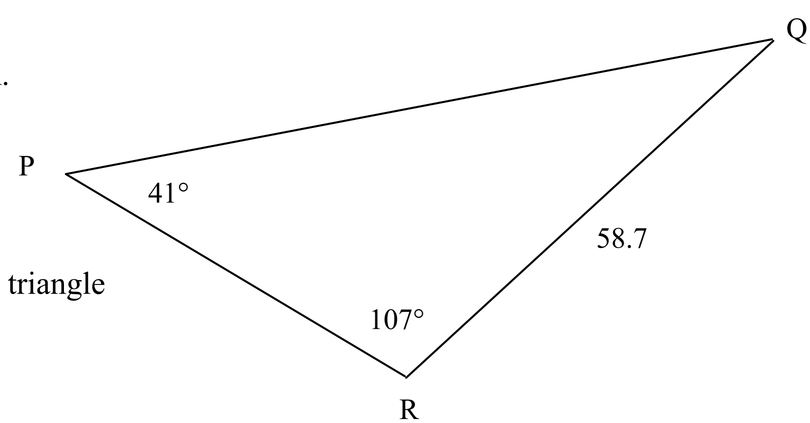


Figure 12-7: Obtuse triangle

$$\frac{PQ}{\sin 107^\circ} = \frac{58.7}{\sin 41^\circ} \qquad PQ = \frac{(58.7)(\sin 107^\circ)}{\sin 41^\circ} = 85.6$$

angle Q = 180° – 107° – 41° = 32°

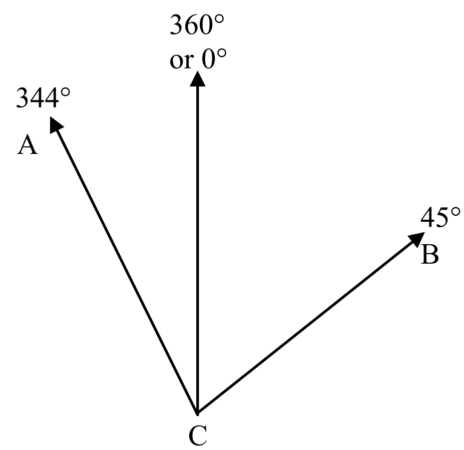
$$\frac{PR}{\sin 32^\circ} = \frac{58.7}{\sin 41^\circ} \qquad PR = \frac{(58.7)(\sin 32^\circ)}{\sin 41^\circ} = 47.4$$

Exercise 12-5

Calculate the internal angles B and C.

Angle B = 297° – 225° = 72°

Angle C is not as straightforward since North, or 360°, is between the direction of side CA (344°) and the direction of side CB (45°). Here is a sketch:



Angle C may be calculated in three steps:

- The angle between North and line CA = 360° – 344° = 16°
- The angle between line CB and North = 45° – 0° = 45°
- The angle between lines CB and CA is the sum of these two angles: Angle C = 16° + 45° = 61°

An alternate solution is to first convert the 45° angle to 360° + 45° = 405°. 405° is equivalent to the direction of 45°. This step then allows you to perform a subtraction without having to account for the North direction as we did above.

Angle C = 405° – 344° = 61°

Note: As a check on your calculations, verify that the three angles add up to 180°.

47° + 72° + 61° = 180°

Exercise 12-6

Now that you know the values for the internal angles, calculate the length of side AC, using the Law of Sines. Estimate the length of side AC first, so that you will be able to recognize whether your calculated value is reasonable.

$$\frac{237.58}{\sin 47^\circ} = \frac{AC}{\sin 72^\circ}$$

$$AC = \frac{(237.58)(\sin 72^\circ)}{\sin 47^\circ} = 308.95 \text{ ft.}$$

Exercise 12-7

Using the given roof pitches, calculate the values for angles D and F. You may express your angles in decimal degrees to two decimal places.

$$\text{Roof angle D} = \tan^{-1}\frac{18}{12} = 56.31°$$

$$\text{Roof angle F} = \tan^{-1}\frac{6}{12} = 26.57°$$

Exercise 12-8

Using the values for angles D and F, calculate the value of angle E.

Angle E = 180° – 56.31° – 26.57° = 97.12°

Exercise 12-9

Using the Law of Sines, calculate the length of web EF. The length of web DE is 21 ft. 6 in. Express your answer to the nearest 1/4 inch.

$$\frac{21.5}{\sin F} = \frac{EF}{\sin D}$$

$$EF = \frac{(21.5)(\sin 56.31°)}{\sin 26.57°} = 40 \text{ ft. } 0 \text{ in.}$$

Exercise 12-10

Using the Law of Sines, calculate the length of web DF. Express your answer to the nearest 1/4 inch.

$$\frac{DF}{\sin E} = \frac{21.5}{\sin F}$$

$$DF = \frac{(21.5)(\sin 97.12°)}{\sin 26.57°} = 47.70 \text{ ft.} = 47 \text{ ft. } 8.36 \text{ in.} = 47 \text{ ft. } 8\tfrac{1}{4} \text{ in.}$$

Exercise 13-1: Calculate the length of side AB, angle A, and angle B.

a. Calculate the length of side AB using the Law of Cosines.

$(AB)^2 = (AC)^2 + (BC)^2 - 2(AC)(BC)(\cos 78° \, 10')$

$AB = \sqrt{(21.1)^2 + (28.6)^2 - 2(21.1)(28.6)(\cos 78° \, 10')} = 31.9$ ft.

b. Calculate the value of angle A, using the Law of Sines.

$$\frac{28.6}{\sin A} = \frac{31.9}{\sin 78° \, 10'}$$

$$\sin A = \frac{(28.6)(\sin 78° \, 10')}{31.9} = 0.877498 \qquad \text{angle } A = \sin^{-1}(0.877498) = 61° \, 20' \, 31''$$

c. Calculate the value of angle B.

angle B = 180° – 78° 10' – 61° 20' 31" = 40° 29' 29"

Exercise 13-2

a. Calculate the length of side RS using the Law of Cosines.

$(RS)^2 = (RT)^2 + (ST)^2 - 2(RT)(ST)(\cos 61° \, 30')$

$RS = \sqrt{(18.75)^2 + (26.88)^2 - 2(18.75)(26.88)(\cos 61° \, 30')}$

RS = 24.35

b. Calculate the value of angle R, using the Law of Sines.

$$\frac{26.88}{\sin R} = \frac{24.35}{\sin 61° \, 30'} \qquad \sin R = \frac{(26.88)(\sin 61° \, 30')}{24.35} = 0.970127474$$

$\theta = \sin^{-1}(0.970127474) = 75.96° = 75° \, 57' \, 37''$

c. Calculate the value of angle S.

angle S = 180° – 61° 30'– 75° 57' 37" = 42° 32' 23"

Exercise 13-3

a. Calculate the length of side JK using the Law of Cosines.

$(JK)^2 = (JL)^2 + (KL)^2 - 2(JL)\,(KL)\,(\cos 14°\ 45')$

$JK = \sqrt{(427)^2 + (403)^2 - 2\,(427)\,(403)\,(\cos 14°\ 45')} = 109$ cm

b. Calculate the value of angle K, using the Law of Sines.

$$\frac{427}{\sin K} = \frac{109}{\sin 14°\ 45'} \qquad \sin K = \frac{(427)\,(\sin 14°\ 45')}{109} = 0.997385614$$

$\theta = \sin^{-1}(0.997385614) = 85.856° = 85°\ 51'\ 22''$

c. Calculate the value of angle J.

angle J = 180° – 14° 45'– 85° 51' 22" = 79° 23' 38"

Exercise 13-4

Using the Law of Cosines, calculate angle B in the triangle shown in Figure 13-8.

$b^2 = a^2 + c^2 - 2ac\,(\cos B)$

We will let a = 25.3, b = 34.3, and c = 38.3

First, solve the basic equation for angle B.

$$\text{Angle B} = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right)$$

Then, enter the known values and solve for angle B.

$$\text{Angle B} = \cos^{-1}\left(\frac{(34.3)^2 - (25.3)^2 - (38.3)^2}{-2(25.3)(38.3)}\right) = \cos^{-1}\ (0.480133954) = 61.3°$$

Practice Worksheet for Chapters 11–13

1. Create an equation for a mechanical wave, given the following information:

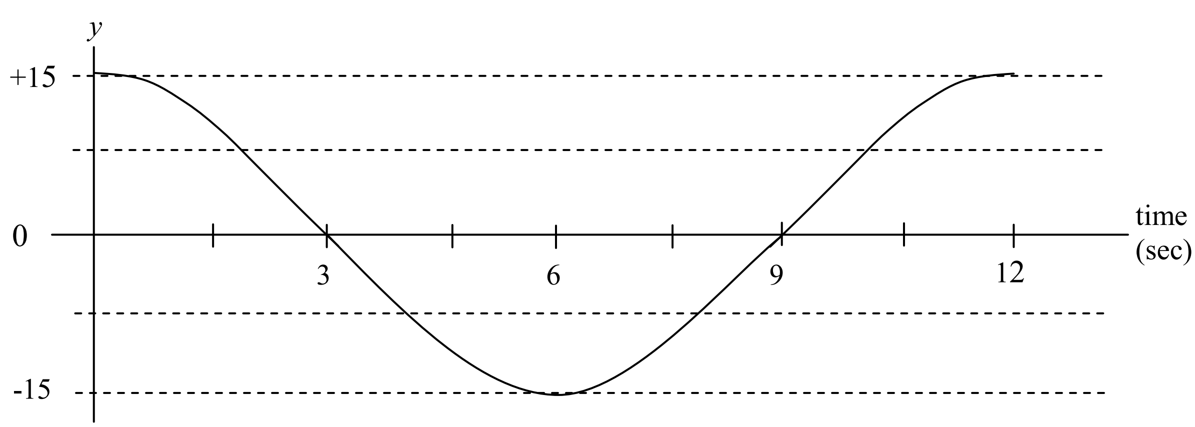
   - The amplitude is 15 cm.
   - The wave period, T is 12 seconds.
   - The positive peak of the wave is *advanced by one-fourth of a wave* (phase shift).
   - Use degrees for the angular units.

   $y = 15 \sin (360° (t/12) + 90°)$

2. Using your equation from problem #1, calculate some y-value points for this wave at the following time intervals:

time, t (seconds)	y
0	15
1.5	10.6
3	0
4.5	-10.6
6	-15
7.5	-10.6
9	0
10.5	10.6
12	15

3. Sketch this wave using your y values from problem #2. Be sure to label the y and x axes.



Practice Worksheet for Chapters 11–13, continued

4. Create an equation for a sound wave, given the following information. Before creating the equation, you will need to calculate the wave period T.

   - The amplitude is 1.7 Pascals (Pa).
   - The wave frequency is 500 Hertz.
   - This wave does not have a phase shift.
   - Use radians for the angular units.

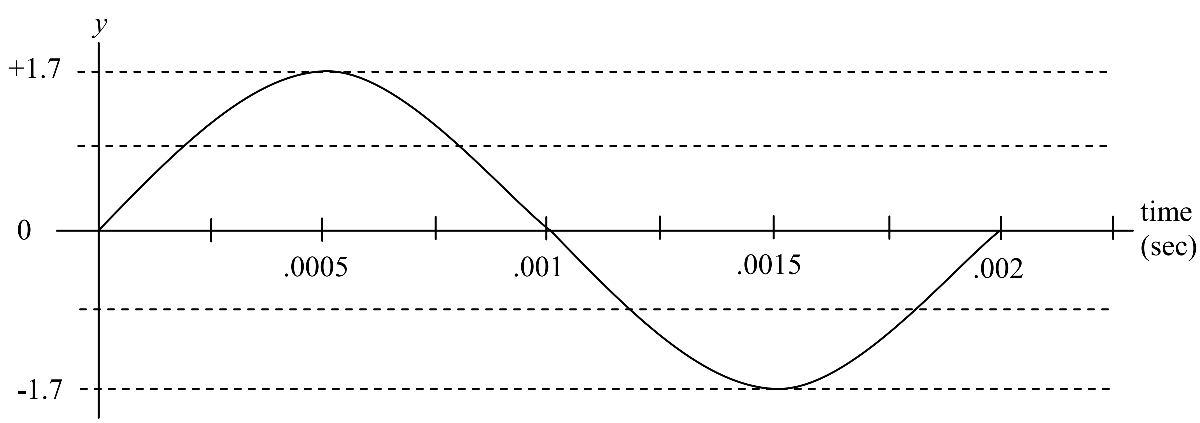
   $$\text{Wave period, T} = \frac{1}{\text{frequency}} = \frac{1 \text{ second}}{500 \text{ cycles}} = \frac{.002 \text{ second}}{1 \text{ cycle}}$$

   Using radians as the angular units, create an equation for this sound wave:
   $y = 1.7 \sin(2\pi\,(t/.002))$

5. Using your equation from problem #4, calculate some y-value points for this wave:

time, t (seconds)	y (Pascals)
0	0
.00025	1.2
.0005	1.7
.00075	1.2
.001	0
.00125	-1.2
.0015	-1.7
.00175	-1.2
.002	0

6. Sketch this wave using your *y* values from problem #5. Be sure to label the *y* and *x* axes.



Practice Worksheet for Chapters 11–13, continued

7. Create an equation for alternating electricity, given the following information. Before creating the equation, you will need to calculate the wave period T using the given frequency.

   - The amplitude is 240 volts and the frequency is 50 Hertz.
   - The positive peak of the wave is *delayed by one-eighth of a full wave* (45°).
   - Use degrees for the angular units.

   $$\text{Wave period, T} = \frac{1}{\text{frequency}} = \frac{1 \text{ second}}{50 \text{ cycles}} = \frac{.02 \text{ second}}{1 \text{ cycle}}$$

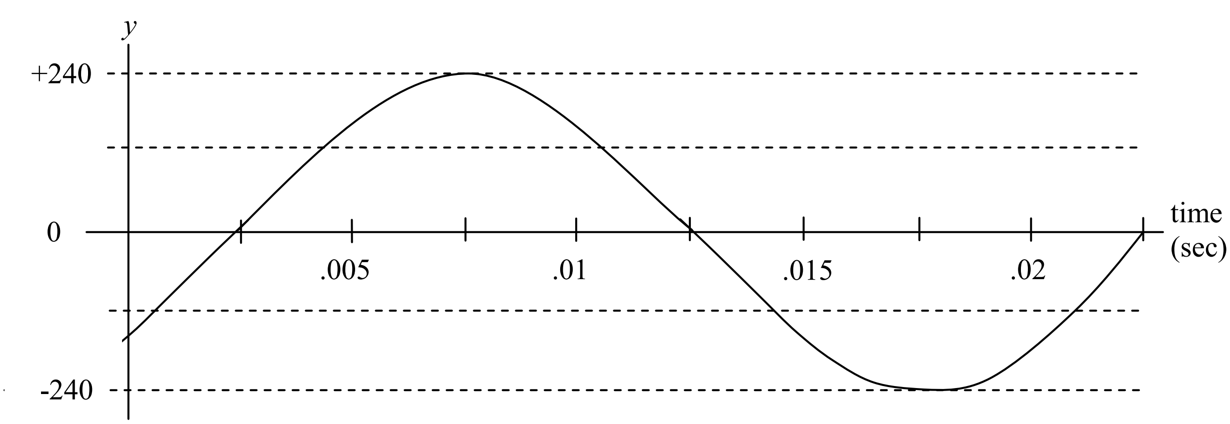
   Using degrees as the angular units, create an equation for this alternating electricity:

   $y = 240 \sin (360° (t/.02) - 45°)$

8. Using your equation from problem #7, calculate some y-value points for this wave:

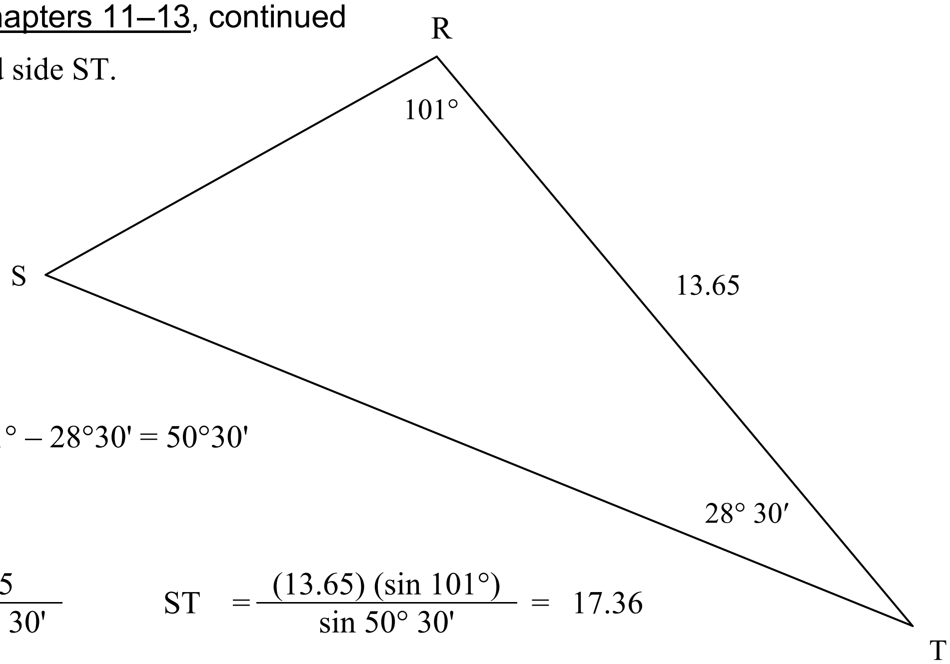
time, t (seconds)	y (volts)
0	-170
.0025	0
.005	170
.0075	240
.01	170
.0125	0
.015	-170
.0175	-240
.02	-170
.0225	0

9. Sketch this wave using your y values from problem #8. Be sure to label the y and x axes.



Practice Worksheet for Chapters 11–13, continued

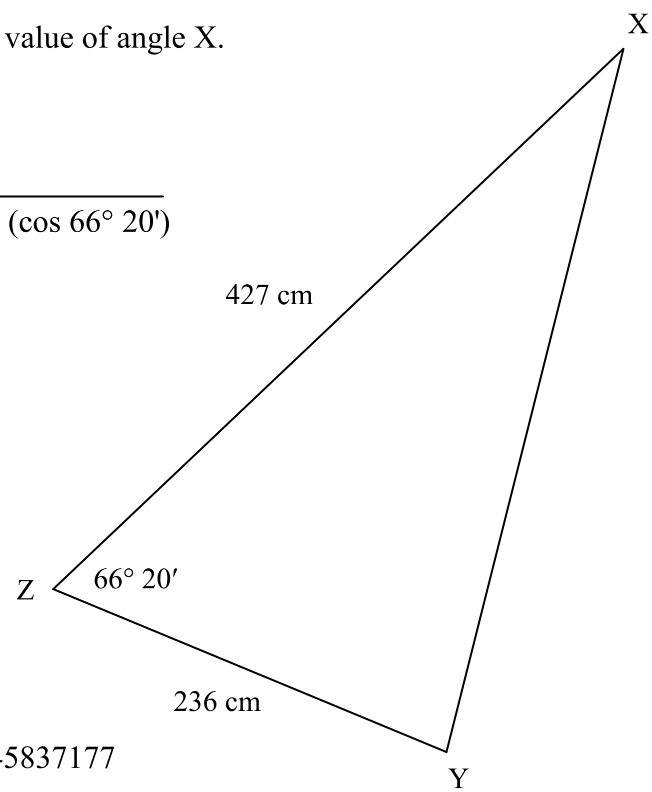
10. Calculate angle S and side ST.



Angle S = 180° – 101° – 28°30' = 50°30'

$$\frac{ST}{\sin 101°} = \frac{13.65}{\sin 50° 30'} \qquad ST = \frac{(13.65)(\sin 101°)}{\sin 50° 30'} = 17.36$$

11. Calculate the length of side XY and the value of angle X.



Use the Law of Cosines

$$XY = \sqrt{(427)^2 + (236)^2 - 2\,(427)\,(236)\,(\cos 66° 20')}$$

XY = 396 cm

Use the Law of Sines

$$\frac{236}{\sin X} = \frac{396}{\sin 66° 20'}$$

$$\sin X = \frac{(236)(\sin 66° 20')}{396} = 0.545837177$$

$$\angle X = \sin^{-1}(0.545837177) = 33° 05'$$

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